Ex - 2.3

Q1. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by :

(i) $x + 1$ (ii) $x - \frac{1}{2}$ (iii) x

(iv) $x + \pi$ (v) $5 + 2x$

Sol. (i) $x + 1$

$x + 1 = 0 \Rightarrow x = -1$

$\therefore$ Remainder $= p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1 + 3 - 3 + 1 = 0$

(ii) $x - \frac{1}{2}$

$x - \frac{1}{2} = 0 \Rightarrow x = \frac{1}{2}$

$\therefore$ Remainder $= p\left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1 = \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1$

$= \frac{27}{8}$

(iii) x

Remainder $= p(0)$

$= 1(0)^3 + 3(0)^2 + 3(0) + 1 = 1$

(iv) $x + \pi$

$x + \pi = 0 \Rightarrow x = -\pi$

$\therefore$ Remainder $= p(-\pi)$

$= 1(-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

(v) $5 + 2x$

$5 + 2x = 0 \Rightarrow x = -5/2$

$\therefore$ Remainder $= p(-5/2)$

$= \left(\frac{-5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1$

$= \frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = -\frac{27}{8}$

Q2. Find the remainder when $x^3 - ax^2 + 6x - a$ divided by $x - a$.

Sol. Let $p(x) = x^3 - ax^2 + 6x - a$

$x - a = 0 \Rightarrow x = a$

$\therefore$ Remainder $= (a)^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a = 5a$

Q3. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$

Sol. $7 + 3x$ will be a factor of $3x^3 + 7x$ only if

$7 + 3x$ divides $3x^3 + 7x$ leaving 0 as remainder.

Let $p(x) = 3x^3 + 7x$

$7 + 3x = 0 \Rightarrow 3x = -7 \Rightarrow x = -7/3$

$\therefore$ Remainder

$$3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right) = \frac{-343}{9} - \frac{49}{3} = \frac{-490}{9} \neq 0$$

so, $7 + 3x$ is not a factor of $3x^3 + 7x$.