NCERT SOLUTIONS

Coordinate Geometry

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Ex - 7.1

Q1. Find the distance between the following pairs of points :
(a) (2,3), (4, 1)
(b) (–5, 7), (–1,3)
(c) (a, b), (– a, – b)

Sol.(a) The given points are : A (2, 3), B (4, 1).

Required distance = AB = BA = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB = $\sqrt{(4-2)^2 + (1-3)^2} = \sqrt{(2)^2 + (-2)^2}$

= $\sqrt{4+4}$ = $\sqrt{8}$ = $2\sqrt{2}$ units

(b) Here $x_1 = -5$, $y_1 = 7$ and $x_2 = -1$, $y_2 = 3$

$\therefore$ The required distance

= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{[-1-(-5)]^2 + (3-7)^2}$

= $\sqrt{(-1+5)^2 + (-4)^2}$

= $\sqrt{16+16} = \sqrt{32} = \sqrt{2 \times 16}$

= $4\sqrt{2}$ units

(c) Here $x_1 = a$, $y_1 = b$ and $x_2 = -a$, $y_2 = -b$

$\therefore$ The required distance

= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(-a-a)^2 + (-b-b)^2}$

= $\sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2}$

= $\sqrt{4(a^2 + b^2)} = 2\sqrt{(a^2 + b^2)}$ units

Q2. Find the distance between the points (0,0) and (36,15).

Sol. **Part-I**

Let the points be A(0, 0) and B(36, 15)

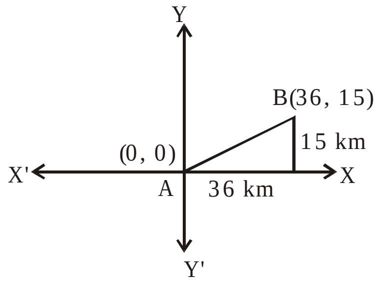
$\therefore$ AB = $\sqrt{(36-0)^2 + (15-0)^2}$

= $\sqrt{(36)^2 + (15)^2}$ = $\sqrt{1296 + 225}$

= $\sqrt{1521} = \sqrt{39^2}$ = 39

Part-II

We have A(0, 0) and B(36, 15) as the positions of two towns



Here $x_1 = 0$, $x_2 = 36$ and $y_1 = 0$, $y_2 = 15$

$\therefore$ $AB = \sqrt{(36-0)^2 + (15-0)^2} = 39$ km

Q3. Determine if the points (1,5), (2,3) and (– 2, – 11) are collinear.

Sol. The given points are :

A(1, 5), B(2, 3) and C(–2, –11).

Let us calculate the distance : AB, BC and CA by using distance formula.

$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2}$

$= \sqrt{1+4} = \sqrt{5}$ units

$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2}$

$= \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$ units

$CA = \sqrt{(-2-1)^2 + (-11-5)^2}$

$= \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$

$= \sqrt{5} \times \sqrt{53}$ units

From the above we see that : $AB + BC \neq CA$

Hence the above stated points A(1, 5), B(2, 3) and C(–2, –11) are not collinear.

Q4. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

Sol. Let the points be A(5, –2), B(6, 4) and C(7, –2).

$\therefore$ $AB = \sqrt{(6-5)^2 + [4-(-2)]^2}$

$= \sqrt{(1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$

$BC = \sqrt{(7-6)^2 + (-2-4)^2}$

$= \sqrt{(1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$

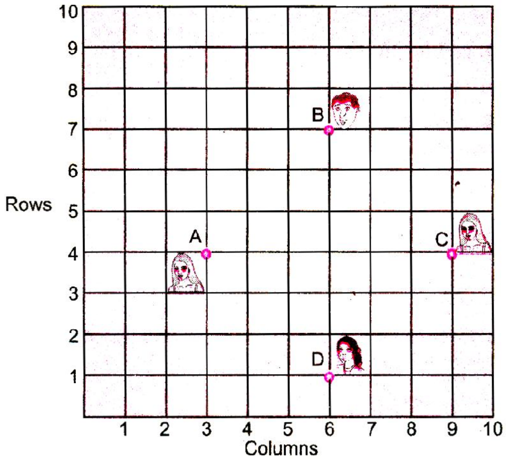
$AC = \sqrt{(7-5)^2 + (-2-(-2))^2}$

$= \sqrt{(+2)^2 + (0)^2} = \sqrt{4+0} = 2$

We have $AB = BC \neq AC$.

$\therefore$ ΔABC is an isosceles triangle.

Q5. In a classroom, 4 friends are seated at the points A, B, C and D as shown in fig. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a rectangle?" Chameli disagrees. Using distance formula, find which of them is correct.



Sol. Let the number of horizontal columns represent the x-coordinates whereas the vertical rows represent the y-coordinates.

$\therefore$ The points are : A(3, 4), B(6, 7), C(9, 4) and D(6, 1)

$\therefore$ $AB = \sqrt{(6-3)^2 + (7-4)^2}$

$= \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(9-6)^2 + (4-7)^2}$

$= \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(6-9)^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$AD = \sqrt{(6-3)^2 + (1-4)^2}$

$= \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Since, AB = BC = CD = AD i.e., All the four sides are equal

Also $AC = \sqrt{(9-3)^2 + (4-4)^2}$

$= \sqrt{(+6)^2 + (0)^2} = 6$ and

$BD = \sqrt{(6-6)^2 + (1-7)^2} = \sqrt{(0)^2 + (-6)^2} = 6$

i.e., BD = AC

$\Rightarrow$ Both the diagonals are also equal.

$\therefore$ ABCD is a square.

Thus, Chameli is correct as ABCD is not a rectangle.

Q6. Name the quadrilateral formed, if any, by the following points, and give reasons for your answer.

(i) (–1, –2), (1, 0), (–1, 2), (– 3, 0)

(ii) (–3, 5), (3, 1), (0, 3), (–1, – 4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Sol. (i) A(–1, –2), B(1, 0), C(–1, 2), D(–3, 0)

Determine distances : AB, BC, CD, DA, AC and BD.

$$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$DA = \sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

AB = BC = CD = DA

The sides of the quadrilateral are equal ....(1)

$$\left.\begin{aligned} AC &= \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = 4 \\ BD &= \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = 4 \end{aligned}\right\}$$

Diagonal AC = Diagonal BD..........(2)

From (1) and (2) we conclude that ABCD is a square.

(ii) Let the points be A(–3, 5), B(3, 1), C(0, 3) and D(–1, –4).

$$\therefore AB = \sqrt{[3-(-3)]^2 + (1-5)^2}$$

$$= \sqrt{6^2 + (-4)^2} = \sqrt{36+16}$$

$$= \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$$

$$CD = \sqrt{(-1-0)^2 + (-4-3)^2} = \sqrt{(-1)^2 + (-7)^2}$$

$$= \sqrt{1+49} = \sqrt{50}$$

$$DA = \sqrt{[-3-(-1)]^2 + [5-(-4)]^2}$$

$$= \sqrt{(-2)^2 + (9)^2}$$

$$= \sqrt{4+81} = \sqrt{85}$$

$$AC = \sqrt{[0-(-3)]^2 + (3-5)^2} = \sqrt{(3)^2 + (-2)^2}$$

$$= \sqrt{9+4} = \sqrt{13}$$

$$BD = \sqrt{(-1-3)^2 + (-4-1)^2} = \sqrt{(-4)^2 + (-5)^2}$$

$$= \sqrt{16+25} = \sqrt{41}$$

We see that $\sqrt{13} + \sqrt{13} = 2\sqrt{13}$

i.e., AC + BC = AB

$\Rightarrow$ A, B and C are collinear. Thus, ABCD is not a quadrilateral.

(iii) Let the points be A(4, 5), B(7, 6), C(4, 3) and D(1, 2).

$\therefore$ AB = $\sqrt{(7-4)^2+(6-5)^2} = \sqrt{3^2+1^2} = \sqrt{10}$

BC = $\sqrt{(4-7)^2+(3-6)^2}$

$= \sqrt{(-3)^2+(-3)^2} = \sqrt{18}$

CD = $\sqrt{(1-4)^2+(2-3)^2}$

$= \sqrt{(-3)^2+(-1)^2} = \sqrt{10}$

DA = $\sqrt{(1-4)^2+(2-5)^2} = \sqrt{9+9} = \sqrt{18}$

AC = $\sqrt{(4-4)^2+(3-5)^2} = \sqrt{0+(-2)^2} = 2$

BD = $\sqrt{(1-7)^2+(2-6)^2} = \sqrt{36+16} = \sqrt{52}$

Since, AB = CD, BC = DA [opposite sides of the quadrilateral are equal]

And AC $\neq$ BD $\Rightarrow$ Diagonals are unequal.

$\therefore$ ABCD is a parallelogram.

Q7. Find the point on the x-axis which is equidistant from (2, – 5) and (– 2, 9).

Sol. We know that any point on x-axis has its ordinate = 0

Let the required point be P(x, 0).

Let the given points be A(2, –5) and B(–2, 9)

$\therefore$ AP = $\sqrt{(x-2)^2+5^2} = \sqrt{x^2-4x+4+25}$

$= \sqrt{x^2-4x+29}$

BP = $\sqrt{[x-(-2)]^2+(-9)^2} = \sqrt{(x+2)^2+(-9)^2}$

$= \sqrt{x^2+4x+4+81} = \sqrt{x^2+4x+85}$

Since, A and B are equidistant from P,

$\therefore$ AP = BP

$\Rightarrow \sqrt{x^2-4x+29} = \sqrt{x^2+4x+85}$

$\Rightarrow x^2 - 4x + 29 = x^2 + 4x + 85$

$\Rightarrow x^2 - 4x - x^2 - 4x = 85 - 29$

$\Rightarrow -8x = 56 \Rightarrow x = \frac{56}{-8} = -7$

$\therefore$ The required point is (–7, 0)

Q8. Find the values of y for which the distance between the points P(2, –3) and Q(10, y) is 10 units.

Sol. Distance between P(2, – 3) and Q(10, y) = 10 units

$\Rightarrow \sqrt{(10-2)^2 + (y+3)^2} = 10$

$\Rightarrow 64 + (y + 3)^2 = 100$

$\Rightarrow (y + 3)^2 = 36$

$\Rightarrow y^2 + 6y + 9 = 36$

$y^2 + 6y - 27 = 0$

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y + 9) - 3(y + 9) = 0$

$\Rightarrow (y + 9)(y - 3) = 0$

$\Rightarrow y + 9 = 0$ or $y - 3 = 0$

$\Rightarrow y = -9$ or 3

Hence, there can be two values of y which are – 9 and 3.

Q9. If Q (0,1) is equidistant from P(5, – 3) and R(x, 6), find the values of x. Also find the distances QR and PR.

Sol. Here, $QP = \sqrt{(5-0)^2 + [(-3)-1]^2} = \sqrt{5^2 + (-4)^2}$

$= \sqrt{25+16} = \sqrt{41}$

$QR = \sqrt{(x-0)^2 + (6-1)^2} = \sqrt{x^2 + 5^2} = \sqrt{x^2 + 25}$

$\because$ QP = QR

$\therefore \sqrt{41} = \sqrt{x^2 + 25}$

Squaring both sides, we have $x^2 + 25 = 41$

$\Rightarrow x^2 + 25 - 41 = 0$

$\Rightarrow x^2 - 16 = 0 \Rightarrow x = \pm\sqrt{16} = \pm 4$

Thus, the point R is (4, 6) or (–4, 6)

Now,

$QR = \sqrt{[(\pm 4)-(0)]^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$

and $PR = \sqrt{(\pm 4 - 5)^2 + (6+3)^2}$

$\Rightarrow PR = \sqrt{(-4-5)^2 + (6+3)^2}$

or $\sqrt{(4-5)^2 + (6+3)^2}$

$\Rightarrow PR = \sqrt{(-9)^2 + 9^2}$ or $\sqrt{1+81}$

$\Rightarrow PR = \sqrt{2 \times 9^2}$ or $\sqrt{82}$

$\Rightarrow PR = 9\sqrt{2}$ or $\sqrt{82}$

Q10. Find a relation between x and y such that the point (x,y) is equidistant from the point (3, 6) and (– 3, 4).

Sol. A(3,6) and B(–3, 4) are the given points. Point P (x, y) is equidistant from the points A and B.

$\Rightarrow PA = PB$

$\Rightarrow \sqrt{(x-3)^2+(y-6)^2} = \sqrt{(x+3)^2+(y-4)^2}$

$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$\Rightarrow (x^2 - 6x + 9) + (y^2 - 12y + 36)$

$= (x^2 + 6x + 9) + (y^2 - 8y + 16)$

$\Rightarrow -6x - 12y + 45 = 6x - 8y + 25$

$\Rightarrow 12x + 4y - 20 = 0 \Rightarrow 3x + y - 5 = 0$

Ex - 7.2

Q1. Find the co-ordinates of the point which divides the line joining of (–1, 7) and (4, –3) in the ratio 2 : 3.

Sol. Let the required point be P(x, y).

Here the end points are (–1, 7) and (4, –3)

$\because$ Ratio = 2 : 3 = $m_1 : m_2$

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{(2\times 4)+3(-1)}{2+3}$$

$$= \frac{8-3}{5} = \frac{5}{5} = 1$$

$$\text{And } y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$= \frac{2\times(-3)+(3\times 7)}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Thus, the required point is (1, 3).

Q2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

Sol.

Points P and Q trisect the line segment joining the points A(4, – 1) and B(–2, – 3), i.e., AP = PQ = QB.

Here, P divides AB in the ratio 1 : 2 and Q divides AB in the ratio 2 : 1.

$$\text{x-coordinate of P} = \frac{1\times(-2)+2\times(4)}{1+2} = \frac{6}{3} = 2 \ ;$$

$$\text{y-coordinate of P} = \frac{1\times(-3)+2\times(-1)}{1+2} = \frac{-5}{3}$$

Thus, the coordinates of P are $\left(2, \frac{-5}{3}\right)$.

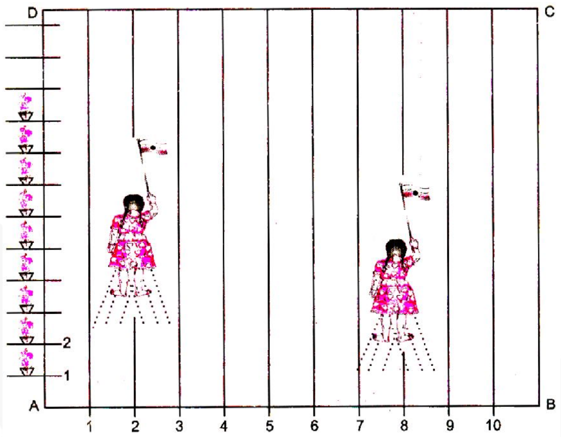
$$\text{Now, x coordinate of Q} = \frac{2\times(-2)+1(4)}{2+1} = 0 \ ;$$

$$\text{y-coordinate of Q} = \frac{2\times(-3)+1\times(-1)}{2+1} = -\frac{7}{3}$$

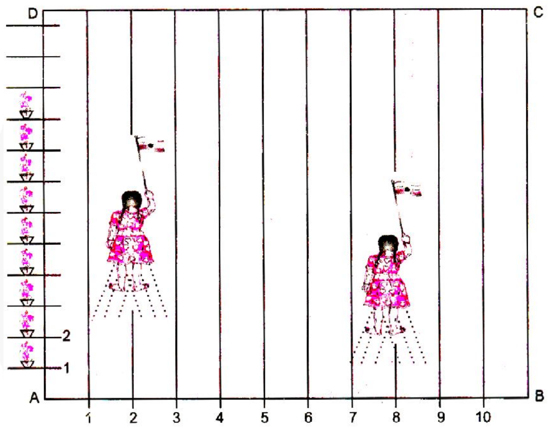
Thus, the coordinates of Q are $\left(0, -\frac{7}{3}\right)$.

Hence, the points of trisection are $P\left(2, \frac{-5}{3}\right)$ and $Q\left(0, -\frac{7}{3}\right)$.

Q3. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in fig. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Sol. Let us consider 'A' as origin, then



AB is the x-axis.

AD is the y-axis.

Now, the position of green flag-post is

$\left(2, \frac{100}{4}\right)$ or (2, 25)

And, the position of red flag-post is

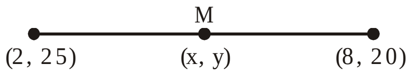
$\left(8, \frac{100}{5}\right)$ or (8, 20)

$\Rightarrow$ Distance between both the flags

$= \sqrt{(8-2)^2 + (20-25)^2}$

$$= \sqrt{6^2 + (-5)^2} = \sqrt{36 + 25} = \sqrt{61}$$

Let the mid-point of the line segment joining the two flags be M(x, y).



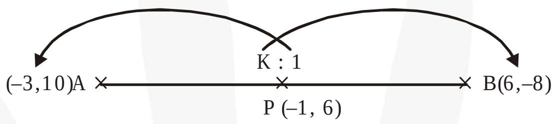
$\therefore \quad x = \dfrac{2+8}{2}$ and $y = \dfrac{25+20}{2}$

or $\quad x = 5$ and $y = 22.5$

Thus, the blue flag is on the 5th line at a distance 22.5 m above AB.

Q4. Find the ratio in which the line segment joining the points (– 3, 10) and (6, –8) is divided by (– 1, 6).

Sol. Let the required ratio be K : 1



Comparing x-coordinate

$$\frac{k \times (6) + 1 \times (-3)}{k+1} = -1$$

$\Rightarrow \quad 6k - 3 = -k - 1$

$\Rightarrow \quad 7k = 2$

$\Rightarrow \quad k = \dfrac{2}{7}$

Comparing y-coordinate

$$\frac{k \times (-8) + 1 \times (10)}{k+1} = 6$$

$\Rightarrow \quad -8k + 10 = 6k + 6$

$\Rightarrow \quad -8K - 6K = 6 - 10$

$\Rightarrow \quad -14K = -4$

$\Rightarrow \quad k = \dfrac{2}{7}$

Q5. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Sol. The given points are : A(1, – 5) and B(–4, 5). Let the required ratio = k : 1 and the required point be P(x, y)

Part-I : To find the ratio

Since, the point P lies on x-axis,

$\therefore$ Its y-coordinate is 0.

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } 0 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$\Rightarrow \quad x = \dfrac{-4k+1}{k+1}$ and $0 = \dfrac{5k-5}{k+1}$

$\Rightarrow \quad x(k + 1) = -4k + 1$

and $5k - 5 = 0 \quad \Rightarrow \quad k = 1$

$\Rightarrow \quad x(k + 1) = -4k + 1$

$\Rightarrow \quad x(1 + 1) = -4 + 1 \qquad [\because \; k = 1]$

$\Rightarrow \quad 2x = -3$

$\Rightarrow\ x = -\frac{3}{2}$

$\therefore$ The required ratio k : 1 = 1 : 1

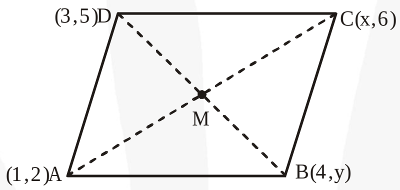
Coordinates of P are (x, 0) $= \left(\frac{-3}{2}, 0\right)$

Q6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Sol. Mid-point of the diagonal AC has x-coordinate

$= \frac{x+1}{2}$ and y-coordinate $= \frac{6+2}{2} = 4$

i.e., $\left(\frac{x+1}{2}, 4\right)$ is the mid-point of AC.



Similarly, mid-point of the diagonal BD is

$\left(\frac{4+3}{2}, \frac{y+5}{2}\right)$, i.e., $\left(\frac{7}{2}, \frac{y+5}{2}\right)$

We know that the two diagonals AC and BD bisect each other at M. Therefore,

$\left(\frac{x+1}{2}, 4\right)$ and $\left(\frac{7}{2}, \frac{y+5}{2}\right)$. Coincide

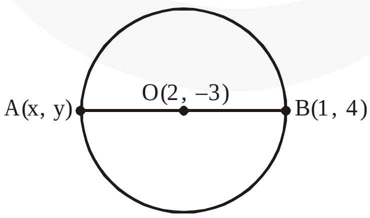
$\Rightarrow \frac{x+1}{2} = \frac{7}{2}$ and $\frac{y+5}{2} = 4$

$\Rightarrow x = 6$ and $y = 3$

Q7. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).

Sol. Here, centre of the circle is O(2, –3)

Let the end points of the diameter be A(x, y) and B(1, 4)



The centre of a circle bisects the diameter.

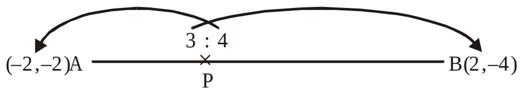
$\therefore\ 2 = \frac{x+1}{2} \Rightarrow x + 1 = 4$ or $x = 3$

And $-3 = \frac{y+4}{2} \Rightarrow y + 4 = -6$ or $y = -10$

Here, the coordinates of A are (3, –10)

Q8. If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

Sol.



$AP = \frac{3}{7}\,AB,$

$BP = AB - AP = AB - \frac{3}{7}AB = \frac{4}{7}AB$

$$\frac{AP}{BP} = \frac{\frac{3}{7}AB}{\frac{4}{7}AB} = \frac{3}{4}$$

Thus, P divides AB in the ratio 3 : 4.

x-coordinate of P $= \frac{3\times(2)+4\times(-2)}{3+4} = -\frac{2}{7}$

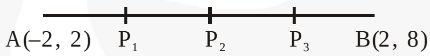
y-coordinate of P $= \frac{3\times(-4)+4\times(-2)}{3+4} = -\frac{20}{7}$

Hence, the coordiantes of P are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

Q9. Find the coordinates of the points which divide the line segment joining A (– 2, 2) and B (2, 8) into four equal parts.

Sol. Here, the given points are A(–2, 2) and B(2, 8)

Let P_1, P_2 and P_3 divide AB in four equal parts.



$\because\ AP_1 = P_1P_2 = P_2P_3 = P_3B$

Obviously, P_2 is the mid-point of AB

$\therefore$ Coordinates of P_2 are

$\left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$ or (0, 5)

Again, P_1 is the mid-point of AP_2.

$\therefore$ Coordinates of P_1 are

$\left(\frac{-2+0}{2}, \frac{2+5}{2}\right)$ or $\left(-1, \frac{7}{2}\right)$

Also P_3 is the mid-point of P_2B.

$\therefore$ Coordinates of P_3 are

$\left(\frac{0+2}{2}, \frac{5+8}{2}\right)$ or $\left(1, \frac{13}{2}\right)$

Thus, the coordinates of P_1, P_2 and P_3 are $\left(-1, \frac{7}{2}\right)$, (0, 5) and $\left(1, \frac{13}{2}\right)$ respectively.

Q10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (–1, 4) and (– 2, – 1) taken in order.

Sol. Diagonals AC and BD bisect each other at right angle to each other at O.

$AC = \sqrt{(-1-3)^2 + (4-0)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$BD = \sqrt{(4+2)^2 + (5+1)^2} = \sqrt{36+36} = 6\sqrt{2}$

Then $OA = \frac{1}{2} AC = \frac{1}{2} \times 4\sqrt{2} = 2\sqrt{2}$

$OB = \frac{1}{2} BD = \frac{1}{2} \times 6\sqrt{2} = 3\sqrt{2}$

Area of $\Delta AOB = \frac{1}{2}(OA) \times (OB) = \frac{1}{2} \times 2\sqrt{2} \times 3\sqrt{2} = 6$ sq. units

Hence, the area of the rhombus ABCD

$= 4 \times$ area of $\Delta AOB = 4 \times 6 = 24$ sq. units.

Ex - 7.3

Q1. Find the area of the triangle whose vertices are :

(i) (2,3), (–1, 0), (2, –4) (ii) (– 5, – 1), (3,–5), (5,2)

Sol. (i) Let the vertices of the triangles be A(2, 3), B (–1, 0) and C(2, –4)

Here $x_1 = 2, y_1 = 3,$

$x_2 = -1, y_2 = 0$

$x_3 = 2, y_3 = -4$

$\because$ Area of a Δ

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\therefore$ Area of a Δ

$= \frac{1}{2}[2\{0-(-4) + (-1)\{-4-(3)\} + 2\{3-0\}]$

$= \frac{1}{2}[2(0+4) + (-1)(-4-3) + 2(3)]$

$= \frac{1}{2}[8+7+6] = \frac{1}{2}[21] = \frac{21}{2}$ sq.units

(ii) A(– 5, – 1), B (3, –5), C (5, 2) are the vertices of the given triangle.

$x_1 = -5, x_2 = 3, x_3 = 5$; $y_1 = -1, y_2 = -5, y_3 = 2.$

Area of the ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-5 \times (-5-2) + 3 \times (2+1) + 5 \times (-1+5)]$

$= \frac{1}{2}[35 + 9 + 20] = \frac{1}{2}[64] = 32$ sq. units

Q2. In each of the following find the value of 'k', for which the points are collinear.

(i) (7, – 2), (5, 1), (3, k) (ii) (8,1), (k – 4), (2,–5).

Sol. The given three points will be collinear if the Δ formed by them has equal to zero area.

(i) Let A(7, –2), B(5, 1) and C(3, k) be the vertices of a triangle.

$\therefore$ The given points will be collinear, if

ar $(\Delta ABC) = 0$

or $7(1-k) + 5(k+2) + 3(-2-1) = 0$

$\Rightarrow 7 - 7k + 5k + 10 + (-6) - 3 = 0$

$\Rightarrow 17 - 9 + 5k - 7k = 0$

$\Rightarrow 8 - 2k = 0 \Rightarrow 2k = 8$

$\Rightarrow k = \frac{8}{2} = 4$

The required value of k = 4.

(ii) A(8, 1), B(k, –4), C(2, –5) are the given points.

$x_1 = 8, x_2 = k, x_3 = 2$

$y_1 = 1, y_2 = -4, y_3 = -5$

the condition for the three points to be collinear is

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$8 \times (-4 + 5) + k \times (-5 - 1) + 2 \times (1 + 4) = 0$

i.e. $8 - 6k + 10 = 0$, i.e., $6k = 18$, i.e., $k = 3$

Q3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, – 1), (2, 1) and (0, 3). Find the ratio of this area of the area of the given triangle.

Sol. Let the vertices of the triangle be A(0, –1), B(2, 1) and C(0, 3).

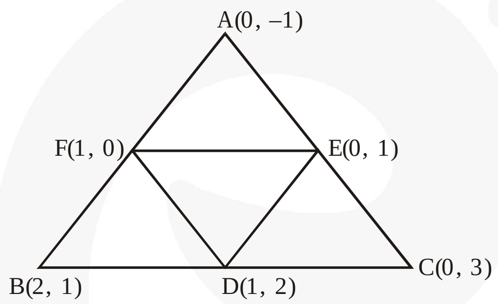
Let D, E and F be the mid-points of the sides BC, CA and AB respectively. Then :

Coordinates of D are

$\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ i.e., $\left(\frac{2}{2}, \frac{4}{2}\right)$ or (1, 2)

Coordinates of E are $\left(\frac{0+0}{2}, \frac{3+(-1)}{2}\right)$ i.e., (0, 1)

Coordinates of F are $\left(\frac{2+0}{2}, \frac{1+(-1)}{2}\right)$ i.e., (1, 0)



Now, ar(ΔABC)

$= \frac{1}{2}[0(1 - 3) + 2\{3 - (-1)\} + 0(-1 - 1)]$

$= \frac{1}{2}[0(-2) + 8 + 0(-2)]$

$= \frac{1}{2}[0 + 8 + 0] = \frac{1}{2} \times 8 = 4$ sq. units

Now, ar (ΔDEF) $= \frac{1}{2}[1(1 - 0) + 0(0 - 2) + 1(2 - 1)]$

$= \frac{1}{2}[1(1) + 0 + 1(1)]$

$= \frac{1}{2}[1 + 0 + 1] = \frac{1}{2} \times 2 = 1$ sq. unit

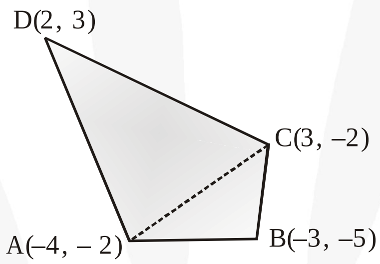
$\therefore \quad \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$

$\therefore$ ar(ΔDEF) : ar(ΔABC) = 1 : 4.

Q4. Find the area of the quadrilateral whose vertices taken in order are (–4, –2), (–3, –5), (3, –2) and (2, 3).

Sol. Join A and C. The given points are

A(–4, –2), B(–3, –5), C(3, –2) and D(2, 3)



Area of ΔABC

$= \frac{1}{2}[(-4)(-5 + 2) - 3(-2 + 2) + 3(-2 + 5)]$

$= \frac{1}{2}[12 + 0 + 9] = \frac{21}{2} = 10.5$ sq. units

Area of ΔACD

$= \frac{1}{2}[(-4)(-2 - 3) + 3(3 + 2) + 2(-2 + 2)]$

$= \frac{1}{2}[20 + 15| = \frac{35}{2} = 17.5$ sq. units.

Area of quadrilateral ABCD

= ar. (ΔABC) + ar. (ΔACD)

= (10.5 + 17.5) sq. units = 28 sq. units

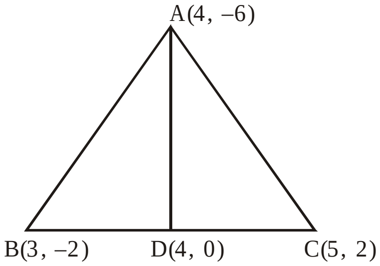
Q5. A median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A(4, – 6), B(3,–2) and C(5,2)

Sol. Here, the vertices of the triangles are A(4, –6), B(3, –2) and C(5, 2).
Let D be the midpoint of BC.

$\therefore$ The coordinates of the mid point D are

$\left\{\frac{3+5}{2}, \frac{-2+2}{2}\right\}$ or (4, 0).

Since, AD divides the triangle ABC into two parts i.e., ΔABD and ΔACD,



Now, ar(ΔABD)

$= \frac{1}{2}[4\{(-2) - 0\} + 3(0 + 6) + 4(-6 + 2)]$

$= \frac{1}{2}[(-8) + 18 + (-16)] = \frac{1}{2}(-6) = -3$ sq. units

= 3 sq. units(numerically) ....(1)

ar(ΔADC) $= \frac{1}{2}[4(0 - 2) + 4(2 + 6) + 5(-6 - 0)]$

$= \frac{1}{2}[-8 + 32 - 30] = \frac{1}{2}[-6] = -3$ sq.units

= 3 sq.units (numerically) ....(2)

From (1) and (2)

ar(ΔABD) = ar(ΔADC)

i.e., A median divides the triangle into two triangles of equal areas.