NCERT SOLUTIONS

Circles

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Ex - 10.1

Q1. Fill in the blanks

(i) The centre of a circle lies in .... of the circle. (exterior, interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in.............of the circle. (exterior/interior)

(iii) The longest chord of a circle is a................of the circle.

(iv) An arc is a...........when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and..........of the circle.

(vi) A circle divides the plane, on which it lies, in .... parts.

Sol.

(i) Interior	(ii) Exterior	(iii) Diameter
(iv) Semicircle	(v) The chord	(vi) Three

Q2. Write true or false : Give reasons for your answers.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

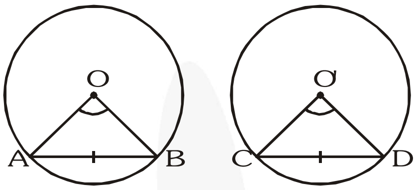
Sol.

(i) True	(ii) False	(iii) False
(iv) True	(v) False	(vi) True

Ex - 10.2

Q1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Sol. **Given :** Two congruent circles C(O, r) and C(O', r) which have chords AB and CD respectively such that AB = CD.



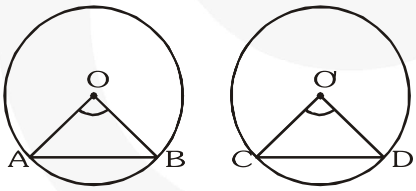
To prove : $\angle AOB = \angle CO'D$

Proof : From ΔAOB and $\Delta CO'D$, we have

	AB = CD	[Given]
	OA = O'C	[Each equal to r]
	OB = O'D	[Each equal to r]
$\therefore$	AOB $\cong$ $\Delta CO'D$	[By SSS-congruence]
$\Rightarrow$	$\angle AOB = \angle CO'D$ [C.P.C.T.]	

Q2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Sol. **Given :** Two congruent circle C(O, r) and C(O', r) which have chords AB and CD respectively, such that $\angle AOB = \angle CO'D$



To prove : AB = CD

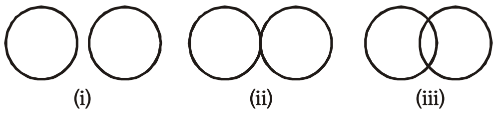
Proof : In Δ AOB and Δ CO'D, we have :

	OA = O'C	[each equal to r]
	OB = O'D	[each equal to r]
	$\angle AOB = \angle CO'D$	[given]
$\therefore$	$\Delta AOB \cong \Delta CO'D$	[by SAS - criterion]
Hence, AB = CD		[C.P.C.T.]

Ex - 10.3

Q1. Draw different pairs of circles. How many points does each pair have in common ? What is the maximum number of common points ?

Sol. Let us draw different pairs of circles as shown below :



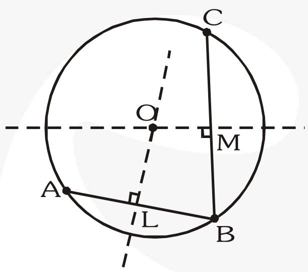
We have

In figure	Maximum number of common points
(i)	nil (zero)
(ii)	one
(iii)	two

Thus, two circles have at the most two points in common.

Q2. Suppose you are given a circle. Give the construction to find its centre.

Sol. We have three points A, B and C on the circle. Join AB and BC. Draw the right bisectors intersect at O. Then, O is the centre of the circle. Note that we can prove OA = OB and OB = OC, i.e., OA = OB = OC is equal to the radius of the circle.



Q3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Sol. Prove that

$\Delta C_1AC_1 = \Delta C_1BC_2$

$\Rightarrow \angle AC_1C_2 = \angle BC_1C_2$ ...(1)

and $\angle AC_2C_1 = \angle BC_2C_1$ . ...(2)

Now, in $\Delta C_1AM = \Delta C_1BM$ ...(2)

$C_1A = C_1B$ (Each = radius)

$C_1M = C_1M$ (Common)

$\angle AC_1M = \angle BC_1M$ (From 1)

$\Rightarrow \Delta C_1AM = \Delta C_1BM$ (SAS congruence)

Then by CPCT, we have

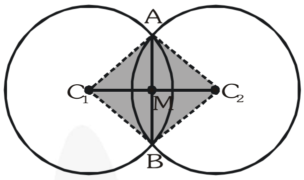
$AM = BM$

and $\angle C_1MA = \angle C_1MB$

Also, $\angle C_1MA + \angle C_1MB = 180°$

$\Rightarrow$ $AM = BM$

and $\angle C_1MA = \angle C_1MB = 90°$



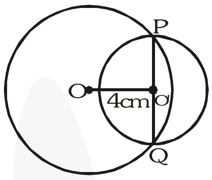
$\Rightarrow$ C_1M is right bisector of chord AB and similarly, C_2M is right bisector of chord AB.

$\Rightarrow$ C_1M is right bisector of chord AB and similarly, C_2M is right bisector of chord AB.

Ex - 10.4

Q1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of common chord.

Sol. We know that if two circles intersect each other at two points, then the line joining their centres is the perpendicular bisector of their common chord.



$\therefore$ Length of the common chord

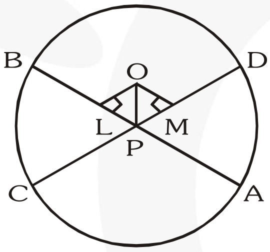
$\Rightarrow$ PQ = 2O'P = 2 × 3 = 6 cm

Q2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Sol. O is the centre of the circle. Chords AB and CD of the circle are equal. P is the point of intersection of AB and CD. Join OP, draw OL ⊥ AB and OM ⊥ OD.

Here, we find OL = OM (∵ AB = CD) ...(1)

In ΔOLP and ΔOMP,



OL = OM (By 1)

OP = OP (Common hypotenuse)

∠OLP = ∠OMP (Each = 90°)

Then we have ΔOLP ≅ ΔOMP (RHS congruence)

By CPCT, or PL = PM ...(2)

Now, AL = BL = ½ AB; CM = DM = ½ CD

$\Rightarrow$ AL = CM (∵ AB = CD) ...(3)

and BL = DM ...(4)

Subtracting (1) from (3),

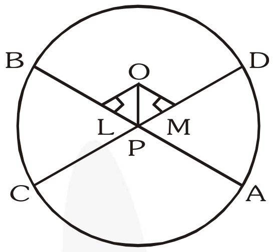
AL – PL = CM – PM $\Rightarrow$ AP = CP

Adding (2) from (4),

PL + BL = PM + DM $\Rightarrow$ PB = PD

Q3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Sol. O is the centre of the circle. Chords AB and CD of the circle are equal. P is the point of intersection of AB and CD. Join OP, draw OL $\perp$ AB and OM $\perp$ OD.



Here, we find OL = OM (∵ AB = CD) ...(1)

In ΔOLP and ΔOMP,

OL = OM (By 1)

OP = OP (Common hypotenuse)

$\angle$OLP = $\angle$OMP (Each = 90°)

Then we have $\Delta OLP \cong \Delta OMP$ (RHS congruence)

By CPCT, or PL = PM ...(2)

Now, AL = BL = ½ AB;

CM = DM = ½ CD

$\Rightarrow$ AL = CM (∵ AB = CD) ...(3)

and BL = DM ...(4)

Subtracting (1) from (3),

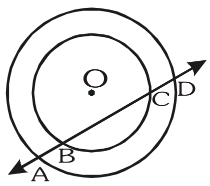
AL – PL = CM – PM

$\Rightarrow$ AP = CP

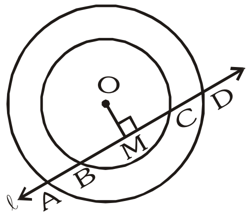
Adding (2) from (4),

PL + BL = PM + DM $\Rightarrow$ PB = PD

Q4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see fig).



Sol. **Given :** Two circles with the common centre O. A line "ℓ" intersects the outer circle at A and D and the inner circle at B and C.



To prove : AB = CD

Construction : Draw OM $\perp$ ℓ.

Proof : OM $\perp$ ℓ [Construction]

For the outer circle,

$\therefore$ AM = MD [Perpendicular from the centre bisects the chord] .... (1)

For the inner circle,

OM $\perp$ ℓ [Construction]

$\therefore$ BM = MC [Perpendicular from the centre to the chord bisects the chord] ...(2)

Subtracting (2) from (1), we have

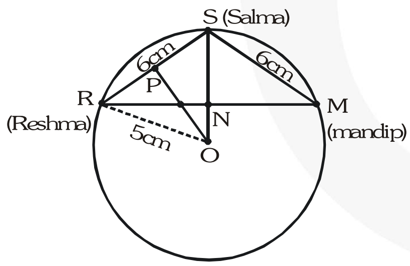
$\Rightarrow$ AM – BM = MD – MC

$\Rightarrow$ AB = CD

Q5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Sol. We draw SN $\perp$ RS.

Now, SN bisects RM and also SN (produced) passes through the centre O.



Put RN = x

$$\text{The ar}(\Delta ORS) = \frac{1}{2} \times OS \times RN$$

$$= \frac{1}{2} \times 5 \times x \quad (\because OS = OR = 5 \text{ m})$$

$$\text{i.e., ar}(\Delta ORS) = \frac{5}{2}x \quad ...(1)$$

Now, draw OP $\perp$ RS, P is mid-point of RS.

$\Rightarrow$ PR = PS = 3 m $\Rightarrow OP^2 = (5)^2 - (3)^2 = 16 \Rightarrow OP = 4$ m

Here, ar(ΔORS) = $\frac{1}{2} \times RS \times OP = \frac{1}{2} \times 6 \times 4$

i.e., ar(ΔORS) = 12 m^2

From (1) and (2),

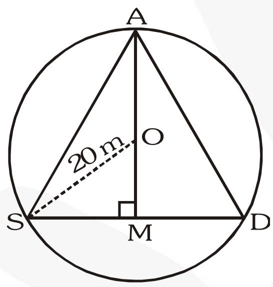
$\frac{5}{2}x = 12 \Rightarrow x = 4.8 \text{ m} \Rightarrow RM = 2x = 2 \times 4.8 \text{ m} \Rightarrow RM = 9.6 \text{ m}$

Thus, distance between Reshma and Mandip is 9.6 m.

Q6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol. Let Ankur, Syed and David are sitting at A, S and D respectively such that AS = SD = AD i.e., ΔASD is an equilateral triangle.

Let the length of each side of the equilateral triangle is 2x metres.



Draw AM $\perp$ SD.

Since, ΔASD is an equilateral triangle,

$\therefore$ AM passes through O.

$\Rightarrow SM = \frac{1}{2}SD = \frac{1}{2}(2x) = x$

Now, in ΔASM, we have $AM^2 + SM^2 = AS^2$

$\Rightarrow AM^2 = AS^2 - SM^2 = (2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$

$\Rightarrow AM = \sqrt{3}x$.

Now, $OM = AM - OA = (\sqrt{3}x - 20)$ m

$\Rightarrow$ (OS = OA = 20 cm)

$\Rightarrow (20)^2 = x^2 + (\sqrt{3}x - 20)^2$

$\Rightarrow 400 = x^2 + 3x^2 - 40\sqrt{3}x + 400$

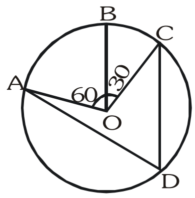
$\Rightarrow 4x^2 = 40\sqrt{3}x \Rightarrow 4x = 40\sqrt{3} \Rightarrow x = 10\sqrt{3}$ m

Now, $SD = 2x = 2 \times 10\sqrt{3} \text{ m} = 20\sqrt{3}$ m

Thus, the length of the string of each phone = $20\sqrt{3}$ m

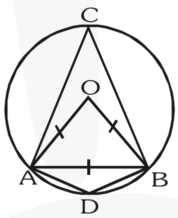
Ex - 10.5

Q1. In Fig. A, B and C are three points on a circle with centre O such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ∠ABC, find ∠ADC.



Sol. $\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2}(60° + 30°) = 45°$

Q2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.



Sol. ∵ OA = OB = AB [Given]

∴ ΔOAB is equilateral

∴ $\angle AOB = 60° \; \angle ACB = \frac{1}{2}\angle AOB$

[The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$= \frac{1}{2} \times 60 = 30°$

∵ ADBC is a cyclic quadrilateral.

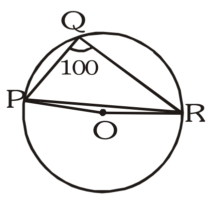
∴ ∠ADB + ∠ACB = 180°

[The sum of either pair of opposite angles of a cyclic quadrilateral is 180°]

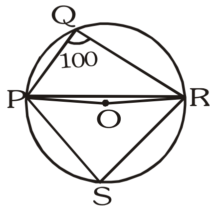
⇒ ∠ADB + 30° = 180° ⇒ ∠ADB = 180° – 30°

⇒ ∠ADB = 50°

Q3. In figure, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.



Sol. Take a point S in the major arc. Join PS and RS.



$\because$ PQRS is a cyclic quadrilateral.

$\therefore$ $\angle PQR + \angle PSR = 180°$

[The sum of either pair of opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow$ $100° + \angle PSR = 180° \Rightarrow \angle PSR = 180° - 100°$

$\Rightarrow$ $\angle PSR = 80°$ ...(i)

Now $\angle PSR = 2\angle PSR$

[The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$= 2 \times 80° = 160°$ ... (2) [Using (i)]

In ΔOPR,

$\because$ OP = OR [radii of a circle]

$\therefore$ $\angle OPR = \angle ORP$ ... (3) [Angles opposite to equal sides of a triangle is 180°]

In ΔOPR,

$\angle OPR + \angle ORP + \angle POR = 180°$ [Sum of all the angles of a triangle is 180°]

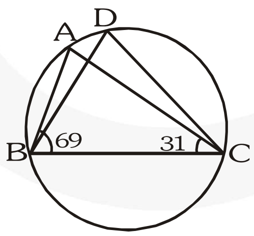
$\Rightarrow$ $\angle OPR + \angle OPR + 160° = 180°$ [Using (2) and (1)]

$\Rightarrow$ $2\angle OPR + 160° = 180°$

$\Rightarrow$ $2\angle OPR = 180° - 160° = 20°$

$\Rightarrow$ $\angle OPR = 10°$

Q4. In fig. $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.



Sol. $\angle ABC + \angle ACB + \angle BAC = 180°$ (By angle sum property)

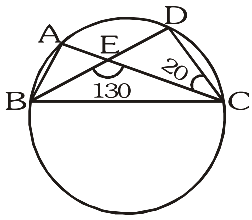
$\Rightarrow$ $69° + 31° + \angle BAC = 180°$

$\Rightarrow$ $\angle BAC = 180° - 100° = 80°$

Since, angles in the same segment are equal

$\angle BDC = \angle BAC$, $\angle BDC = 80°$.

Q5. In figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130°$ and $\angle ECD = 20°$. Find $\angle BAC$.



Sol. $\angle CED + \angle BEC = 180°$ [Linear Pair]

$\Rightarrow$ $\angle CED + 130° = 180°$

$\Rightarrow$ $\angle CED + 180° – 130° = 50°$ ...(i)

$\angle ECD = 20°$ ...(ii)

In ΔCED, $\angle CED + \angle ECD + \angle CDE = 180°$

[Sum of all the angles of a triangle is 180°]

$\Rightarrow$ $50° + 20° + \angle CDE = 180°$ [Using (i) and (ii)]

$\Rightarrow$ $70° + \angle CDE = 180°$

$\Rightarrow$ $\angle CDE = 180° – 70°$

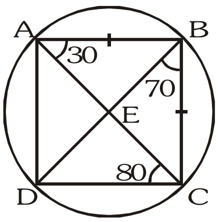
$\Rightarrow$ $\angle CDE = 110°$ ...(iii)

Now $\angle BAC = \angle CDE = 110°$

[Angle in the same segment of a circle are equal]

Q6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC$ = is 30°, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.

Sol.



Since angles in the same segment of a circle are equal

$\therefore$ $\angle BAC$ = BDC

$\Rightarrow$ BDC = 30°

Also $\angle DBC = 70°$ (Given)

$\therefore$ In $\angle BCD$, we have

$\Rightarrow$ $\angle BCD + \angle DBC + \angle CDB = 180°$ [sum of angles of a triangle is 180°]

$\Rightarrow$ $\angle BCD + 70° + 30° = 180°$

$\Rightarrow$ $\angle BCD = 80°$

Now, in ΔABC, $AB = BC$ (given)

$\therefore \angle BCA = \angle BAC$

(angles opp. to equal sides of a triangle are equal)

$\Rightarrow \angle BCA = 30°$ $[\angle BAC = 30°]$

Now, $\angle BCA + \angle ECD = \angle BCD$

$\Rightarrow 30° + \angle ECD = 80°$

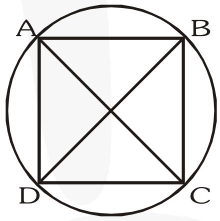
$\Rightarrow \angle ECD = 50°$

Q7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Sol. Since, AC and BD are diameters.

$\Rightarrow AC = BD$ [all diameters of a circle are equal]

Also, $\angle BAD = 90°$



[angle formed in a semicircle is 90°]

Similarly, $\angle ABC = 90°$, $\angle BCD = 90°$

and $\angle CDA = 90°$.

Now, in right ΔABC and ΔBAD, we have

$AC = BD$ (from (1))

$AB = BA$ (common)

$\angle ABC = \angle BAD$ (each equal to 90°)

$\therefore \Delta ABC \cong \Delta BAD$ (By RHS congruence)

$\Rightarrow BC = AD$ (CPCT)

Similarly, $AB = DC$

Thus, ABCD is a rectangle.

Q8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Sol. **Given :** ABCD is a trapezium whose two non-parallel sides AD and BC are equal.

To Prove : Trapezium ABCD is a cyclic.

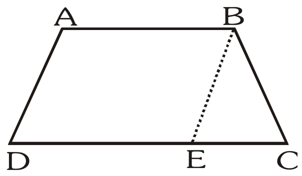
Construction : Draw $BE \| AD$

Proof :

$\because AB \| DE$ [Given]

$AD \| BE$ [By construction]

$\therefore$ Quadrilateral ABCD is a parallelogram.



$\therefore$ $\angle BAD = \angle BED$ ...(i) [Opp. $\angle$s of a || gm]

and AD = BE ...(ii) [Opp. sides of a || gm]

But AD = BC ...(iii) [Given]

From (ii) and (iii)

BE = BC

$\therefore$ $\angle BEC = \angle BCE$ ...(iv) [Angle opposite to equal sides]

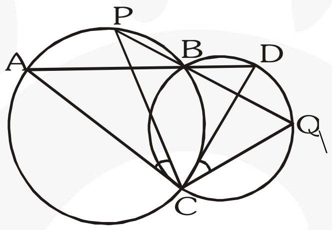
$\angle BEC + \angle BED = 180°$ [Linear pair]

$\Rightarrow$ $\angle BCE + \angle BAD = 180°$ [From (iv) and (i)]

$\Rightarrow$ Trapezium ABCD is cyclic.

[$\because$ If a pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic]

Q9. Two circles intersect at two points B and C. Through B, two line segment ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that $\angle ACP = \angle QCD$.



Sol. Given : Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.

To Prove : $\angle ACP = \angle QCD$

Proof : $\angle ACP = \angle ABP$ ...(i) [Angles in the same segment of a circle are equal]

$\angle QCD = \angle QBD$ ...(ii) [Angles in the same segment of a circle are equal]

$\angle ABP = \angle QBD$ ...(iii) [Vertically Opposite Angles]

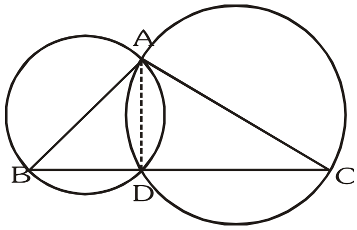
From (i), (ii) and (iii),

$\angle ACP = \angle QCD$.

Q10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Sol. We have ΔABC, and two circles described with diameter as AB and AC respectively. They intersect at a point D, other than A.

Let us join A and D.



AB is a diameter

∴ $\angle ADB$ is an angle formed in a semicircle.

⇒ $\angle ADB = 90°$ ......(1)

Similarly, $\angle ADC = 90°$ ......(2)

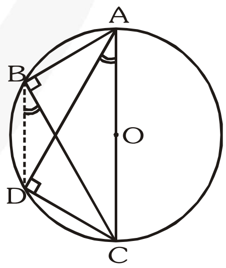
adding (1) and (2) $\angle ADB + \angle ADC = 180°$

i.e., B, D and C are collinear points BC is a straight line. Thus, D lies on BC.

Q11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Sol. AC is a hypotenuse

$\angle ADC = 90° = \angle ABC$



∴ Both the triangles are in the same semicircle.

⇒ A, B, C and D are concyclic.

Join BD

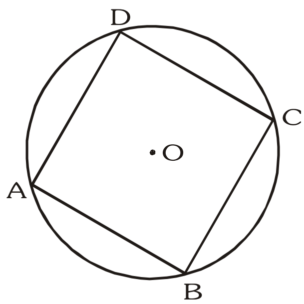
DC is chord

∴ $\angle CAD$ and $\angle CBD$ are formed on the same segment

∴ $\angle CAD = \angle CBD$

Q12. Prove that a cyclic parallelogram is a rectangle.

Sol. We have a cyclic parallelogram ABCD.



∴ Sum of its opposite angles is 180°

∴ $\angle A + \angle C = 180°$ ......(1)

But $\angle A = \angle C$ ......(2)

From (1) and (2), we have

$\angle A = \angle C = 90°$

Similarly, $\angle B = \angle D = 90°$

⇒ Each angle of the parallelogram ABCD is 90°

Thus, ABCD is a rectangle.