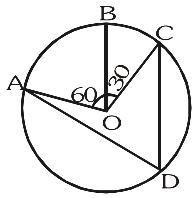
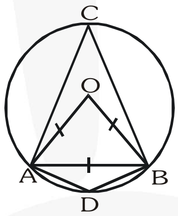
Ex - 10.5

Q1. In Fig. A, B and C are three points on a circle with centre O such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ∠ABC, find ∠ADC.



Sol. $\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2}(60° + 30°) = 45°$

Q2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.



Sol. ∵ OA = OB = AB [Given]

∴ ΔOAB is equilateral

∴ $\angle AOB = 60°$ $\angle ACB = \frac{1}{2}\angle AOB$

[The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$= \frac{1}{2} \times 60 = 30°$

∵ ADBC is a cyclic quadrilateral.

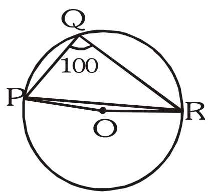
∴ ∠ADB + ∠ACB = 180°

[The sum of either pair of opposite angles of a cyclic quadrilateral is 180°]

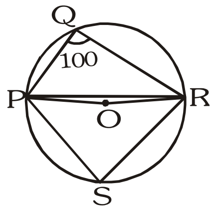
⇒ ∠ADB + 30° = 180° ⇒ ∠ADB = 180° – 30°

⇒ ∠ADB = 50°

Q3. In figure, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.



Sol. Take a point S in the major arc. Join PS and RS.



∵ PQRS is a cyclic quadrilateral.

∴ $\angle PQR + \angle PSR = 180°$

[The sum of either pair of opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow$ $100° + \angle PSR = 180° \Rightarrow \angle PSR = 180° - 100°$

$\Rightarrow$ $\angle PSR = 80°$ ...(i)

Now $\angle PSR = 2\angle PSR$

[The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$= 2 \times 80° = 160°$ ... (2) [Using (i)]

In ΔOPR,

∵ OP = OR [radii of a circle]

∴ $\angle OPR = \angle ORP$ ... (3) [Angles opposite to equal sides of a triangle is 180°]

In ΔOPR,

$\angle OPR + \angle ORP + \angle POR = 180°$ [Sum of all the angles of a triangle is 180°]

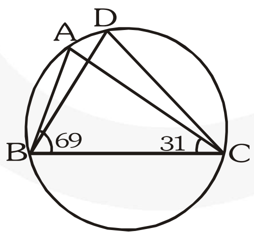
$\Rightarrow$ $\angle OPR + \angle OPR + 160° = 180°$ [Using (2) and (1)]

$\Rightarrow$ $2\angle OPR + 160° = 180°$

$\Rightarrow$ $2\angle OPR = 180° - 160° = 20°$

$\Rightarrow$ $\angle OPR = 10°$

Q4. In fig. $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.



Sol. $\angle ABC + \angle ACB + \angle BAC = 180°$ (By angle sum property)

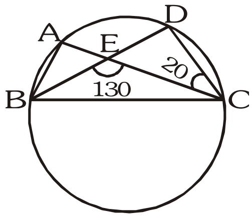
$\Rightarrow$ $69° + 31° + \angle BAC = 180°$

$\Rightarrow$ $\angle BAC = 180° - 100° = 80°$

Since, angles in the same segment are equal

$\angle BDC = \angle BAC$, $\angle BDC = 80°$.

Q5. In figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.



Sol. ∠CED + ∠BEC = 180° [Linear Pair]

⇒ ∠CED + 130° = 180°

⇒ ∠CED + 180° – 130° = 50° ...(i)

∠ECD = 20° ...(ii)

In ΔCED, ∠CED + ∠ECD + ∠CDE = 180°

[Sum of all the angles of a triangle is 180°]

⇒ 50° + 20° + ∠CDE = 180° [Using (i) and (ii)]

⇒ 70° + ∠CDE = 180°

⇒ ∠CDE = 180° – 70°

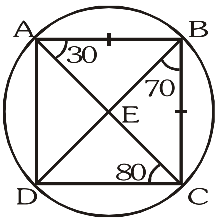
⇒ ∠CDE = 110° ...(iii)

Now ∠BAC = ∠CDE = 110°

[Angle in the same segment of a circle are equal]

Q6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70°, ∠BAC = is 30°, find ∠BCD. Further, if AB = BC, find ∠ECD.

Sol.



Since angles in the same segment of a circle are equal

∴ ∠BAC = BDC

⇒ BDC = 30°

Also ∠DBC = 70° (Given)

∴ In ∠BCD, we have

⇒ ∠BCD + ∠DBC + ∠CDB = 180° [sum of angles of a triangle is 180°]

⇒ ∠BCD + 70° + 30° = 180°

⇒ ∠BCD = 80°

Now, in ΔABC, $AB = BC$ (given)

$\therefore$ $\angle BCA = \angle BAC$

(angles opp. to equal sides of a triangle are equal)

$\Rightarrow$ $\angle BCA = 30°$ $[\angle BAC = 30°]$

Now, $\angle BCA + \angle ECD = \angle BCD$

$\Rightarrow$ $30° + \angle ECD = 80°$

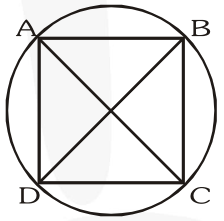
$\Rightarrow$ $\angle ECD = 50°$

Q7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Sol. Since, AC and BD are diameters.

$\Rightarrow$ $AC = BD$ [all diameters of a circle are equal]

Also, $\angle BAD = 90°$



[angle formed in a semicircle is 90°]

Similarly, $\angle ABC = 90°$, $\angle BCD = 90°$

and $\angle CDA = 90°$.

Now, in right ΔABC and ΔBAD, we have

$AC = BD$ (from (1))

$AB = BA$ (common)

$\angle ABC = \angle BAD$ (each equal to 90°)

$\therefore$ $\Delta ABC \cong \Delta BAD$ (By RHS congruence)

$\Rightarrow$ $BC = AD$ (CPCT)

Similarly, $AB = DC$

Thus, ABCD is a rectangle.

Q8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Sol. **Given :** ABCD is a trapezium whose two non-parallel sides AD and BC are equal.

To Prove : Trapezium ABCD is a cyclic.

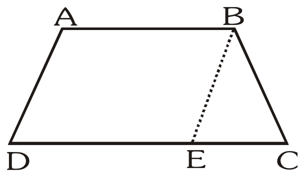
Construction : Draw BE||AD

Proof :

$\because$ AB||DE [Given]

AD||BE [By construction]

$\therefore$ Quadrilateral ABCD is a parallelogram.



$\therefore$ $\angle BAD = \angle BED$ ...(i) [Opp. $\angle$s of a || gm]

and AD = BE ...(ii) [Opp. sides of a || gm]

But AD = BC ...(iii) [Given]

From (ii) and (iii)

BE = BC

$\therefore$ $\angle BEC = \angle BCE$ ...(iv) [Angle opposite to equal sides]

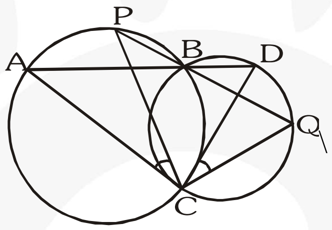
$\angle BEC + \angle BED = 180°$ [Linear pair]

$\Rightarrow$ $\angle BCE + \angle BAD = 180°$ [From (iv) and (i)]

$\Rightarrow$ Trapezium ABCD is cyclic.

[$\because$ If a pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic]

Q9. Two circles intersect at two points B and C. Through B, two line segment ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that $\angle ACP = \angle QCD$.



Sol. Given : Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.

To Prove : $\angle ACP = \angle QCD$

Proof : $\angle ACP = \angle ABP$ ...(i) [Angles in the same segment of a circle are equal]

$\angle QCD = \angle QBD$ ...(ii) [Angles in the same segment of a circle are equal]

$\angle ABP = \angle QBD$ ...(iii) [Vertically Opposite Angles]

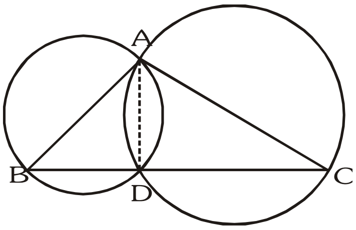
From (i), (ii) and (iii),

$\angle ACP = \angle QCD$.

Q10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Sol. We have ΔABC, and two circles described with diameter as AB and AC respectively. They intersect at a point D, other than A.

Let us join A and D.



AB is a diameter

∴ ∠ADB is an angle formed in a semicircle.

⇒ ∠ADB = 90° ......(1)

Similarly, ∠ADC = 90° ......(2)

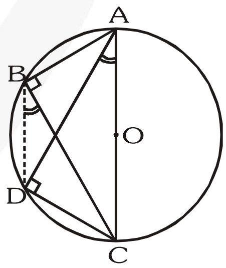
adding (1) and (2) ∠ADB + ∠ADC = 180°

i.e., B, D and C are collinear points BC is a straight line. Thus, D lies on BC.

Q11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD.

Sol. AC is a hypotenuse

∠ADC = 90° = ∠ABC



∴ Both the triangles are in the same semicircle.

⇒ A, B, C and D are concyclic.

Join BD

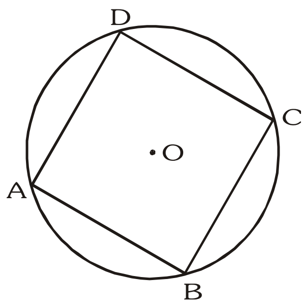
DC is chord

∴ ∠CAD and ∠CBD are formed on the same segment

∴ ∠CAD = ∠CBD

Q12. Prove that a cyclic parallelogram is a rectangle.

Sol. We have a cyclic parallelogram ABCD.



∴ Sum of its opposite angles is 180°

∴ ∠A + ∠C = 180° ......(1)

But ∠A = ∠C ......(2)

From (1) and (2), we have

∠A = ∠C = 90°

Similarly, ∠B = ∠D = 90°

⇒ Each angle of the parallelogram ABCD is 90°

Thus, ABCD is a rectangle.