Ex - 3.2

Q1. Form the pair of linear equations in the following problems, and find their solutions graphi cally.

(i) 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost ` 50, whereas 7 pencils and 5 pens together cost 46.

Find the cost of one pencil and that of one pen.

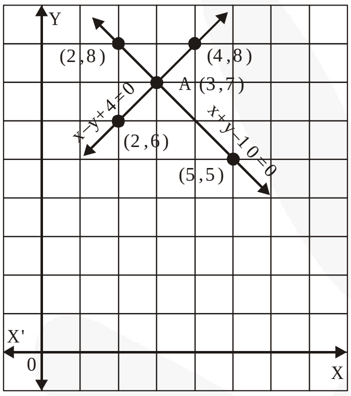
Sol. (i) Let the number of boys be x and the number of girls be y.

According to the given conditions

$x + y = 10$ and $y = x + 4$

We get the required pair of linear equations as

$x + y - 10 = 0,\ x - y + 4 = 0$



Graphical Solution

$x + y - 10 = 0$ ...(i)

x	2	5
y = 10 – x	8	5

$x - y + 4 = 0$ ...(ii)

x	2	4
y = x + 4	6	8

From the graph, we have : x = 3, y = 7 common solution of the two linear equations.

Hence, the number of boys = 3 and the number of girls = 7.

(ii) Let the cost of 1 pencil be Rs x and cost of 1 pen be Rs. y.

$5x + 7y = 50$

$7x + 5y = 46$

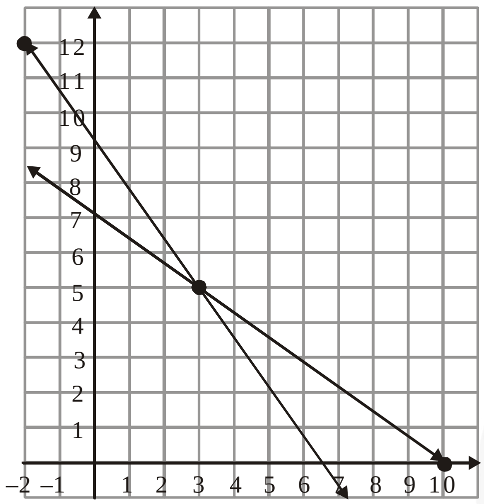
Graphical solution

$5x + 7\ y = 50°\ 7x + 5x = 46$

$y = \frac{50 - 5x}{7}$ $\quad$ $y = \frac{46 - 7x}{5}$

x	3	10
y	5	0

x	3	–2
y	5	12



From the graph we have x = 3, y = 5.
Hence, cost of one pencil = Rs.3 and cost of one pen = Rs.5

Q2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident.

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

Sol. (i) $5x - 4y + 8 = 0$ ...(i)

$7x + 6y - 9 = 0$ ...(ii)

$\frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = -\frac{2}{3}$ $\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow$ Lines represented by (i) and (ii) intersect at a point

(ii) $9x + 3y + 12 = 0$ .....(i)

$18x + 6y + 24 = 0$ ....(ii)

$\frac{a_1}{a_2} = \frac{9}{18}, \frac{b_1}{b_2} = \frac{3}{6}, \frac{c_1}{c_2} = \frac{12}{24}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ Lines represented by (i) and (ii) are coincident.

(iii) $6x - 3y + 10 = 0$ .....(i)

$2x - y + 9 = 0$ .....(ii)

$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}, \frac{c_1}{c_2} = \frac{10}{9}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ Lines represented by (i) and (ii) are parallel

Q3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pairs of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5; 2x - 3y = 7$

(ii) $2x - 3y = 8; 4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7; 9x - 10y = 14$

(iv) $5x - 3y = 11; -10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

Sol. (i) $3x + 2y - 5 = 0$ ...(i)

$2x - 3y - 7 = 0$ ...(ii)

$\frac{a_1}{a_2} = \frac{3}{2}; \frac{b_1}{b_2} = \frac{2}{-3} = -\frac{2}{3}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow$ The equations have a unique solution.

Hence, consistent.

(ii) $2x - 3y = 8$ .....(i)

$4x - 6y = 9$ ....(ii)

$\frac{a_1}{a_2} = \frac{2}{4}, \frac{b_1}{b_2} = \frac{-3}{-6}, \frac{c_1}{c_2} = \frac{8}{9}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ The equations have no solution. Hence inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$ ....(i)

$9x - 10y = 14$ ....(ii)

$$\frac{a_1}{a_2} = \frac{3/2}{9} = \frac{1}{6}, \frac{b_1}{b_2} = \frac{5/3}{-10} = \frac{-1}{6}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow$ The equations have a unique solutions

Hence, consistent.

(iv) $5x - 3y = 11$ ....(i)

$-10x + 6y = -22$ ....(ii)

$$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2},$$

$$\frac{c_1}{c_2} = \frac{11}{-22} = \frac{-1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

The equations have infinite solutions.

Hence, consistent.

(v) $\frac{4}{3}x + 2y = 8$ ....(i)

$2x + 3y = 12$ ....(ii)

$$\frac{a_1}{a_2} = \frac{4/3}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3} = \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

The equations have infinite solutions.

Hence, consistent.

Q4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically :

(i) $x + y = 5$, $2x + 2y = 10$

(ii) $x - y = 8$, $3x - 3y = 16$

(iii) $2x + y - 6 = 0$, $4x - 2y - 4 = 0$

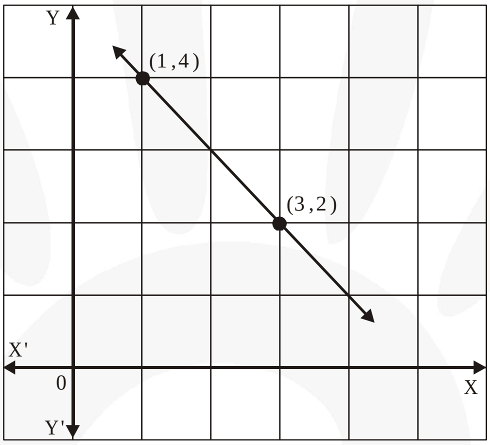
(iv) $2x - 2y - 2 = 0$, $4x - 4y - 5 = 0$

Sol. (i) $x + y = 5$ ...(i)

$2x + 2y = 10$ ...(ii)

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$$

i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$



Hence, the pair of linear equations is consistent.

(i) and (ii) are same equations and hence the graph is coincident straight line.

x	1	3
y = 5 – x	4	2

(ii) $x - y = 8$ ....(i)

$3x - 3y = 16$ ....(ii)

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, lines have no solution

Hence, inconsistent.

(iii) $2x + y = 6$ ....(i)

$4x - 2y = 4$ ....(ii)

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{-2} = \frac{-1}{2}, \frac{c_1}{c_2} = \frac{6}{4} = \frac{3}{2}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

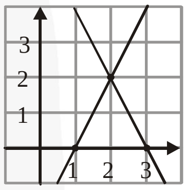
Therefore, lines have unique solution.

Hence, consistent

from (i) from (ii)

x	2	3
y	2	0

x	2	1
y	2	0



from graph $x = 2, y = 2$

(iv) $2x - 2y = 2$ ....(i)

$4x - 4y = 5$ ....(ii)

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{2}{5}$$

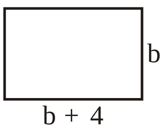
$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, lines have no solution.

Hence, Inconsistent.

Q5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden

Sol.



Length, $\ell = b + 4$ and Breadth = b

Perimeter of rectangle = $2(\ell + b)$

$$\frac{1}{2}[2(\ell + b)] = 36$$

$(\ell + b) = 36$ ....(i)

As, ℓ = b + 4, so puting the value of ℓ

in equation (i), we get

$\Rightarrow$ b + 4 + b = 36

2b + 4 = 36

2b = 32

b = 16m, ℓ = b + 4 = 16 + 4 = 20m

Thus, length of garden = 20m and breadth of garden = 16 m

Q6. Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(i) Intersecting lines

(ii) Parallel lines

(iii) Coincident lines

Sol. (i) 2x + 3y – 8 = 0 (Given equation)

3x + 2y + 4 = 0 (New equation)

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the graph of the two equations will be two intersecting lines.

(ii) 2x + 3y – 8 = 0 (given equation)

4x + 6y – 10 = 0 (New equation)

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the graph of the two equations will be two parallel lines.

(iii) 2x + 3y – 8 = 0 (given equation)

4x + 6y – 16 = 0 (New equation)

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the graph of the two equations will be two conicident lines.

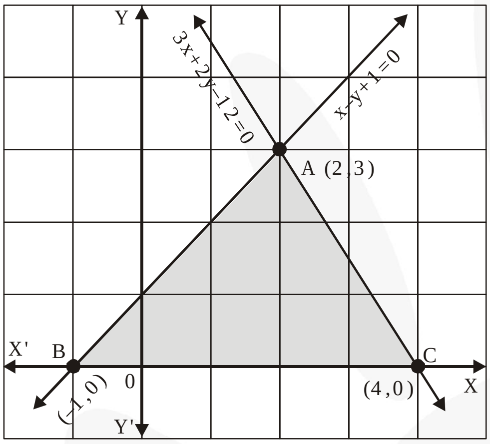
Q7. Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Sol. $x - y + 1 = 0$ ...(i)

x	–1	2
y = x + 1	0	3

$3x + 2y - 12 = 0$ ...(ii)

x	2	4
$y = \frac{12 - 3x}{2}$	3	0



The vertices of the triangle are
A (2, 3), B (–1, 0) and C (4, 0)