Ex - 1.3

Q1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Sol. (i) $\frac{36}{100} = 0.36$ (Terminating)

(ii) $\frac{1}{11} = 0.090909.....$ (Non terminating Repeating)

$$
\begin{array}{r|l}
11 & 1.00000 \\
 & -99 \\
\hline
 & 100 \\
 & 99 \\
\hline
 & 100 \\
 & 99 \\
\hline
 & 1
\end{array}
\quad 0.090909....
$$

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$ (Terminating decimal)

(iv) $\frac{3}{13} = 0.230769230769.....$

$= 0.\overline{230769}$ (Non Terminating repeating)

(v) $\frac{2}{11} = 0.1818..... = 0.\overline{18}$ (Non Terminating repeating)

(vi) $\frac{329}{400}$

$$
\begin{array}{r|l}
400 & 329.0000 \\
 & 320\,0 \\
\hline
 & 900 \\
 & 800 \\
\hline
 & 1000 \\
 & 800 \\
\hline
 & 2000 \\
 & 2000 \\
\hline
 &
\end{array}
\quad 0.8225
$$

$\frac{329}{400} = 0.8225 \Rightarrow$ (Terminating)

Q2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division ? If so, how ?

Sol. Yes, we can predict decimal explain without actually doing long division method as

$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times .\overline{142857} = .\overline{428571}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times .\overline{142857} = .\overline{571428}$

$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times .\overline{142857} = .\overline{714285}$

$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times .\overline{142857} = .\overline{857142}$

Q3. Express the following in the form p/q, where p and q are integers and q ≠ 0.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Sol. (i) Let x = 0.6666..... (1)

Multiplying both the sides by 10.

10 x = 6.666..... (2)

Subtract (1) from (2)

10x – x = (6.6666.....) – (0.6666.....)

$\Rightarrow 9x = 6 \Rightarrow x = \frac{6}{9} = \frac{2}{3}$

(ii) Let x = $0.4\overline{7}$ = .4777...

Multiply both sides by 10

$10x = 4.\overline{7}$ ...(1)

Multiply both sides by 10

$100\ x = 47.\overline{7}$ ...(2)

Subtract (1) from (2)

90x = 43

$x = \frac{43}{90}$

(iii) Let x = $0.\overline{001} = 0.001001001...$ ...(1)

Multiply both sides by 1000

$1000x = 1.\overline{001}$ ...(2)

Subtract (1) from (2)

999x = 1

$x = \frac{1}{999}$

Q4. Express 0.99999 ..... in the form p/q. Are you surprised by your answer ? With your teacher and classmates discuss why the answer makes sense.

Sol. Let x = 0.999.... ...(1)

Multiply both sides by 10 we get

10x = 9.99.... ...(2)

Subtract (1) from (2)

$9x = 9 \Rightarrow x = 1$

$.9999.... = 1 = \frac{1}{1}$

$\therefore$ p = 1, q = 1

Q5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17 ? Perform the division to check your answer.

Sol. Maximum no. of digits in the repeating block of digits in decimal expansion of $\frac{1}{17}$ can be 16.

```
      0.0588235294117647
17 ) 1.00000000000000000000000000000000
     85
     150
     136
      140
      136
       40
       34
        60
        51
         90
         85
          50
          34
           160
           153
            70
            68
             20
             17
              30
              17
               130
               119
                110
                102
                 80
                 68
                  120
                  119
                   100
                    85
                     150
                     136
                      4
```

Ans. $.\overline{0588235294117647}$

Q6. Look at several examples of rational numbers in the form p/q (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy ?

Sol. There is a property that q must satisfy rational no. of form $\frac{p}{q}$ (q ≠ 0) where p, q are integers with no common factors other than 1 having terminating decimal representation (expansions) is that the prime factorization of q has only powers of 2 or powers of 5 or both [i.e., q must be of the form $2^m \times 5^n$]. Here m,n are whole numbers.

Q7. Write three numbers whose decimal expansion are non-terminating non-recurring.

Sol. 0.01001000100001...
0.202002000200002...
0.003000300003...

Q8. Find three different irrational numbers between the rational numbers 5/7 and 9/11.

Sol.
$$7\overline{)5.000000}(0.714285...$$
49
10
7
30
28
20
14
60
56
40
35
5

Thus, $\frac{5}{7} = 0.\overline{714285}$

$$\frac{9}{11} = 11\overline{)9.0000}(0.8181...$$
88
20
11
90
88
20
11
9

Thus, $\frac{9}{11} = 0.\overline{81}$

Three different irrational numbers between

$\frac{5}{7}$ and $\frac{9}{11}$ are taken as

0.750750075000750000...

0.780780078000780000...

0.80800800080000800000...

Q9. Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 ..... (v) 1.101001000100001.....

Sol. (i) $\sqrt{23}$ = irrational number

(ii) $\sqrt{225}$ = 15 = Rational number

(iii) 0.3796 decimal expansion is terminating

$\Rightarrow$ .3796 = Rational number.

(iv) 7.478478...

= $7.\overline{478}$ which is non terminating recurring.

= Rational number.

(v) 1.101001000100001.....

decimal expansion is non terminating and non repeating.

= Irrational number