Ex - 5.1

Q1. Which of the following statements are true and which are false ? Give reasons for your answers.

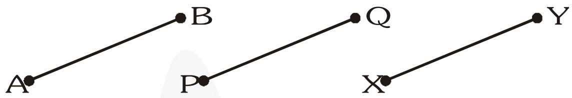
(i) Only one line can pass through a single point.

(ii) There are infinite number of lines which pass through two distinct points.

(iii) A terminated line can be produced indefinitely on both the sides.

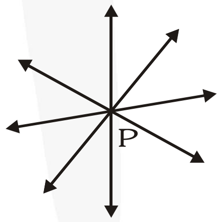
(iv) If two circles are equal, then their radii are equal.

(v) In Fig., if AB = PQ and PQ = XY, then AB = XY.



Sol. (i) False, because infinitely many lines can pass through a single point.

This is self evident and can be seen visually by the student as follows :

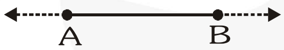


(ii) False, because the given statement contradicts the postulate I of the Euclid that assures that there is a unique line that passes through two distinct points.

Through two points P and Q, a unique line can be drawn.

(iii) True.

Evidence : According to Euclid's postulate 2; a terminated line can be produced indefinitely.



(iv) True.

Evidence : According to the axiom 4 of Euclid; that "the things which coincide with one another are equal to one another". If we superimpose the region bounded by one circle on the other, then they coincide. So, their centres and boundaries coincide. Therefore, their radii will coincide.

(v) True.

Evidence : Euclid's axiom I states that the things which, are equal to the same thing are equal to one another.

Q2. Give a definition for each of the following terms. Are there other terms that need to be defined first ? What are they and how might you define them ?

(i) parallel lines (ii) perpendicular lines

(iii) line segment (iv) radius of a circle

(v) square.

Sol. (i) Parallel lines : Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction. Other term involved is the "plane". We keep the Plane as undefined term. The only thing is that we can represent it intuitively or explain it with the help of physical model.

(ii) Perpendicular lines : When a straight line set up on a straight line makes the adjacent angles equal to one another, each of equal angle is right and the straight line standing on the other is called a perpendicular to that on which it stands. The other terms that need to be defined first is (1) Angle (2) Adjacent angles (3) Right angle. Let us define these.

(1) Angle : A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.

(2) Adjacent angles : The two angles with the same vertex, one arm common and other arms lying on the opposite sides of the common arm are called adjacent angles.

(3) Right angle : An angle equal to one quarter of a complete angle is called a right angle.

(iii) Line segment : A line segment which extends indefinitely in both directions gives a line. Other term involved is line. We keep the line as undefined term. The only thing is that we can represent it intuitively or explain it with the help of physical model.

(iv) Radius of a circle : A line segment joining the centre to any point on the circle is called the radius of the circle.

Other terms that need to be defined first are :

(1) Circle (2) Centre.

Let us define these :

(1) Circle : A circle is a closed curve on a plane, all points on which are at the same distance from a fixed point with in it.

(2) Centre of circle : The fixed point from which all the points on a circle are equidistant is called its centre.

(v) Square : Of quadrilateral figures, a square is that which is both equilateral and right-angled. Other terms that need to be defined first are

(1) equilateral

(2) right angle. Let us define these.

(1) Equilateral : A figure having all its sides equal is called an equilateral.

(2) Right angle : An angle equal to one quarter of a complete angle is called a right angle.

Q3. Consider two 'postulates' given below.

(i) Given any two distinct points A and B, there exists a third point C which is between A and B.

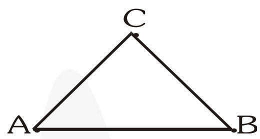
(ii) There exist at least three points that are not on the same line.

Do these postulates contain any undefined terms ? Are these postulates consistent ? Do they follow from Euclid's postulates ? Explain.

Sol. There are several undefined terms which are to be listed by a student. These two postulates (i) and (ii) are consistent because they deal with two different situations.

Postulate (i) states that given two points A and B, there is a point C lying on the line in between them.

Postulate (ii) states that for given two points A and B, we can take point C not lying on the line through A and B.



Hence, we observe that the postulates do not follow from Euclid's postulates, however they follow from Axiom 1.

Q4. If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}$ AB. Explain by drawing the figure.

Sol. Given that C lies between A and B

and AC = BC

So, AC + AC = BC + AC

[∵ according to Euclid's definition, if equals are added to equals, the whole is equal]

i.e., 2AC = AB

[BC + AC coincides with AB]

Therefore, AC = $\frac{1}{2}$ AB

Q5. In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

Sol. Let C and D be the two mid-points of line segment AB.

So, according to Euclid's axiom (4) when line is folded about point C we observe that part BC superimposes over the part AC.

It implies that

AC = BC ...(1)

Similarly, D is the mid point of AB implies that AD = BD ...(2)

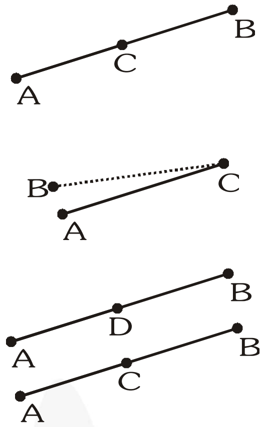
we have, AB = AB

or AC + BC = AD + BD

or AC + AC = AD + AD [Using (1) and (2)]

or 2AC = 2AD

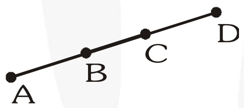
or AC = AD ...(3)



When we superimpose AD over AC and BD over BC we find that D exactly lies over C. It implies that D and C are not two different points but the same.

Hence, we conclude that mid-point of a line segment is unique.

Q6. In Fig., if AC = BD, then prove that AB = CD.



Sol. Given that AC = BD ...(1)

AC = AB + BC

[point B lies between A and C] ...(2)

BD = BC + CD

[Point C lies between B and DI ...(3)

Substituting (2) and (3) in (1), we get :

AB + BC = BC + CD

Subtracting BC from both sides, we get :

AB + BC – BC = BC + CD – BC = BC – BC + CD

So, AB = CD

[∵ if equals are subtracted from equals, the remaining are equal]

Q7. Why is axiom 5, in the list of Euclid's axioms; considered as a 'universal truth' ? (Note that the question is not about the fifth postulate.)

Sol. Euclid's Axiom 5 states that "The whole is greater than the part."

Since, this is true for anything in any part of the world.

So, this is a universal truth.