NCERT SOLUTIONS

Heron's Formula

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Ex - 12.1

Q1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board ?

Sol. The equilateral triangle each side = a

Its semiperimeter = $\frac{a+a+a}{2} = \frac{3}{2}a$

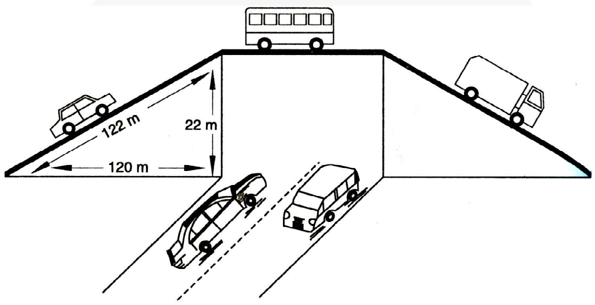
By Heron's formula, the area of the triangle

$$= \sqrt{\frac{3}{2}a \times \left(\frac{3}{2}a - a\right) \times \left(\frac{3}{2}a - a\right) \times \left(\frac{3}{2}a - a\right)} = \frac{\sqrt{3}}{4}a^2$$

When perimeter of the triangle is 180 cm, we have 3a = 180 cm i.e., a = 60 cm. Then the area of the triangle

$$= \frac{\sqrt{3}}{4}(60)^2 \text{ cm}^2 = 900\sqrt{3} \text{ cm}^2$$

Q2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig.). The advertisements yield as earning of Rs. 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay ?



Sol. Sides of the two equal triangular walls below the bridge are 122m, 22m and 120m.

$$s = \frac{122m + 22m + 120m}{2} = 132m$$

Area of one triangular wall

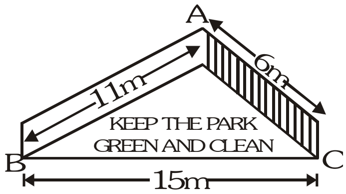
$$= \sqrt{132 \times (132 - 122) \times (132 - 22) \times (132 - 120)} \text{ m}^2$$

$$= \sqrt{132 \times 10 \times 110 \times 12} \text{ m}^2 = 1320 \text{ m}^2$$

Company hired only one wall for 3 months. Thus, earning from advertisements for 3 months at the rate of Rs.5000 per m^2 per year.

$$= \text{Rs.}5000 \times \frac{3}{12} \times 1320 = \text{Rs.}16,500,00$$

Q3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig.). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.



Sol. The sides of the triangular wall be 15m, 11m and 6m.

$$s = \frac{15m + 11m + 6m}{2} = 16m$$

Area of the wall $= \sqrt{16 \times (16-15) \times (16-11) \times (16-6)}\ m^2$

$= \sqrt{16 \times 1 \times 5 \times 10}\ m^2 = 20\sqrt{2}\ m^2$

Q4. Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Sol. a = 18 cm,

b = 10 cm

Perimeter = 42

we have a + b + c = 42

$\Rightarrow$ c = 14

$$s = \frac{a+b+c}{2} = \frac{42}{2} = 21$$

Area of

$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{21(3)(11)(7)} = 3.7\sqrt{11}$

$= 21\sqrt{11}\ cm^2$

Q5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area.

Sol. Let the sides of triangle be 12k, 17k, 25k

Perimeter = 12k + 17k + 25k = 54k.

$\Rightarrow$ 54k = 540

k = 10

$\Rightarrow$ a = 12 × k = 12 × 10 = 120

b = 17 × k = 17 × 10 = 170

c = 25 × k = 25 × 10 = 250

$$s = \frac{a+b+c}{2} = \frac{540}{2} = 270$$

$$\therefore \quad \text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

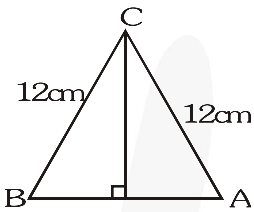
$$= \sqrt{270(270-120)(270-170)(270-250)}$$

$$= \sqrt{270(150)(100)(20)}$$

$$= 3 \times 30 \times 5 \times 20 \text{ cm}^2 = 9000 \text{ cm}^2$$

Q6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of triangle.

Sol. a = 12 cm



b = 12 cm

Perimeter = 30 cm

$\Rightarrow$ c = 30 – 24 = 6 cm

$$s = \frac{30}{2} = 15 \text{ cm}$$

$$\therefore \quad \text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

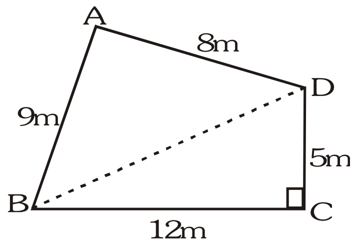
$$= \sqrt{15(15-12)(15-12)(15-6)}$$

$$= \sqrt{15.3.3.9} = 9\sqrt{15} \text{ cm}^2$$

Ex - 12.2

Q1. A park, in the shape of a quadrilateral ABCD has $\angle C = 90°$, AB = 9 m, BC = 12 m, CD = 5 m, AD = 8 m. How much area does it occupy ?

Sol. Join the diagonal AD of the quadrilateral ABCD.



$BD^2 = BC^2 + CD^2 = (12)^2 + (5)^2 = 169$

$\Rightarrow$ BD = $\sqrt{169}$ = 13 m

Area of right angle ΔBCD

$= \frac{1}{2} \times BC \times CD = \frac{1}{2} \times 12 \times 5m^2 = 30\ m^2$

Now, the sides of the ΔABD are 9m in,13 m and 8 m.

Semiperimeter of ΔABD

$S = \frac{9m + 13m + 8m}{2} = 15$ m.

The area of the $\Delta ABD = \sqrt{15 \times (15-9) \times (15-13) \times (15-8)}\ m^2$

$\Rightarrow \sqrt{15 \times 6 \times 2 \times 7}\ m^2 = \sqrt{5 \times 3 \times 3 \times 2 \times 2 \times 7}$

$\Rightarrow$ $6 \times 5.916\ m^2$ (approx.) = $35.495\ m^2$ (approx.)

= 35.5 (approx.)

Thus area of the quadrilateral ABCD

= ar (ΔBCD) + ar (ΔABD)

= $30\ m^2 + 35.5\ m^2$ (approx.) = $65.5\ m^2$ (approx.)

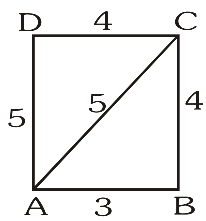
Q2. Find the area of quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Sol. a = 4

b = 5

c = 3

$a^2 + c^2 = b^2$



Area of ΔABC

$= \frac{1}{2} \times \text{Base} \times \text{Height.} = \frac{1}{2} \times 3 \times 4 = 6\ cm^2$

For ΔACD $\quad a = 4,\ b = 5,\ c = 5$

$$s = \frac{a+b+c}{2} = \frac{14}{2} = 7 \text{ cm}$$

$\therefore$ Area of ΔACD

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{7(7-4)(7-5)(7-5)}$$

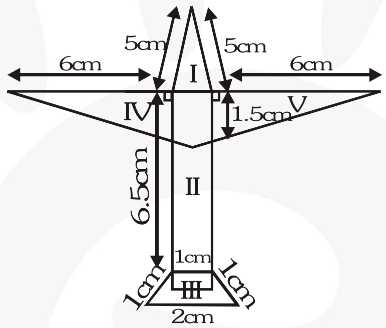
$$= \sqrt{7 \times 3 \times 2 \times 2}$$

$$= 2\sqrt{21} \text{ cm}^2$$

area of quadrilateral ABCD $= 6+2\sqrt{21}$

$\cong 15.2 \text{ cm}^2$

Q3. Radha made a picture of an aeroplane with coloured paper as shown in Fig. find the total area of the paper used.



Sol. It is triangular part and its sides are 5 cm, 5 cm, 1 cm. Here, semiperimeter of the triangle

$$s = \frac{5\text{cm} + 5\text{cm} + 1\text{cm}}{2} = \frac{11}{2} \text{ cm}$$

Area of part I in figure

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

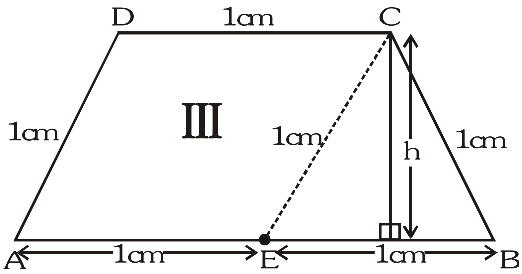
$$= \sqrt{\frac{11}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{9}{2}} \text{ cm}^2$$

$$= \frac{3}{4}\sqrt{11} \text{ cm}^2 = \frac{3}{4} \times 3.31 \text{ cm}^2 = 2.482 \text{ (approx.)}$$

Area of part II in figure

$$= 6.5 \times 1 \text{ cm}^2 = 6.5 \text{ cm}^2$$

Area of part III in figure



Area of $\Delta BEC = \frac{\sqrt{3}}{4}(1)^2 \text{ cm}^2 = \frac{\sqrt{3}}{4} \text{ cm}^2$

Let h be the height of the ΔBEC i.e., height of the trapezium.

$\frac{1}{2} \times BE \times h = \frac{\sqrt{3}}{4}$

$\Rightarrow \frac{1}{2} \times 1 \times h = \frac{\sqrt{3}}{4} \quad \Rightarrow h = \frac{\sqrt{3}}{2} \text{ cm}$

Area of Trapezium ABCD

$= \frac{1}{2}$(sum of parallel sides) (height)

$= \frac{1}{2}(1+2)\left(\frac{\sqrt{3}}{2}\right)\text{cm}^2$

$= \frac{3}{4}\sqrt{3} \text{ cm}^2 = \frac{3}{4} \times 1.732$ (approx.)

$= 1.299$ (approx.) $= 1.3$ (approx.)

Area of part IV in figure

$= \frac{1}{2} \times 1.5 \times 6 \text{ cm}^2 = 4.5 \text{ cm}^2$

Area of part V in figure

$= \frac{1}{2} \times 1.5 \times 6 \text{ cm}^2 = 4.5 \text{ cm}^2$

Total area of the paper used

= part (I + II + III + IV + V)

$= 2.482 \text{ cm}^2 + 6.5 \text{ cm}^2 + 1.3 \text{ cm}^2 + 4.5\text{cm}^2 + 4.5\text{cm}^2$

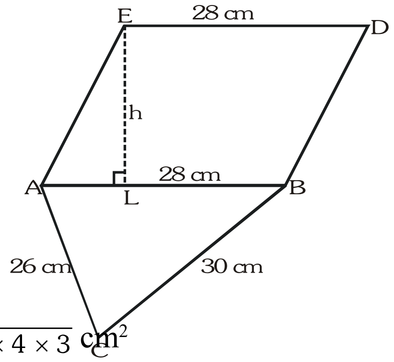
$= (10.282 + 9) \text{ cm}^2$ (approx.)

$= 19.282 \text{ cm}^2$ (approx.)

$= 19.3 \text{ cm}^2$ (approx.)

Q4. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Sol. Semiperimeter of ΔABC



$$s = \frac{26+28+30}{2} = 42 \text{ cm}$$

$s - a = 42 - 26 = 16$ cm

$s - b = 42 - 28 = 14$ cm

$s - c = 42 - 30 = 12$ cm

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42 \times 16 \times 14 \times 12}\ \text{cm}^2 = \sqrt{14 \times 3 \times 4 \times 4 \times 14 \times 4 \times 3}\ \text{cm}^2$

$= 14 \times 4 \times 3 \times 2\ \text{cm}^2 = 336\ \text{cm}^2$

$\therefore$ Area of parallelogram = Area of triangle [Given]

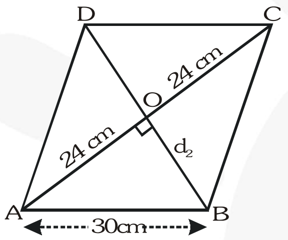
$h \times AB = 336$

$h \times 28 = 336\ \text{cm}^2$

$$h = \frac{336}{28} = 12\text{cm}$$

Q5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 cm and its longer diagonal is 48 cm, how much area of grass field will each cow be getting?

Sol. Since the diagonals of a Rhombus bisect each other at right angles



$\therefore$ $OB = \sqrt{(30)^2 - (24)^2} = \sqrt{324} = 18$ cm

$\Rightarrow$ diagonal of $d_2 = 2 \times OB = 2 \times 18 = 36$ cm

$\therefore$ Area of a Rhombus $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 48 \times 36 = 864\ \text{cm}^2$ [$d_1 = 48$ cm (given)]

Total area of grass field for 18 cows = $864\ \text{cm}^2$

Area of grass grazed by each cow $= \frac{864}{18} = 48\ \text{cm}^2$.

Q6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig.) each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella ?

Sol. Sides of a triangular piece of coloured cloth are 20 cm, 50 cm and 50 cm.

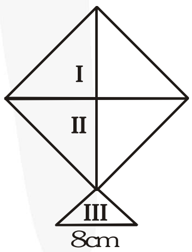
Its semiperimeter $= \dfrac{20\text{cm} + 50\text{cm} + 50\text{cm}}{2} = 60\text{cm}$

Then, the area of one triangular piece $= \sqrt{60 \times 10 \times 10 \times 40}\ \text{cm}^2 = 200\sqrt{6}\ \text{cm}^2$

There are 5 triangular pieces one colour and 5 of the other colour.

Then total area of cloth of each colour (two colours) $= 5 \times 200\sqrt{6}\ \text{cm}^2 = 1000\sqrt{6}\ \text{cm}^2$

Q7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in Fig. How much paper of each shade has been used in it ?



Sol. Area of paper shade I

$= \dfrac{1}{2}\left[\dfrac{1}{2} \times 32 \times 32\right]$

$= 256\ \text{cm}^2$

Area of paper shade II = $256\ \text{cm}^2$

Area of paper of shade III

a = 8 cm

b = 6 cm

c = 6 cm

$s = \dfrac{a+b+c}{2} = \dfrac{8+6+6}{2} = 10\text{cm}$

$\therefore$ Area of paper of shade III

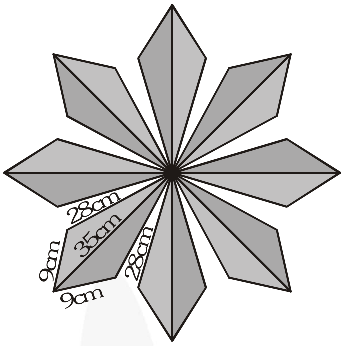
$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10(10-8)(10-6)(10-6)}$

$= \sqrt{10.2.4.4} = 8\sqrt{5}$

$= 17.85\ \text{cm}^2$

Q8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (See Fig.). Find the cost of polishing the tiles at the rate of 50 p per cm^2.



Sol. Sides of a triangular tile are 9cm, 28 cm and 35 cm.

Its semiperimeter = $\frac{9\text{cm}+28\text{cm}+35\text{cm}}{2}$ =36cm

Area of one tile = $\sqrt{36\times27\times8\times1}$ cm^2 = $36\sqrt{6}$ cm^2

Total area of 16 tiles

= $576\sqrt{6}$ cm^2

= 576 × 2.45 cm^2 (approx.)

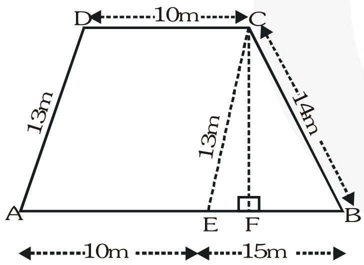
= 1411.20 cm^2 (approx.)

Total cost of polishing at the rate of 50p per cm^2

= Rs.1411.20 × cm^2 = Rs.705.60

Q9. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Sol. Through C draw CE || DA



Draw CF ⊥ AB

In ΔBCE, we have

s = $\frac{15+14+13}{2}$ = 21 m

s – a = 21 – 15 = 6 m

s – b = 21 – 14 = 7 m

s – c = 21 – 13 = 8 m

Area of $\Delta BCE = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21 \times 6 \times 7 \times 8}\ m^2$

$= 7 \times 4 \times 3 = 84\ m^2$

Now, area of $\Delta BCE = 84\ m^2$

$\Rightarrow \frac{1}{2} \times \text{Base} \times \text{Altitude} = 84\ m^2$

$\Rightarrow \frac{1}{2} \times 15 \times h = 84$

$\Rightarrow h = \frac{84 \times 2}{15}\ m$

$\Rightarrow$ h = Distance between parallel sides of trapezium $= \frac{168}{15}$ m

Area of parallelogram, AECD = base × height

$= 10 \times \frac{168}{15} = 56 \times 2 = 112\ m^2$

$\therefore$ Area of trapezium ABCD = Area of parallelogram AECD + Area of ΔBCE

$= 112 + 84 = 196\ m^2$