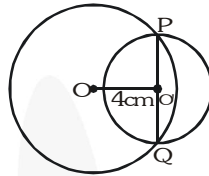


Ex - 10.4

Q1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of common chord.

Sol. We know that if two circles intersect each other at two points, then the line joining their centres is the perpendicular bisector of their common chord.



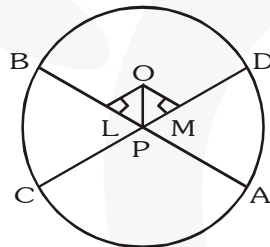
\therefore Length of the common chord
 $\Rightarrow PQ = 2O'P = 2 \times 3 = 6 \text{ cm}$

Q2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Sol. O is the centre of the circle. Chords AB and CD of the circle are equal. P is the point of intersection of AB and CD. Join OP, draw $OL \perp AB$ and $OM \perp CD$.

Here, we find $OL = OM$ ($\because AB = CD$) ... (1)

In $\triangle OLP$ and $\triangle OMP$,

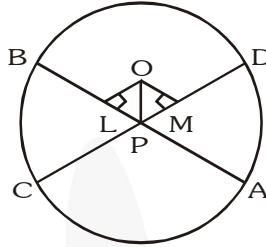


$OL = OM$	(By 1)
$OP = OP$	(Common hypotenuse)
$\angle OLP = \angle OMP$	(Each = 90°)
Then we have $\triangle OLP \cong \triangle OMP$	(RHS congruence)
By CPCT, or $PL = PM$...(2)
Now, $AL = BL = \frac{1}{2} AB$; $CM = DM = \frac{1}{2} CD$	
$\Rightarrow AL = CM$ ($\because AB = CD$)	...(3)
and $BL = DM$...(4)

Subtracting (1) from (3),
 $AL - PL = CM - PM \Rightarrow AP = CP$
 Adding (2) from (4),
 $PL + BL = PM + DM \Rightarrow PB = PD$

Q3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Sol. O is the centre of the circle. Chords AB and CD of the circle are equal. P is the point of intersection of AB and CD. Join OP, draw $OL \perp AB$ and $OM \perp OD$.



Here, we find $OL = OM$ $(\because AB = CD) \dots(1)$

In $\triangle OLP$ and $\triangle OMP$,

$OL = OM$ (By 1)

$OP = OP$ (Common hypotenuse)

$\angle OLP = \angle OMP$ (Each = 90°)

Then we have $\triangle OLP \cong \triangle OMP$ (RHS congruence)

By CPCT, or $PL = PM$ $\dots(2)$

Now, $AL = BL = \frac{1}{2} AB$;

$CM = DM = \frac{1}{2} CD$

$\Rightarrow AL = CM$ ($\because AB = CD$) $\dots(3)$

and $BL = DM$ $\dots(4)$

Subtracting (1) from (3),

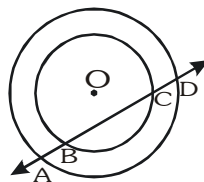
$AL - PL = CM - PM$

$\Rightarrow AP = CP$

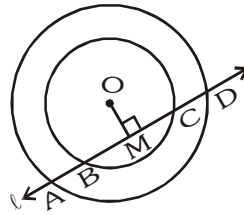
Adding (2) from (4),

$PL + BL = PM + DM \Rightarrow PB = PD$

Q4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$ (see fig).



Sol. **Given :** Two circles with the common centre O. A line “ ℓ ” intersects the outer circle at A and D and the inner circle at B and C.



To prove : $AB = CD$

Construction : Draw $OM \perp \ell$.

Proof : $OM \perp \ell$ [Construction]

For the outer circle,

$$\therefore AM = MD \quad [\text{Perpendicular from the centre bisects the chord}] \quad \dots (1)$$

For the inner circle,

$OM \perp \ell$ [Construction]

$$\therefore BM = MC \quad [\text{Perpendicular from the centre to the chord bisects the chord}] \quad \dots(2)$$

Subtracting (2) from (1), we have

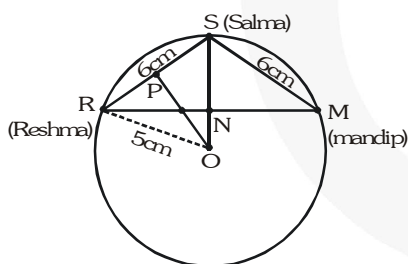
$$\Rightarrow AM - BM = MD - MC$$

$$\Rightarrow AB = CD$$

Q5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Sol. We draw $SN \perp RS$.

Now, SN bisects RM and also SN (produced) passes through the centre O.



Put $RN = x$

$$\text{The ar}(\triangle ORS) = \frac{1}{2} \times OS \times RN$$

$$= \frac{1}{2} \times 5 \times x \quad (\because OS = OR = 5 \text{ m})$$

$$\text{i.e., ar}(\triangle ORS) = \frac{5}{2}x \quad \dots(1)$$

Now, draw $OP \perp RS$, P is mid-point of RS.

$$\Rightarrow PR = PS = 3 \text{ m} \Rightarrow OP^2 = (5)^2 - (3)^2 = 16 \Rightarrow OP = 4 \text{ m}$$

$$\text{Here, ar}(\Delta ORS) = \frac{1}{2} \times RS \times OP = \frac{1}{2} \times 6 \times 4$$

$$\text{i.e., ar}(\Delta ORS) = 12 \text{ m}^2$$

From (1) and (2),

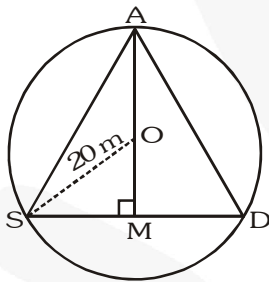
$$\frac{5}{2}x = 12 \Rightarrow x = 4.8 \text{ m} \Rightarrow RM = 2x = 2 \times 4.8 \text{ m} \Rightarrow RM = 9.6 \text{ m}$$

Thus, distance between Reshma and Mandip is 9.6 m.

Q6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol. Let Ankur, Syed and David are sitting at A, S and D respectively such that $AS = SD = AD$ i.e., ΔASD is an equilateral triangle.

Let the length of each side of the equilateral triangle is $2x$ metres.



Draw $AM \perp SD$.

Since, ΔASD is an equilateral triangle,

\therefore AM passes through O.

$$\Rightarrow SM = \frac{1}{2} SD = \frac{1}{2} (2x) = x$$

Now, in ΔASM , we have $AM^2 + SM^2 = AS^2$

$$\Rightarrow AM^2 = AS^2 - SM^2 = (2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$$

$$\Rightarrow AM = \sqrt{3}x.$$

Now, $OM = AM - OA = (\sqrt{3}x - 20) \text{ m}$

$$\Rightarrow (OS = OA = 20 \text{ cm})$$

$$\Rightarrow (20)^2 = x^2 + (\sqrt{3}x - 20)^2$$

$$\Rightarrow 400 = x^2 + 3x^2 - 40\sqrt{3}x + 400$$

$$\Rightarrow 4x^2 = 40\sqrt{3}x \Rightarrow 4x = 40\sqrt{3} \Rightarrow x = 10\sqrt{3} \text{ m}$$

Now, $SD = 2x = 2 \times 10\sqrt{3} \text{ m} = 20\sqrt{3} \text{ m}$

Thus, the length of the string of each phone = $20\sqrt{3} \text{ m}$