



# **NCERT SOLUTIONS**

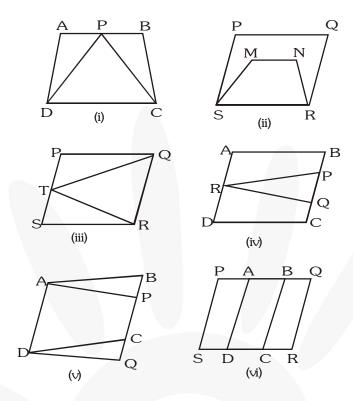
**Area of Parallelogram and Triangle** 

**<sup>\*</sup>Saral** हैं, तो शब शरल हैं।



## Ex - 9.1

Q1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

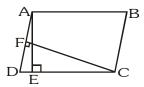


- **Sol.** (i)  $\Delta PDC$  and quadrilateral ABCD have same base DC. The two figures are between the parallels AB and DC.
  - (ii) Figure PQRS and MNRS are not between the same parallels.
  - (iii) ΔQTR & parallelogram PQRS have same base QR & between same parallels QR & PS
  - (v) Parallelogram ABCD and APQD have common base AD and also, between same parallel AD and BQ.
  - (vi) No two figures have same base and between same parallels.



### Ex - 9.2

Q1. In fig, ABCD is a parallelogram, AE  $\perp$  DC and CF  $\perp$  AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



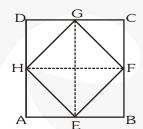
Sol. In figure

$$AD \times CF = CD \times AE$$

$$\Rightarrow$$
 AD  $\times$  10 = 16  $\times$  8 cm

$$\Rightarrow$$
 AD = 12.8 cm

- Q2. If E,F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) =  $\frac{1}{2}$  ar (ABCD).
- Sol. ∴ E and G are the mid-points of AB and CD respectively∴ EG is parallel to BC or AD.



Also ar ( $\parallel$  gm EBCG) = ar ( $\parallel$  gm AEGD)

$$=\frac{1}{2}\operatorname{ar}\left(\|\operatorname{gm}\operatorname{ABCD}\right) \qquad \qquad \dots (1)$$

$$\therefore \text{ ar } (\Delta EFG) = \frac{1}{2} \text{ ar}(\parallel \text{gm EBCG}) \qquad \dots (2)$$

Similarly, ar 
$$(\Delta EHG) = \frac{1}{2} \operatorname{ar}(\|\operatorname{gm AEGD})$$
 ....(3)

Adding (2) and (3), we get:

$$\Rightarrow$$
 ar ( $\triangle$ EFG) + ar ( $\triangle$ EHG) =  $\frac{1}{2}$  [ar(||gm EBCG) + (||gm AEGD)]

$$\Rightarrow$$
 ar(||gm EFGH)

$$\frac{1}{2} \left[ \frac{1}{2} ( \parallel \mathsf{gm} \; \mathsf{ABCD} ) + \frac{1}{2} \mathsf{ar} ( \parallel \mathsf{gm} \; \mathsf{ABCD} ) \right]$$

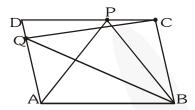


From (1)

$$\Rightarrow$$
 ar (EFGH) =  $\frac{1}{2}$  [ar (||gm ABCD)]

$$\Rightarrow$$
 Thus, ar(||gm EFGH) =  $\frac{1}{2}$  ar (||gm ABCD).

- Q3. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar(BQC).
- **Sol.**  $ar(\Delta APB) = \frac{1}{2}ar(ABCD)$  ...(1)



$$ar (\Delta BQC) = \frac{1}{2} ar (ABCD) ...(2)$$

$$ar (\Delta APB) = ar (\Delta BQC)$$

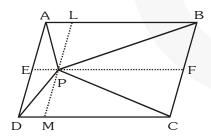
Q4. In figure, P is point in interior of a parallelogram ABCD. Show that:

(i) 
$$ar (\Delta APB) + ar (\Delta PCD)$$

$$= \frac{1}{2} \operatorname{ar} (\|^{gm} \operatorname{ABCD})$$

(ii) 
$$ar(\Delta APD) + ar(\Delta PBC)$$

= 
$$ar (\Delta APB) + ar (\Delta PCD)$$
.



**Sol.** (i) Through P, draw a line EF||AB.

Since  $\Delta ABP$  and parallelogram ABFE are on the same base AB and between the same parallels AB and EF.

$$\therefore \text{ ar } (\Delta ABP) = \frac{1}{2} \text{ ar } (\parallel^{gm} ABFE) \qquad ...(1)$$



Similarly,  $\Delta DCP$  and paralleogram DCFE are on the same base DC and between the same parallel DC and EF.

$$\therefore \text{ ar } (\Delta DCP) = \frac{1}{2} \text{ ar } (\parallel^{gm} DCFE) \dots (2)$$

Adding eqn. (1) and eqn. (2), we get

$$ar (\Delta ABP) + ar (\Delta DCP)$$

= 
$$\frac{1}{2}$$
 ar( $\|^{gm}$  ABFE) +  $\frac{1}{2}$  ar( $\|^{gm}$  DCFE)

$$\Rightarrow$$
 ar  $(\triangle ABP)$  + ar  $(\triangle DCP) = \frac{1}{2}$ 

$$\{ar(\parallel^{gm} ABFE)+ar(\parallel^{gm} DCFE)\}$$

$$\Rightarrow$$
 ar  $(\triangle ABP)$  + ar  $(\triangle DCP) = \frac{1}{2} ar(||^{gm} ABCD)$ 

(ii) Through P, draw a line LM || AD,

Since  $\Delta APD$  and parallelogram ALMD are on the same base AD and between the same parallels AD and LM.

$$\therefore \operatorname{ar}(\Delta APD) = \frac{1}{2} \operatorname{ar} (\|g^{m} ALMD) \qquad ...(3)$$

Similarly,

$$ar(\Delta PBC) = \frac{1}{2} ar (\parallel^{gm} BLMC)$$
 ...(4)

Adding eqns. (3) and (4), we get

$$ar(\Delta APD) + ar(\Delta PBC)$$

$$= \frac{1}{2} \operatorname{ar}(\|^{gm} \operatorname{ALMD}) + \frac{1}{2} \operatorname{ar}(\|^{gm} \operatorname{BLMC})$$

$$= \frac{1}{2} \left\{ ar(\|^{gm} ALMD) + ar(\|^{gm} BLMC) \right\}$$

$$= \frac{1}{2} \operatorname{ar}(\|^{gm} \operatorname{ABCD}).$$

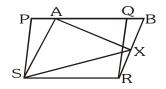
Hence, ar  $(\Delta APD)$  + ar  $(\Delta PBC)$ 

= ar 
$$(\Delta APB)$$
 + ar  $(\Delta PCD)$ .

**Q5.** In fig, PQRS and ABRS are parallelograms and X is any point on side BR. Show that :

(i) 
$$ar(PQRS) = ar(ABRS)$$

(ii) ar (AX S) = 
$$\frac{1}{2}$$
 ar (PQRS)





- **Sol.** (i) Parallelograms PQRS and ABRS are on the same base RS and between the same parallels RS and PB
  - $\therefore$  ar(|| gm PQRS) = ar (|| gm ABRS).
  - (ii)  $\Delta AXS$  and parallelogram ABRS are on the same base AS and between the same parallels AS and BR.

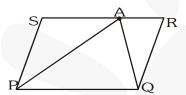
$$\therefore \text{ ar } (\Delta AXS) = \frac{1}{2} \text{ ar } (ABRS) \qquad \dots (1)$$

But ar 
$$(PQRS) = ar (ABRS)$$
 [Proved in (i)] ....(2)

From (1) and (2), we have

$$ar(\Delta AXS) = \frac{1}{2}ar(PQRS)$$

- **Q6.** A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?
- **Sol.** The field is divided into three parts. The parts are:  $\Delta APS$ ,  $\Delta APQ$  and  $\Delta AQR$ .



We have ar 
$$(\Delta APQ) = \frac{1}{2} \operatorname{ar} (PQRS)$$

Therefore, ar  $(\Delta APS)$  + ar  $(\Delta AQR)$ 

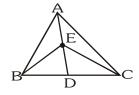
= ar 
$$(\Delta APQ) = \frac{1}{2}$$
 ar  $(PQRS)$ 

She can sow wheat in  $\triangle APQ$  and pulses in  $\triangle APS$  and  $\triangle AQR$  or vice-versa.



### Ex - 9.3

Q1. In fig, E is any point on median AD of a  $\triangle$ ABC. Show that ar (ABE) = ar (ACE).



**Sol.** AD is a median of  $\triangle$ ABC.

Therefore, we have 
$$ar(\Delta ABD) = ar(\Delta ACD)$$
 ...(1)

ED is a median of  $\triangle$ EBC

Therefore, we have 
$$ar(\Delta EBD) = ar(\Delta ECD)$$
 ...(2)

Subtracting (2) from (1),

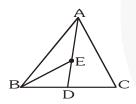
$$ar(\Delta ABD) - ar(\Delta EBD) = ar(\Delta ACD) - ar(\Delta ECD)$$

$$\Rightarrow$$
 ar ( $\triangle$ ABE) = ar( $\triangle$ ACE).

Q2. In a triangle ABC, E is the mid-point of median AD. Show that ar (BED) =  $\frac{1}{4}$  ar(ABC).

**Sol.** AD is a median of  $\triangle ABC$ 

$$\Rightarrow$$
 ar( $\triangle ABD$ ) =  $\frac{1}{2}$  ar( $\triangle ABC$ ) ...(1)



BE is a median of  $\triangle BAD$ 

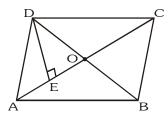
$$\Rightarrow$$
 ar( $\triangle BED$ ) =  $\frac{1}{2}$ ( $\triangle ABD$ ) ...(2)

From (1) and (2),

$$ar(\Delta BED) = \frac{1}{2} \left\{ \frac{1}{2} ar(\Delta ABC) \right\} = \frac{1}{4} ar(\Delta ABC).$$



- Q3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.
- Sol. We know, Diagonals of a ||gm bisect each other,



$$\Rightarrow$$
 AO' = OC and OB = OD

ar (
$$\triangle AOD$$
)= $\frac{1}{2} \times AO \times DE$ 

$$ar(\Delta DOC) = \frac{1}{2} \times OC \times DE$$

Since 
$$AO = OC$$

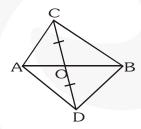
$$\therefore \text{ ar } (\Delta AOD) = \text{ar } (\Delta DOC) \qquad \dots (1)$$

$$ar(\Delta DOC) = ar(\Delta BOC)$$
 ....(2)

and ar 
$$(\Delta AOB) = ar (\Delta BOC)$$
 ....(3)

$$ar(\Delta AOB) = ar(\Delta BOC) = ar(\Delta COD) = ar(AOD)$$

**Q4.** In fig, ABC and ABD are two triangles on the same base AB. If line- segment CD is bisected by AB at O, show that ar(ABC) = ar (ABD).



**Sol.** 
$$CO = DO$$
 [Given]

Now, AO is a median of  $\triangle$ ACD

$$\Rightarrow \operatorname{ar}(\Delta AOC) = \operatorname{ar}(\Delta AOD)$$
 ...(1)

Similarly, 
$$ar(\Delta BOC) = ar(\Delta BOD)$$
 ...(2)

Adding (1) and (2), we have  $ar(\Delta ABC) = ar(\Delta ABD)$ .

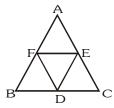
- **Q5.** D, E and F are respectively the mid-points of the sides BC, CA and AB of a  $\triangle$ ABC. Show that
  - (i) BDEF is a parallelogram.

(ii) ar (DEF) = 
$$\frac{1}{4}$$
 ar (ABC)

(iii) ar (BDEF) = 
$$\frac{1}{2}$$
 ar (ABC)



**Sol.** (i)



EF || BC

[Mid point theorem]

$$\Rightarrow$$
 EF || BD

Also, 
$$EF = \frac{1}{2}BC$$
,

$$EF = BD$$

[: D is the midpoint of BC]

:. BDEF is a parallelogram.

(ii) ar(BDEF) = ar(DCEF)

$$\Rightarrow \frac{1}{2} \operatorname{ar} (BDEF) = \frac{1}{2} \operatorname{ar} (DCEF)$$

$$\Rightarrow$$
 ar ( $\triangle$ BDF) = ar ( $\triangle$ CDE)

...(1)

Similarly, ar 
$$(\Delta CDE) = ar (\Delta DEF)$$

...(2)

$$ar(\Delta AEF) = ar(\Delta DEF)$$

....(3)

From (1), (2) and (3) we have

$$ar(\Delta AEF) = ar(\Delta FBD) = ar(\Delta DEF)$$

$$= ar (\Delta CDE)$$

$$\therefore ar(\Delta ABC) = ar(\Delta AEF) + ar(\Delta FBD) + ar(\Delta DEF) + ar(\Delta CDE)$$

 $= 4 \text{ ar } (\Delta \text{DEF})$ 

$$\Rightarrow$$
 ar  $(\Delta DEF) = \frac{1}{4}$  ar  $(\Delta ABC)$ .

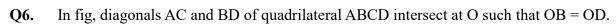
(iii) 
$$ar(BDEF) = ar(\Delta BDF) + ar(\Delta DEF)$$

=  $ar (\Delta DEF) + ar (\Delta DEF)$ 

= 2 ar ( $\Delta$ DEF).

$$= 2 \left[ \frac{1}{4} \operatorname{ar}(\Delta ABC) \right] = \frac{1}{2} \operatorname{ar}(\Delta ABC)$$

$$\therefore$$
 ar (BDEF) =  $\frac{1}{2}$  ar ( $\triangle$ ABC)

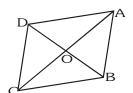


If AB = CD, then show that :

(i) 
$$ar(DOC) = ar(AOB)$$

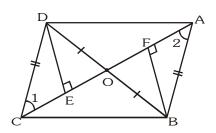
(ii) 
$$ar(DCB) = ar(ACB)$$

(iii) DA||CB or ABCD is a parallelogram.





#### **Sol.** (i) In $\triangle$ DEO and $\triangle$ BFO,



We have DO = BO [given]

 $\angle DOE = \angle BOF$  [Vertically opposite angles]

 $\angle DEO = \angle BFO$  [each 90°]

 $\therefore$   $\triangle$ DEO  $\cong$   $\triangle$ BFO [By ASA congruency]

 $\Rightarrow DE = BF$  [By CPCT]

and ar  $(\Delta DEO) = ar (\Delta BFO)$  ....(1)

Now, in  $\triangle DEC$  and  $\triangle BFA$ , we have  $\angle DEC = \angle BFA$  [each 90°]

DE = BF [proved above]

DC = BA [given]

 $\therefore \quad \Delta DEC \cong \Delta BFA \qquad [By RHS congruency]$ 

 $\Rightarrow$  ar ( $\triangle$ DEC) = ar ( $\triangle$ BFA) ....(2)

Adding (1) and (2), we have

 $ar (\Delta DEO) + ar (\Delta DEC)$ 

=  $ar (\Delta BFO) + ar (\Delta BFA)$ 

 $\Rightarrow$  ar ( $\triangle$ DOC) = ar ( $\triangle$ AOB).

(ii) :  $\operatorname{ar}(\Delta DOC) = \operatorname{ar}(\Delta AOB)$  [proved above]

Adding ar ( $\triangle BOC$ ) on both the sides, we have

 $ar(\Delta DCB) = ar(\Delta ACB)$ 

(iii) Since  $\Delta DCB$  and  $\Delta ACB$  are on the same base CB and having equal areas.

 $\Rightarrow$  CB || DA [ABCD is a ||gm]

Also  $\angle 1 = \angle 2$  [By c.p.c.t]

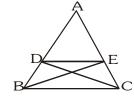
So, AB || CD [alternate interior angles]

 $\Rightarrow$  ABCD is a parallelogram.

Q7. D and E are points on sides AB and AC respectively of  $\triangle$ ABC such that ar (DBC) = ar (EBC). Prove that DE || BC.

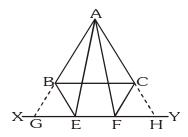
**Sol.**  $\triangle$ DBC and  $\triangle$ EBC have equal areas and same base BC.

- ⇒ The two triangles are between the same parallels.
- ⇒ DE||BC





- **Q8.** XY is a line parallel to side BC of a triangle ABC. If BE||AC and CF||AB meet XY at E and F respectively, show that ar  $(\triangle ABE) = ar(\triangle ACF)$ .
- **Sol.** In figure, AB (produced) and AC(produced) meet XY and G and H respectively.



Now, BGFC and BEHC are parallelograms.

$$\Rightarrow$$
 BC = GF and BC = EH

$$\Rightarrow$$
 GF = EH  $\Rightarrow$  GE = FH

$$\Rightarrow$$
 ar( $\triangle$ BGE) = ar( $\triangle$ CFH) ...(1)

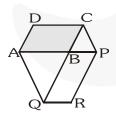
Also, we find that

$$ar(\Delta AGE) = ar(\Delta AHF)$$
 ...(2)

Subtracting (1) from (2), we have

$$ar(\Delta ABE) = ar(\Delta ACF).$$

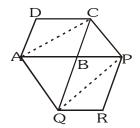
**Q9.** The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed Show that ar (ABCD) = ar (PBQR).



**Sol.** We join AC and PQ in figure.

we are given that AQ||CP.

Now,  $\triangle$ ACQ and  $\triangle$ APQ have same base AQ. The two triangles are between same parallels.





Therefore, we have

$$ar(\Delta ACQ) = ar(\Delta APQ)$$

$$\Rightarrow$$
 ar( $\triangle ABC$ ) + ar ( $\triangle ABQ$ )

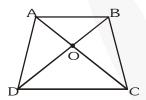
= 
$$ar(\Delta BPQ) + ar(\Delta ABQ)$$

$$\Rightarrow$$
 ar( $\triangle$ ABC) = ar ( $\triangle$ BPQ)

$$\Rightarrow \frac{1}{2} \operatorname{ar} (ABCD) = \frac{1}{2} \operatorname{ar} (PBQR)$$

$$\Rightarrow$$
 ar(ABCD) = ar (PBQR).

- Q10. Diagonals AC and BD of a trapezium ABCD with AB $\parallel$ DC intersect each other at O. Prove that ar (AOD) = ar (BOC).
- **Sol.** :  $\operatorname{ar}(\Delta ABD) = \operatorname{ar}(\Delta ABC)$

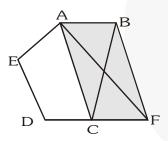


Subtracting ar ( $\triangle$ AOB) from both sides, we get

$$\Rightarrow$$
 ar ( $\triangle$ ABD) – ar ( $\triangle$ AOB) = ar ( $\triangle$ ABC) – ar ( $\triangle$ AOB)

$$\Rightarrow$$
 ar ( $\triangle$ AOD) = ar ( $\triangle$ BOC)

Q11. In fig, ABCDE is a pentagon. A line through B parallel to



AC meets DC produced at F. Show that

(i) 
$$ar(ACB) = ar(ACF)$$

(ii) 
$$ar(AEDF) = ar(ABCDE)$$

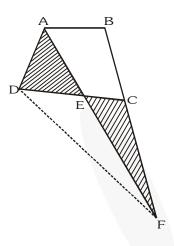
**Sol.** (i)  $\triangle$ ACB and  $\triangle$ ACF in figure, have same base AC and also, between the same parallels. Therefore,  $ar(\triangle ACB) = ar(\triangle ACF)$ 

(ii) 
$$ar(ACDE) + ar(\Delta ACB) = ar(ACDE) + ar(\Delta ACF)$$
  
 $\Rightarrow ar(ABCDE) = ar(AEDF).$ 



- Q12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.
- **Sol.** Let us draw DF  $\parallel$  AC and join A and F.

$$\therefore$$
 ar  $(\Delta DAF) = ar (\Delta DCF)$ .



Substracting ar ( $\Delta$ DEF) from both sides, we get

$$ar (\Delta DAF) - ar (\Delta DEF)$$

$$= ar (\Delta DCF) - ar (\Delta DEF)$$

$$\Rightarrow$$
 ar ( $\triangle$ ADE) = ar ( $\triangle$ CEF)

Let us prove that ar  $(\Delta ABF) = ar (ABCD)$ ,

We have:

$$ar(\Delta CEF) = ar(\Delta ADE)$$

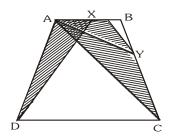
[proved above]

Adding ar (ABCE) to both sides, we get:

$$ar(\Delta CEF) + ar(ABCE) = ar(\Delta ADE) + ar(ABCE)$$

$$\Rightarrow$$
 ar ( $\triangle$ ABF) = ar (ABCD)

- Q13. ABCD is a trapezium with AB $\parallel$ DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).
- **Sol.** We have a trapezium ABCD such that AB || DC.





AB || DC

[Given]

....(1)

$$\therefore \quad \text{ar } (\Delta ADX) = \text{ar } (\Delta ACX)$$

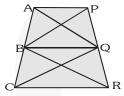
$$ar(\Delta ACX) = ar(\Delta ACY)$$

....(2)

From (1) and (2), we have

$$ar(\Delta ADX) = ar(\Delta ACY)$$

**Q14.** In fig,  $AP \parallel BQ \parallel CR$ . Prove that ar (AQC) = ar (PBR).



Sol. In figure, AP||BQ and BQ||CR

$$\Rightarrow$$
 ar( $\triangle ABQ$ ) = ar( $\triangle PBQ$ )

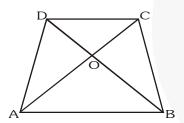
and 
$$ar(\Delta BQC) = ar(\Delta BQR)$$

Adding (1) and (2), we have

$$ar(\Delta AQC) = ar(\Delta PBR)$$

Q15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

Sol.



 $ar(\Delta AOD) = ar(\Delta BOC)$ 

Adding ar ( $\triangle$ AOB) to both the sides, we have

$$\Rightarrow$$
 ar ( $\triangle$ AOD) + ar ( $\triangle$ AOB) = ar ( $\triangle$ BOC) + ar ( $\triangle$ AOB)

$$ar(\Delta ABD) = ar(\Delta ABC)$$

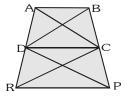
Since, they are on the same base AB,

Now, ABCD is quadrilateral having a pair of opposite sides parallel.

:. ABCD is trapezium.



**Q16.** In fig, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



- **Sol.** ar  $(\Delta DRC)$  = ar  $(\Delta DPC)$ . [Given]
  - $\therefore$   $\triangle$ DRC and  $\triangle$ DPC must lie between the same parallels.

So, DC || RP.

:. Quadrilateral DCPR is a trapezium.

Again, we have

$$ar (\Delta BDP) = ar (\Delta ARC)$$
 [Given] ...(1)

Also, ar 
$$(\Delta DPC)$$
 = ar  $(\Delta DRC)$  [Given] ...(2)

Substracting (2) from (1) we get:

$$ar (\Delta BDP) - ar (\Delta DPC) = ar (\Delta ARC) - ar (\Delta DRC)$$

ar (
$$\triangle BDC$$
) = ar ( $\triangle ADC$ ).

Since, they are on the same base DC

 $\therefore$  ABDC and AADC must lie between the same parallels

So; AB  $\parallel$  DC.

:. Quadrilateral ABCD is a trapezium.