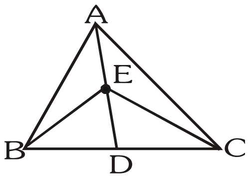
Ex - 9.3

Q1. In fig, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE).



Sol. AD is a median of ΔABC.

Therefore, we have $ar(\Delta ABD) = ar(\Delta ACD)$ ...(1)

ED is a median of ΔEBC

Therefore, we have $ar(\Delta EBD) = ar(\Delta ECD)$ ...(2)

Subtracting (2) from (1),

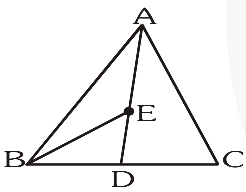
$ar(\Delta ABD) - ar(\Delta EBD) = ar(\Delta ACD) - ar(\Delta ECD)$

$\Rightarrow$ $ar(\Delta ABE) = ar(\Delta ACE)$.

Q2. In a triangle ABC, E is the mid-point of median AD. Show that ar (BED) = $\frac{1}{4}$ ar(ABC).

Sol. AD is a median of ΔABC

$\Rightarrow$ $ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC)$ ...(1)



BE is a median of ΔBAD

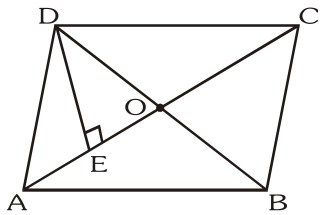
$\Rightarrow$ $ar(\Delta BED) = \frac{1}{2}(\Delta ABD)$ ...(2)

From (1) and (2),

$ar(\Delta BED) = \frac{1}{2}\left\{\frac{1}{2} ar(\Delta ABC)\right\} = \frac{1}{4} ar(\Delta ABC)$.

Q3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Sol. We know, Diagonals of a ||gm bisect each other,



$\Rightarrow$ AO' = OC and OB = OD

ar (ΔAOD)$=\frac{1}{2}\times$ AO $\times$ DE

ar (ΔDOC)$=\frac{1}{2}\times$ OC $\times$ DE

Since AO = OC

$\therefore$ ar (ΔAOD) = ar (ΔDOC) ....(1)

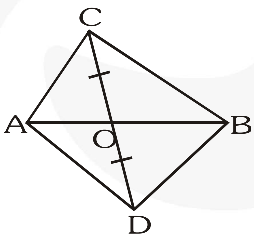
ar (ΔDOC) = ar (ΔBOC) ....(2)

and ar (ΔAOB) = ar (ΔBOC) ....(3)

From (1), (2) and (3) we have

ar (ΔAOB) = ar (ΔBOC) = ar (ΔCOD) = ar (AOD)

Q4. In fig, ABC and ABD are two triangles on the same base AB. If line- segment CD is bisected by AB at O, show that ar(ABC) = ar (ABD).



Sol. CO = DO [Given]

Now, AO is a median of ΔACD

$\Rightarrow$ ar(ΔAOC) = ar(ΔAOD) ...(1)

Similarly, ar(ΔBOC) = ar(ΔBOD) ...(2)

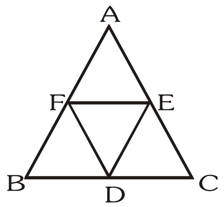
Adding (1) and (2), we have ar(ΔABC) = ar(ΔABD).

Q5. D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that

(i) BDEF is a parallelogram. (ii) ar (DEF) = $\frac{1}{4}$ ar (ABC)

(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC)

Sol. (i)



EF || BC [Mid point theorem]

$\Rightarrow$ EF || BD

Also, EF = $\frac{1}{2}$ BC,

EF = BD

[$\because$ D is the midpoint of BC]

$\therefore$ BDEF is a parallelogram.

(ii) ar (BDEF) = ar (DCEF)

$\Rightarrow \frac{1}{2}$ ar (BDEF) = $\frac{1}{2}$ ar (DCEF)

$\Rightarrow$ ar (ΔBDF) = ar (ΔCDE) ...(1)

Similarly, ar (ΔCDE) = ar (ΔDEF) ...(2)

ar (ΔAEF) = ar (ΔDEF) ....(3)

From (1), (2) and (3) we have

ar (ΔAEF) = ar (ΔFBD) = ar (ΔDEF)

= ar (ΔCDE)

$\therefore$ ar (ΔABC) = ar (ΔAEF) + ar(ΔFBD) + ar(ΔDEF) + ar (ΔCDE)

= 4 ar (ΔDEF)

$\Rightarrow$ ar (ΔDEF) = $\frac{1}{4}$ ar (ΔABC).

(iii) ar (BDEF) = ar (ΔBDF) + ar (ΔDEF)

= ar (ΔDEF) + ar (ΔDEF)

= 2 ar (ΔDEF).

$= 2\left[\frac{1}{4}\text{ar}(\Delta ABC)\right] = \frac{1}{2}$ ar (ΔABC)

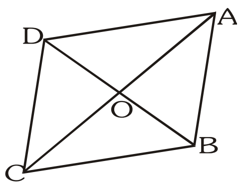
$\therefore$ ar (BDEF) = $\frac{1}{2}$ ar (ΔABC)

Q6. In fig, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that :

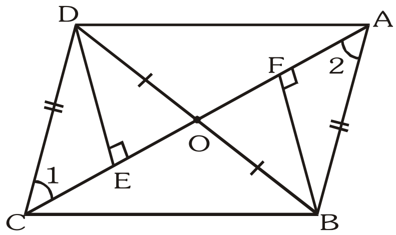
(i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA || CB or ABCD is a parallelogram.



Sol. (i) In ΔDEO and ΔBFO,



We have DO = BO [given]

∠DOE = ∠BOF [Vertically opposite angles]

∠DEO = ∠BFO [each 90°]

∴ ΔDEO ≅ ΔBFO [By ASA congruency]

⇒ DE = BF [By CPCT]

and ar (ΔDEO) = ar (ΔBFO) ....(1)

Now, in ΔDEC and ΔBFA, we have

∠DEC = ∠BFA [each 90°]

DE = BF [proved above]

DC = BA [given]

∴ ΔDEC ≅ ΔBFA [By RHS congruency]

⇒ ar (ΔDEC) = ar (ΔBFA) ....(2)

Adding (1) and (2), we have

ar (ΔDEO) + ar (ΔDEC)

= ar (ΔBFO) + ar (ΔBFA)

⇒ ar (ΔDOC) = ar (ΔAOB).

(ii) ∵ ar (ΔDOC) = ar (ΔAOB) [proved above]

Adding ar (ΔBOC) on both the sides, we have

ar (ΔDCB) = ar (ΔACB)

(iii) Since ΔDCB and ΔACB are on the same base CB and having equal areas.

⇒ CB || DA [ABCD is a ||gm]

Also ∠1 = ∠2 [By c.p.c.t]

So, AB || CD [alternate interior angles]

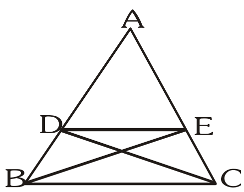
⇒ ABCD is a parallelogram.

Q7. D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE||BC.

Sol. ΔDBC and ΔEBC have equal areas and same base BC.

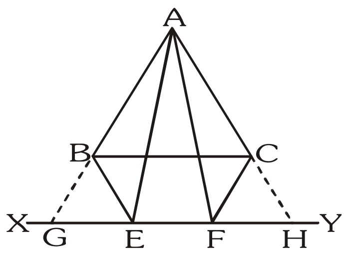
⇒ The two triangles are between the same parallels.

⇒ DE||BC



Q8. XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that ar (ΔABE) = ar (ΔACF).

Sol. In figure, AB (produced) and AC(produced) meet XY and G and H respectively.



Now, BGFC and BEHC are parallelograms.

⇒ BC = GF and BC = EH

⇒ GF = EH ⇒ GE = FH

⇒ ar(ΔBGE) = ar(ΔCFH) ...(1)

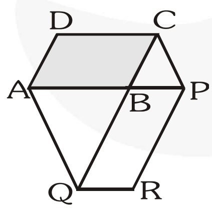
Also, we find that

ar(ΔAGE) = ar (ΔAHF) ...(2)

Subtracting (1) from (2), we have

ar(ΔABE) = ar (ΔACF).

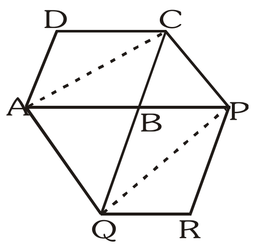
Q9. The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed Show that ar (ABCD) = ar (PBQR).



Sol. We join AC and PQ in figure.

we are given that AQ||CP.

Now, ΔACQ and ΔAPQ have same base AQ. The two triangles are between same parallels.



Therefore, we have

ar(ΔACQ) = ar (ΔAPQ)

$\Rightarrow$ ar(ΔABC) + ar (ΔABQ)

= ar(ΔBPQ) + ar (ΔABQ)

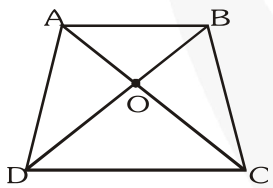
$\Rightarrow$ ar(ΔABC) = ar (ΔBPQ)

$\Rightarrow$ $\frac{1}{2}$ ar (ABCD) = $\frac{1}{2}$ ar(PBQR)

$\Rightarrow$ ar(ABCD) = ar (PBQR).

Q10. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).

Sol. $\because$ ar (ΔABD) = ar (ΔABC)

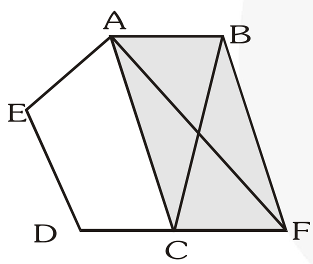


Subtracting ar (ΔAOB) from both sides, we get

$\Rightarrow$ ar (ΔABD) – ar (ΔAOB) = ar (ΔABC) – ar (ΔAOB)

$\Rightarrow$ ar (ΔAOD) = ar (ΔBOC)

Q11. In fig, ABCDE is a pentagon. A line through B parallel to



AC meets DC produced at F. Show that

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

Sol. (i) ΔACB and ΔACF in figure, have same base AC and also, between the same parallels.

Therefore, ar(ΔACB) = ar (ΔACF)

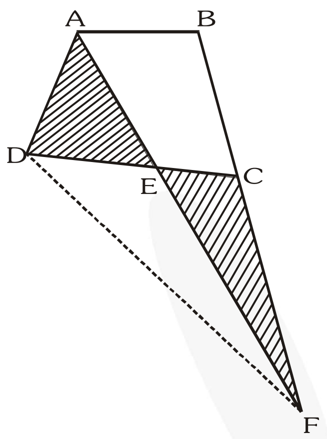
(ii) ar(ACDE) + ar(ΔACB) = ar(ACDE) + ar(ΔACF)

$\Rightarrow$ ar(ABCDE) = ar(AEDF).

Q12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Sol. Let us draw DF || AC and join A and F.

$\therefore$ ar (ΔDAF) = ar (ΔDCF).



Substracting ar (ΔDEF) from both sides, we get

ar (ΔDAF) – ar (ΔDEF)

= ar (ΔDCF) – ar (ΔDEF)

$\Rightarrow$ ar (ΔADE) = ar (ΔCEF)

Let us prove that ar (ΔABF) = ar (ABCD),

We have:

ar (ΔCEF) = ar (ΔADE) [proved above]

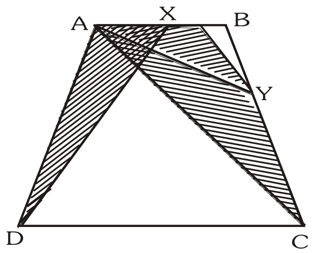
Adding ar (ABCE) to both sides, we get :

ar (ΔCEF) + ar (ABCE) = ar (ΔADE) + ar (ABCE)

$\Rightarrow$ ar (ΔABF) = ar (ABCD)

Q13. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).

Sol. We have a trapezium ABCD such that AB || DC.



AB || DC [Given]

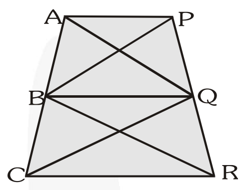
∴ ar (ΔADX) = ar (ΔACX) ....(1)

ar (ΔACX) = ar (ΔACY) ....(2)

From (1) and (2), we have

ar (ΔADX) = ar (ΔACY)

Q14. In fig, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).



Sol. In figure, AP||BQ and BQ||CR

⇒ ar(ΔABQ) = ar(ΔPBQ) ...(1)

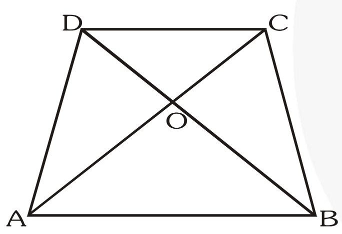
and ar(ΔBQC) = ar(ΔBQR) ...(2)

Adding (1) and (2), we have

ar(ΔAQC) = ar(ΔPBR)

Q15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

Sol.



ar (ΔAOD) = ar (ΔBOC)

Adding ar (ΔAOB) to both the sides, we have

⇒ ar (ΔAOD) + ar (ΔAOB) = ar (ΔBOC) + ar (ΔAOB)

ar(ΔABD) = ar (ΔABC)

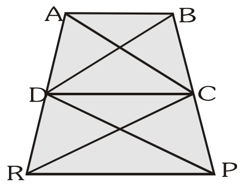
Since, they are on the same base AB,

∴ AB || DC

Now, ABCD is quadrilateral having a pair of opposite sides parallel.

∴ ABCD is trapezium.

Q16. In fig, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



Sol. ar (ΔDRC) = ar (ΔDPC). [Given]

∴ ΔDRC and ΔDPC must lie between the same parallels.

So, DC || RP.

∴ Quadrilateral DCPR is a trapezium.

Again, we have

ar (ΔBDP) = ar (ΔARC) [Given] ...(1)

Also, ar (ΔDPC) = ar (ΔDRC) [Given] ...(2)

Substracting (2) from (1) we get :

ar (ΔBDP) – ar (ΔDPC) = ar (ΔARC) – ar (ΔDRC)

ar (ΔBDC) = ar (ΔADC).

Since, they are on the same base DC

∴ ΔBDC and ΔADC must lie between the same parallels

So; AB || DC.

∴ Quadrilateral ABCD is a trapezium.