NCERT SOLUTIONS

Number Systems

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Ex - 1.1

Q1. Is zero a rational number? Can you write it in the form p/q, where p and q are integers and $q \neq 0$?

Sol. Yes, zero is a rational number. We can write zero in the form p/q whose p and q are integers and $q \neq 0$.

so, 0 can be written as $\frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ etc.

Q2. Find six rational numbers between 3 and 4.

Sol. First rational number between 3 and 4 is $= \frac{3+4}{2} = \frac{7}{2}$

Similarly other numbers

$$\frac{3+\frac{7}{2}}{2} = \frac{13}{4}$$

$$\frac{3+\frac{13}{4}}{2} = \frac{25}{8}$$

$$\frac{3+\frac{25}{8}}{2} = \frac{49}{16}$$

$$\frac{3+\frac{49}{16}}{2} = \frac{97}{32}$$

$$\frac{\frac{97}{32}+3}{2} = \frac{193}{64}$$

So, numbers are

$$\frac{7}{2}, \frac{13}{4}, \frac{25}{8}, \frac{49}{16}, \frac{97}{32}, \frac{193}{64}$$

Q3. Find five rational numbers between 3/5 and 4/5.

Sol. Let $a = \frac{3}{5}$ $b = \frac{4}{5}$ $n = 5$

then, $d = \frac{b-a}{n+1} = \frac{\frac{4}{5}-\frac{3}{5}}{5+1} = \frac{1}{30}$

So, rational numbers are

$$\frac{3}{5}+\frac{1}{30}=\frac{19}{30}$$

$$\frac{3}{5}+\frac{2}{30}=\frac{20}{30}$$

$$\frac{3}{5}+\frac{3}{30}=\frac{21}{30}$$

$$\frac{3}{5}+\frac{4}{30}=\frac{22}{30}$$

$$\frac{3}{5}+\frac{5}{30}=\frac{23}{30}$$

Thus, numbers are

$$\frac{19}{30},\frac{20}{30},\frac{21}{30},\frac{22}{30},\frac{23}{30}$$

Q4. State whether the following statements are true or false? Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Sol. (i) True, the collection of whole numbers contains all natural numbers.

(ii) False, –2 is not a whole number

(iii) False, $\frac{1}{2}$ is a rational number but not a whole number.

Ex - 1.2

Q1. State whether the following statements are true or false ? Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Sol. (i) True, since collection of real numbers consists of rationals and irrationals.

(ii) False, because no negative number can be the square root of any natural number.

(iii) False, 2 is real but not irrational.

Q2. Are the square roots of all positive integers irrational ? If not, give an example of the square root of a number that is a rational number.

Sol. No, $\sqrt{4} = 2$ is a rational number.

Q3. Show how $\sqrt{5}$ can be represented on the number line.

Sol. $\sqrt{5}$ on Number line.

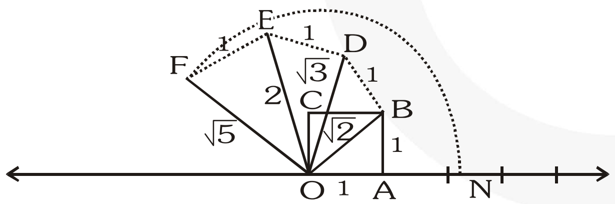
OABC is unit square

So, OB = $\sqrt{1^2+1^2} = \sqrt{2}$

OD = $\sqrt{\left(\sqrt{2}\right)^2+1} = \sqrt{3}$

OE = $\sqrt{\left(\sqrt{3}\right)^2+1} = 2$

OF = $\sqrt{(2)^2+1} = \sqrt{5}$



Using compass we can cut arc with centre O and radius = OF on number line. ON is required result.

Ex - 1.3

Q1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Sol. (i) $\frac{36}{100} = 0.36$ (Terminating)

(ii) $\frac{1}{11} = 0.090909.......$ (Non terminating Repeating)

```
11)1.00000(0.090909....
  -99
   100
    99
    100
     99
      1
```

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$ (Terminating decimal)

(iv) $\frac{3}{13} = 0.230769230769......$

$= 0.\overline{230769}$ (Non Terminating repeating)

(v) $\frac{2}{11} = 0.1818....... = 0.\overline{18}$ (Non Terminating repeating)

(vi) $\frac{329}{400}$

```
400)329.0000(0.8225
    320 0
      900
      800
      1000
       800
       2000
       2000
```

$\frac{329}{400} = 0.8225 \Rightarrow$ (Terminating)

Q2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division ? If so, how ?

Sol. Yes, we can predict decimal explain without actually doing long division method as

$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times .\overline{142857} = .\overline{428571}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times .\overline{142857} = .\overline{571428}$

$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times .\overline{142857} = .\overline{714285}$

$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times .\overline{142857} = .\overline{857142}$

Q3. Express the following in the form p/q, where p and q are integers and q ≠ 0.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Sol. (i) Let x = 0.6666........ (1)

Multiplying both the sides by 10.

10 x = 6.666.......... (2)

Subtract (1) from (2)

10x – x = (6.6666......) – (0.6666.....)

$\Rightarrow 9x = 6 \Rightarrow x = \frac{6}{9} = \frac{2}{3}$

(ii) Let x = $0.4\overline{7}$ = .4777...

Multiply both sides by 10

10x = $4.\overline{7}$ ...(1)

Multiply both sides by 10

100 x = $47.\overline{7}$ ...(2)

Subtract (1) from (2)

90x = 43

$x = \frac{43}{90}$

(iii) Let x = $0.\overline{001} = 0.001001001...$ ...(1)

Multiply both sides by 1000

1000x = $1.\overline{001}$ ...(2)

Subtract (1) from (2)

999x = 1

$x = \frac{1}{999}$

Q4. Express 0.99999 ..... in the form p/q. Are you surprised by your answer ? With your teacher and classmates discuss why the answer makes sense.

Sol. Let x = 0.999.... ...(1)

Multiply both sides by 10 we get

10x = 9.99.... ...(2)

Subtract (1) from (2)

$9x = 9 \Rightarrow x = 1$

$.9999.... = 1 = \frac{1}{1}$

$\therefore$ p = 1, q = 1

Q5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17 ? Perform the division to check your answer.

Sol. Maximum no. of digits in the repeating block of digits in decimal expansion of $\frac{1}{17}$ can be 16.

```
        0.05882352941176470 5
17 ) 1.00000000000000000000000000000
      85
      150
      136
       140
       136
         40
         34
          60
          51
           90
           85
            50
            34
            160
            153
              70
              68
               20
               17
                30
                17
                130
                119
                 110
                 102
                   80
                   68
                   120
                   119
                     100
                      85
                      150
                      136
                        4
```

Ans. $.\overline{0588235294117647}$

Q6. Look at several examples of rational numbers in the form p/q ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy ?

Sol. There is a property that q must satisfy rational no. of form $\frac{p}{q}$ ($q \neq 0$) where p, q are integers with no common factors other than 1 having terminating decimal representation (expansions) is that the prime factorization of q has only powers of 2 or powers of 5 or both [i.e., q must be of the form $2^m \times 5^n$]. Here m,n are whole numbers.

Q7. Write three numbers whose decimal expansion are non-terminating non-recurring.

Sol. 0.01001000100001...
0.202002000200002...
0.003000300003...

Q8. Find three different irrational numbers between the rational numbers 5/7 and 9/11.

Sol. 7) 5.000000 (0.714285...
49
10
7
30
28
20
14
60
56
40
35
5

Thus, $\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11}$ = 11) 9.0000 (0.8181...
88
20
11
90
88
20
11
9

Thus, $\frac{9}{11} = 0.\overline{81}$

Three different irrational numbers between

$\frac{5}{7}$ and $\frac{9}{11}$ are taken as

0.75075007500075000...

0.78078007800078000...

0.80800800080000800000...

Q9. Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 ...... (v) 1.101001000100001......

Sol. (i) $\sqrt{23}$ = irrational number

(ii) $\sqrt{225}$ = 15 = Rational number

(iii) 0.3796 decimal expansion is terminating

⇒ .3796 = Rational number.

(iv) 7.478478...

= $7.\overline{478}$ which is non terminating recurring.

= Rational number.

(v) 1.101001000100001.....

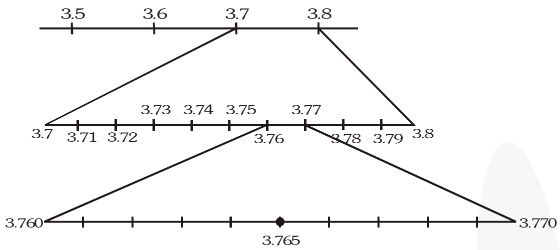
decimal expansion is non terminating and non repeating.

= Irrational number

Ex - 1.4

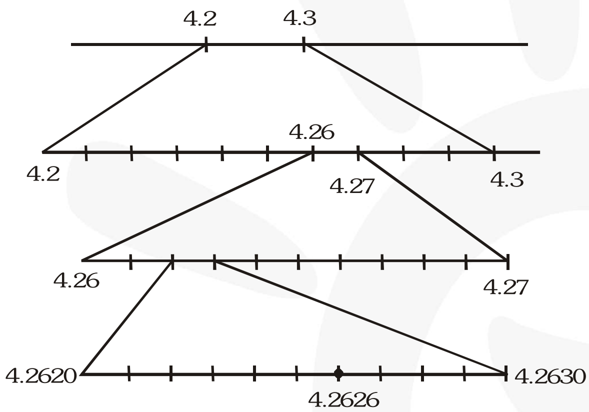
Q1. Visualise on the number line, using successive magnification.

Sol. n = 3.765



Q2. Visualize $4.\overline{26}$ on the number line, upto 4 decimal places.

Sol. n = $4.\overline{26}$



Ex - 1.5

Q1. Classify the following numbers as rational or irrational :

(i) $2-\sqrt{5}$ (ii) $(3+\sqrt{23})-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Sol. (i) $\because$ 2 is a rational number and $\sqrt{5}$ is an irrational number.

$\therefore 2-\sqrt{5}$ is an irrational number.

(ii) $(3+\sqrt{23})-\sqrt{23} \Rightarrow (3+\sqrt{23})-\sqrt{23} = 3$ is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$ Rational number.

(iv) $\frac{1}{\sqrt{2}}$

$\because$ 1 is a rational number and $\sqrt{2}$ is an irrational number.

So, $\frac{1}{\sqrt{2}}$ is irrational number.

(v) 2π

$\because$ 2 is a rational number and π is an irrational number

So, 2π is irrational number.

Q2. Simplify each of the following expressions :

(i) $(3+\sqrt{3})(2+\sqrt{2})$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Sol. (i) $(3+\sqrt{3})(2+\sqrt{2}) = 3(2+\sqrt{2})+\sqrt{3}(2+\sqrt{2})$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3}) = (3)^2-(\sqrt{3})^2 = 9-3 = 6$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$

$= \left(\sqrt{5}\right)^2+2\sqrt{10}+\left(\sqrt{2}\right)^2$

$= 7+2\sqrt{10}$

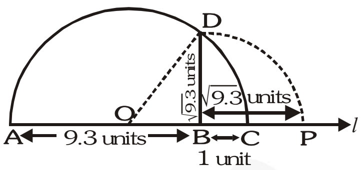
(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right) = 5-2 = 3$

Q3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = c/d$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction ?

Sol. There is no contradiction. When we measure a length with a scale or any other device, we only get an approximate rational value. Therefore, we may not realise that c or d is irrational.

Q4. Represent $\sqrt{9.3}$ on the number line.

Sol.



Let l be the number line.

Draw a line segment AB = 9.3 units and BC = 1 unit. Find the mid point O of AC.

Draw a semicircle with centre O and radius OA or OC.

Draw BD $\perp$ AC intersecting the semicircle at D. Then, BD = $\sqrt{9.3}$ units. Now, with centre B and radius BD, draw an arc intersecting the number line ℓ at P.

Hence, BD = BP = $\sqrt{9.3}$

Q5. Rationalise the denominators of the following :

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$ (iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Sol. (i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \frac{\sqrt{7}+\sqrt{6}}{1} = \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

$\frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{3}$

(iv) $\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$

$= \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$

Ex - 1.6

Q1. Find : (i) $(64)^{1/2}$ (ii) $32^{1/5}$ (iii) $125^{1/3}$

Sol. (i) $(64)^{1/2} = (8^2)^{1/2} = (8^{2\times\frac{1}{2}}) = 8^1 = 8$

(ii) $32^{1/5} = (2^5)^{1/5} = (2^{5\times\frac{1}{5}}) = 2^1 = 2$

(iii) $(125)^{1/3} = (5^3)^{1/3} = 5^{3\times\frac{1}{3}} = 5$

Q2. Find : (i) $9^{3/2}$ (ii) $32^{2/5}$ (iii) $16^{3/4}$ (iv) $125^{1/3}$

Sol. (i) $9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = (3)^3 = 27$

(ii) $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = 2^2 = 4$

(iii) $16^{3/4} = (2^4)^{3/4} = 2^3 = 8$

(iv) $125^{1/3} = (5^3)^{1/3} = 5$

Q3. Simplify : (i) $2^{2/3}.2^{1/5}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{1/2}}{11^{1/4}}$ (iv) $7^{1/2}$. $8^{1/2}$

Sol. (i) $2^{\frac{2}{3}} . 2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{(3^3)^7} = \frac{1}{3^{21}} = 3^{-21}$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}$

$= 11^{\frac{1}{4}} = \sqrt[4]{11}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

$= (7 \times 8)^{1/2} = (56)^{1/2}$