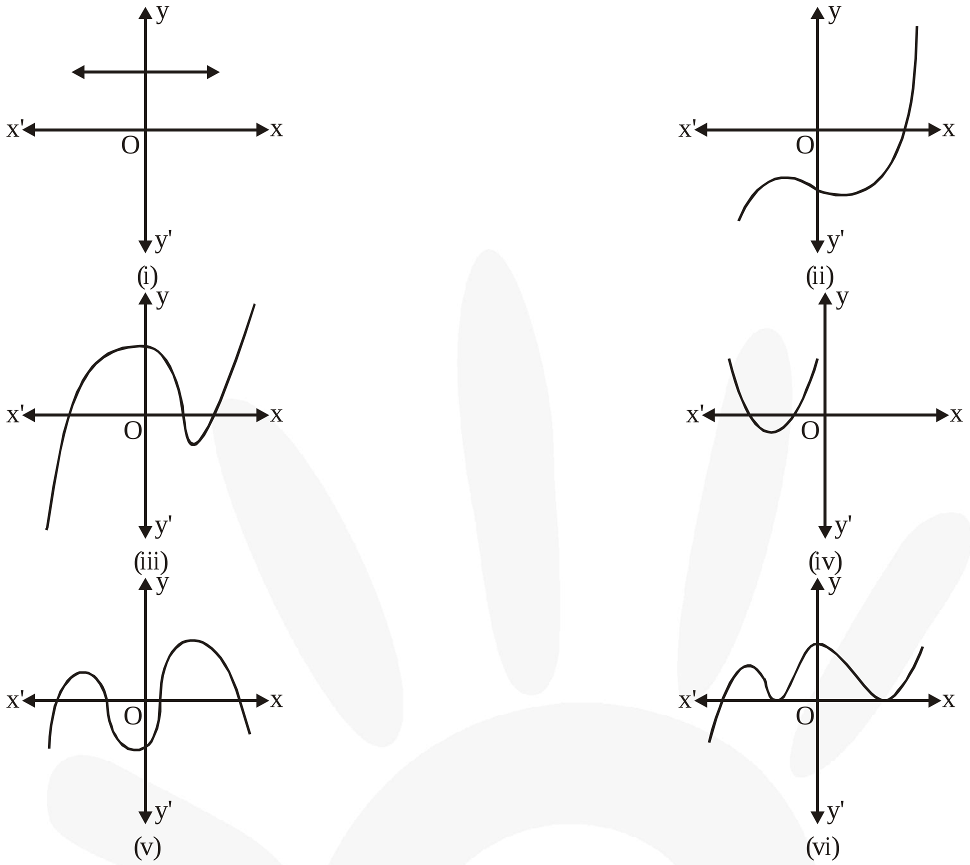

# NCERT SOLUTIONS

## Polynomial

eSaral है, तो सब सरल है।

# EX-2.1

**Q1.** The graph of y = p(x) are given in fig below, for some polynomials p(x). Find the number of zeros of p(x), in each case.




**Sol.** (i) Graph of y = p(x) does not intersect the x-axis. Hence, polynomial p(x) has no zero.

(ii) Graph of y = p(x) intersects the x-axis at one and only one point.
Hence, polynomial p(x) has **one and only one** real zero.

(iii) Graph of y = p(x) intersects the x-axis at 3 points. Hence, polynomial p(x) has 3 zeros.

(iv) Graph of y = p(x) intersects the x-axis at 2 points. Hence, polynomial p(x) has 2 zeros.

(v) Graph of y = p(x) intersects the x-axis at 4 points. Hence, polynomial p(x) has 4 zeros.

(vi) Graph of y = p(x) intersects the x-axis at 1 points and touch x-axis at 2 points. Hence, p(x) has 3 zeros.

# EX-2.2

**Q1.** Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients.

(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$
(iii) $6x^2 - 3 - 7x$ (iv) $4u^2 + 8u$
(v) $t^2 - 15$ (vi) $3x^2 - x - 4$

**Sol.** (i) $x^2 - 2x - 8 = x^2 - 4x + 2x - 8$
$= x(x - 4) + 2(x - 4) = (x + 2)(x - 4)$
Zeroes are $-2$ and $4$.
Sum of the zeros
$= (-2) + (4) = 2 = \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$
Product of the zeros
$= (-2)(4) = -8 = \frac{(-8)}{1} = \frac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$

(ii) $4s^2 - 4s + 1 = (2s - 1)^2$
The two zeros are $\frac{1}{2}, \frac{1}{2}$
Sum of the two zeros
$= \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$
Product of two zeros
$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4} = \frac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$

(iii) $6x^2 - 7x - 3$
$= 6x^2 - 9x + 2x - 3$
$= 3x(2x - 3) + 1(2x - 3)$
$= (2x - 3)(3x + 1)$
zeros are $\frac{3}{2}, \frac{-1}{3}$
Sum of zeros $= \frac{3}{2} + \left(\frac{-1}{3}\right)$
$= \frac{9-2}{6} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$
Product of zeros
$= \frac{3}{2} \times \left(\frac{-1}{3}\right) = \frac{-1}{2} = \frac{-(3)}{6}$
$= \frac{(\text{constant term})}{(\text{coefficient of } x^2)}$

(iv) $4u^2 + 8u = 4u(u + 2)$

zeros are 0, –2

Sum of zeros

$= 0 + (-2) = -2 = \frac{-(8)}{4}$

$= -\frac{(\text{coefficient of } u)}{(\text{coefficient of } u^2)}$

Product of zeros

$= 0 \times (-2) = 0 = \frac{0}{4}$

$= \frac{\text{constant term}}{\text{coefficient of } u^2}$

(v) $t^2 - 15 = (t - \sqrt{15})(t + \sqrt{15})$

zeros are $\sqrt{15}$, $-\sqrt{15}$

sum of zeros

$= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{0}{1}$

$= -\frac{(\text{coefficient of } t)}{(\text{coefficient of } t^2)}$

Product of zeros

$= \left(\sqrt{15}\right)\left(-\sqrt{15}\right) = -15 = \frac{-15}{1}$

$= \frac{\text{constant term}}{\text{coefficient of } t^2}$

(vi) $3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

zeros are $\frac{4}{3}, -1$

Sum of zeros

$= \frac{4}{3} - 1 = \frac{1}{3} = -\frac{(-1)}{3}$

$= -\frac{(\text{coefficient of } x)}{\text{coefficient of } x^2}$

Product of zeros $= \frac{4}{3} \times (-1) = -\frac{4}{3}$

$= \frac{(\text{constant term})}{\text{coefficient of } x^2}$

**Q2.** Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

(i) $\frac{1}{4}, -1$ (ii) $\sqrt{2}, \frac{1}{3}$

(iii) $0, \sqrt{5}$ (iv) 1, 1

(v) $-\frac{1}{4}, \frac{1}{4}$ (vi) 4, 1

**Sol.** (i) Required polynomial =
$x^2$ – (sum of zeros) x + product of zeros

$= x^2 - \frac{1}{4}x - 1$

$= \frac{1}{4}(4x^2 - x - 1)$.

(ii) Required polynomial =
$x^2$ – (sum of zeros) x + product of zeros

$= x^2 - \sqrt{2}\,x + \frac{1}{3}$

$= \frac{1}{3}(3x^2 - 3\sqrt{2}\,x + 1)$.

(iii) Required polynomial =
$x^2$ – (sum of zeros) x + product of zeros

$= x^2 - 0\,x + \sqrt{5}$

$= x^2 + \sqrt{5}$.

(iv) Required polynomial =
$x^2$ – (sum of zeros) x + product of zeros

$= x^2 - 1\,x + 1$

$= x^2 - x + 1$.

(v) Required polynomial =
$x^2$ – (sum of zeros) x + product of zeros

$= x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}$

$= x^2 + \frac{1}{4}x + \frac{1}{4}$

$= \frac{1}{4}(4x^2 + x + 1)$.

(vi) Required polynomial =
$x^2$ – (sum of zeros) x + product of zeros $= x^2 - 4x + 1$.

# EX-2.3

**Q1.** Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$.

**Sol.** (i)

$$
\begin{array}{r|l}
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \;\big(\; q(x) = (x - 3) \\
 & \underline{\overset{}{\underset{-}{x^3}} \qquad\quad \underset{+}{-2x}} \\
 & -3x^2 + 7x - 3 \\
 & \underline{\underset{+}{-3x^2} \qquad\quad \underset{-}{+6}} \\
 & r(x) = (7x - 9)
\end{array}
$$

Hence, Quotient $q(x) = x - 3$ and Remainder $r(x) = 7x - 9$

(ii)

$$
\begin{array}{r|l}
 & q(x) = x^2 + x - 3 \\
x^2 - x + 1 & x^4 - 3x^2 + 4x + 5 \\
 & \underline{\underset{-}{x^4} \underset{-}{+ x^2} \qquad \underset{\mp}{} x^3} \\
 & x^3 - 4x^2 + 4x + 5 \\
 & \underline{\underset{-}{x^3} \underset{+}{- x^2} \underset{-}{+ x}} \\
 & -3x^2 + 3x + 5 \\
 & \underline{\underset{+}{-3x^2} \underset{-}{+ 3x} \underset{+}{- 3}} \\
 & r(x) = 8
\end{array}
$$

Hence, Quotient, $q(x) = x^2 + x - 3$

and remainder, $r(x) = 8$

(iii)

$$
\begin{array}{r|l}
 & q(x) = -x^2 - 2 \\
-x^2 + 2 & x^4 - 5x + 6 \\
 & \underline{\underset{-}{x^4} \qquad\qquad \mp 2x^2} \\
 & 2x^2 - 5x + 6 \\
 & \underline{\underset{-}{2x^2} \qquad\quad \mp 4} \\
 & r(x) = -5x + 10
\end{array}
$$

Hence, Quotient, $q(x) = -x^2 - 2$

Remainder, $r(x) = -5x + 10$

**Q2.** Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1,\ x^5 - 4x^3 + x^2 + 3x + 1$

**Sol.** (i)

$$q(t) = 2t^2 + 3t + 4$$

$$\begin{array}{r|l}
t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & \underline{\overset{-}{2t^4} \qquad \overset{+}{-6t^2}} \\
 & 3t^3 + 4t^2 - 9t - 12 \\
 & \underline{\overset{-}{3t^3} \qquad \overset{+}{-9t}} \\
 & 4t^2 - 12 \\
 & \underline{\overset{-}{4t^2} \overset{+}{-12}} \\
 & \text{Remainder} = 0
\end{array}$$

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii)

$$q(x) = 3x^2 - 4x + 2$$

$$\begin{array}{r|l}
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & \underline{\overset{-}{3x^4} \overset{-}{+} 9x^3 \overset{-}{+} 3x^2} \\
 & -4x^3 - 10x^2 + 2x + 2 \\
 & \underline{\overset{+}{-} 4x^3 \overset{+}{-} 12x^2 \overset{+}{-} 4x} \\
 & 2x^2 + 6x + 2 \\
 & \underline{\overset{-}{2x^2} \overset{-}{+} 6x \overset{-}{+} 2} \\
 & \text{Remainder} = 0
\end{array}$$

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii)

$$q(x) = x^2 - 1$$

$$\begin{array}{r|l}
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & \underline{\overset{-}{x^5} \overset{+}{-} 3x^3 \overset{-}{+} x^2} \\
 & -x^3 + 3x + 1 \\
 & \underline{\overset{+}{-} x^3 \overset{-}{+} 3x \overset{+}{-} 1} \\
 & \text{Remainder} = 2
\end{array}$$

Hence, $x^2 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$

**Q3.** Obtain all other zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

**Sol.** Two of the zeros of $3x^4 + 6x^3 - 2x^2 - 10\,x - 5$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$$\Rightarrow \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)$$

is a factor of the polynomial.

i.e., $x^2 - \frac{5}{3}$ is a factor.

i.e., $(3x^2 - 5)$ is a factor of the polynomial. Then we apply the division algorithm as below :

$$\begin{array}{r|l}
 & q(x) = x^2 + 2x + 1 \\ \hline
3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & \underline{\underset{-}{3x^4} \qquad \underset{+}{-5x^2}} \\
 & 6x^3 + 3x^2 - 10x - 5 \\
 & \underline{\underset{-}{6x^3} \qquad \underset{+}{-10x}} \\
 & 3x^2 - 5 \\
 & \underline{\underset{-}{3x^2} \underset{+}{-5}} \\
 & \times
\end{array}$$

The other two zeros will be obtained from the quadratic polynomial $q(x) = x^2 + 2x + 1$

Now $x^2 + 2x + 1 = (x + 1)^2$.

Its zeros are $-1, -1$.

Hence, all other zeros are $-1, -1$.

**Q4.** On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).

**Sol.** $(x^3 - 3x^2 + x + 2) = g(x) \times (x - 2) + (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 + 2x + -4 = g(x) \times (x - 2)$

$\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x) \times (x - 2)$

$$g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x-2 & x^3 - 3x^2 + 3x - 2 \\ & \underline{\overset{-}{}x^3 \mp 2x^2} \\ & \quad -x^2 + 3x - 2 \\ & \quad \underline{\mp x^2 \pm 2x} \\ & \qquad\quad x - 2 \\ & \qquad\quad \underline{x \mp 2} \\ & \qquad\qquad 0 \end{array}$$

So, $g(x) = x^2 - x + 1$

**Q5.** Give examples of polynomials p(x), g(x),q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)

(ii) deg q(x) = deg r(x)

(iii) deg r(x) = 0.

**Sol.** (i) $p(x) = 2x^2 + 2x + 8$, $g(x) = 2x^0 = 2$;

$q(x) = x^2 + x + 4$ ; $r(x) = 0$

(ii) $p(x) = 2x^2 + 2x + 8$ ; $g(x) = x^2 + x + 9$ ;

$q(x) = 2$ ; $r(x) = -10$

(iii) $p(x) = x^3 + x + 5$ ; $g(x) = x^2 + 1$ ;

$q(x) = x$ ; $r(x) = 5$.