Ex - 2.5

Q1. Use suitable identities to find the following products :

(i) $(x + 4)(x + 10)$ (ii) $(x + 8)(x - 10)$ (iii) $(3x + 4)(3x - 5)$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$ (v) $(3 - 2x)(3 + 2x)$

Sol. (i) $(x + 4)(x + 10)$

$= x^2 + (4 + 10)x + (4)(10) = x^2 + 14x + 40$

(ii) $(x + 8)(x - 10)$

$= (x + 8)\{x + (-10)\}$

$= x^2 + \{8 + (-10)\}x + 8(-10)$

$= x^2 - 2x - 80$

(iii) $(3x + 4)(3x - 5)$

$= (3x + 4)(3x - 5) = (3x + 4)(3x + (-5))$

$= (3x)^2 + \{4 + (-5)\}(3x) + 4(-5)$

$= 9x^2 - 3x - 20$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

Let, $y^2 = x$

$\Rightarrow \left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = \left(x + \frac{3}{2}\right)\left(x - \frac{3}{2}\right)$

$= x^2 - \frac{9}{4}$

(using idenity) $(a + b)(a - b) = a^2 - b^2$

$\Rightarrow (y^2)^2 - \frac{9}{4}$

$\Rightarrow y^4 - \frac{9}{4}$

(v) $(3 - 2x)(3 + 2x)$

$(3)^2 - (2x)^2 = 9 - 4x^2$

(using idenity) $(a + b)(a - b) = a^2 - b^2$

Q2. Evaluate the following product without multiplying directly :

(i) 103×107 (ii) 95×96 (iii) 104×96

Sol. (i) $103 \times 107 = (100 + 3) \times (100 + 7)$

$= (100)^2 + (3 + 7)(100) + (3)(7)$

$= 10000 + 1000 + 21 = 11021$

Alternate solution :

$103 \times 107 = (105 - 2) \times (105 + 2)$

$= (105)^2 - (2)^2 = (100 + 5)^2 - 4$

$= (100)^2 + 2(100)(5) + (5)^2 - 4$

$= 10000 + 1000 + 25 - 4$

$= 11021.$

(ii) 95×96

$= (90 + 5) \times (90 + 6)$

$= (90)^2 + (5 + 6)\,90 + (5)(6)$

$= 8100 + 990 + 30 = 9120$

(iii) 104×96

$= (100 + 4) \times (100 - 4)$

(using idenity) $(a + b)(a - b) = a^2 - b^2$

$= (100)^2 - (4)^2 = 10000 - 16$

$= 9984$

Q3. Factorise the following using appropriate identities :

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Sol. (i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)^2$

$= (3x + y)(3x + y)$

(ii) $4y^2 - 4y + 1$

$= (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)^2 = (2y - 1)(2y - 1)$

(iii) $x^2 - \frac{y^2}{100}$

(using idenity) $a^2 - b^2 = (a + b)(a - b)$

$$x^2 - \left(\frac{y}{10}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$$

Q4. Expand each of the following using suitable identities :

(i) $(x + 2y + 4z)^2$ (ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$ (iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$ (vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Sol. (i) $(x + 2y + 4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y)$

$2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x - y + z)^2$

$= (2x - y + z)(2x - y + z)$

$= (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(-2x)(2z) + 2(3y)(2z)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy - 8xz + 12yz$

(iv) $(3a - 7b - c)^2 = (3a - 7b - c)(3a - 7b - c)$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) +$

$2(3a)(-c) + 2(-7b)(-c)$

$= 9a^2 + 49b^2 + c^2 - 42ab - 6ac + 14bc$

(v) $(-2x + 5y - 3z)^2$

$= (-2x + 5y - 3z)(-2x + 5y - 3z)$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) +$

$2(-2x)(-3z) + 2(-3z)(5y)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy + 12xz - 30yz$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

$= \left(\frac{1}{4}a - \frac{1}{2}b + 1\right)\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)$

$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(\frac{1}{4}a\right)(1)^2 + 2\left(-\frac{1}{2}b\right)(1)$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

Q5. Factorise :

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8zx$

Sol. (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(-2x)$

$= \{2x + 3y + (-4z)\}^2 = (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8zx$

$= \left(-\sqrt{2}x\right)^2 + y^2 + \left(2\sqrt{2}z\right)^2 + 2\left(-\sqrt{2}x\right)y + 2y\left(2\sqrt{2}z\right) + 2\left(2\sqrt{2}z\right)\left(-\sqrt{2}x\right)$

$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$

Q6. Write the following cubes in expanded form :

(i) $(2x + 1)^3$ (ii) $(2a - 3b)^3$

(iii) $\left[\frac{3}{2}x + 1\right]^3$ (iv) $\left[x - \frac{2}{3}y\right]^3$

Sol. (i) $(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1)$

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$

$= 8a^3 - 27b^3 - 18ab(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $\left[\frac{3}{2}x + 1\right]^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x + 1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$

$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(iv) $\left(x - \frac{2}{3}y\right)^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3x\left(\frac{2}{3}y\right)\left(x - \frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2xy\left(x - \frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

Q7. Evaluate the following using suitable identities :

(i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$

Sol. (i) $(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208.$

(iii) $(998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$

$= 1000000000 - 8 - 6000(1000 - 2)$

$= 994011992$

Q8. Factorise each of the following :

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Sol. (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3(2a)(b)(2a + b)$

$= (2a + b)^3 = (2a + b)(2a + b)(2a + b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 + (-b)^3 + 3(2a)^2(-b) + 3(2a)(-b)^2$

$= (2a - b)^3$

(iii) $27 - 125a^3 - 135a + 225a^2$

$= 3^3 - (5a)^3 - 3(3)(5a)(3 - 5a)$

$= (3 - 5a)^3$

(iv) $64a^3 - 27b^3 - 144a^2b + 180ab^2$

$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$

$= (4a - 3b)^3$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

$= \left(3p - \frac{1}{6}\right)^3 = \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

Q9. Verify : (i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Sol. (i) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)\{(x + y)^2 - 3xy\}$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 + 2xy + y^2 - 3xy)$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$\Rightarrow x^3 - y^3 = (x - y)^3 + 3xy(x - y)$

$\Rightarrow x^3 - y^3 = (x - y)[(x - y)^2 + 3xy]$

$\Rightarrow x^3 - y^3 = (x - y)[x^2 + y^2 - 2xy + 3xy]$

$\Rightarrow x^3 - y^3 = (x - y)[x^2 + y^2 + xy]$

Q10. Factorise each of the following :

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Sol. (i) $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)\{(3y)^2 - (3y)(5z) + (5z)^2\}$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(ii) $64m^3 - 343n^3$

$= (4m)^3 - (7n)^3$

$= [4m - 7n][16m^2 + 4m.7n + (7n)^2]$

$= (4m - 7n)[16m^2 + 28mn + 49n^2]$

Q11. Factorise : $27x^3 + y^3 + z^3 - 9xyz$

Sol. $27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)((3x)^2 + (y)^2 + (z)^2 - (3x)(y) - (y)(z) - (z)(3x))$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

Q12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Sol. $\frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}(x + y + z)[(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2zx + x^2)$

$= \frac{1}{2}(x + y + z)[2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx]$

$= \frac{1}{2}(x + y + z)\, 2(x^2 + y^2 + z^2 - xy - yz - zx]$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx]$

$= x^3 + y^3 + z^3 - 3xyz$

Q13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

Sol. We know

$x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$x + y + z = 0$ [given]

$\Rightarrow (0)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= 0$

or $x^3 + y^3 + z^3 = 3xyz$

Q14. Without actually calculating the cubes, find the value of each of the following :

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Sol. (i) $(-12)^3 + (7)^3 + (5)^3$

$= \{(-12)^3 + (7)^3 + (5)^3 - 3(-12)(7)(5)\} + 3(-12)(7)(5)$

$= (-12 + 7 + 5)\{(-12)^2 + (7)^2 + (5)^2 - (-12)(7) - (7)(5) - (5)(-12)\} + 3(-12)(7)(5)$

$= 0 + 3(-12)(7)(5) = -1260$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

$\because 28 - 15 - 13 = 0$

$(28)^3 + (-15)^3 + (-13)^3$

$= 3(28)(-15)(-13) = 16380$

(using identity)

if $a + b + c = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given :

(i) Area : $25a^2 - 35a + 12$

(i) Area : $35y^2 + 13y - 12$

Sol. (i) Area $= 25a^2 - 35a + 12$

$= 25a^2 - 20a - 15a + 12$

$= 5a(5a - 4) - 3(5a - 4)$

$= (5a - 3)(5a - 4)$

Here, Length $= 5a - 3$, Breadth $= 5a - 4$

(ii) $35y^2 + 13y - 12$

$= 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (5y + 4)(7y - 3)$

Here, Length $= 5y + 4$, Breadth $= 7y - 3$.

Q16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2 - 12x$

(ii) Volume : $12ky^2 + 8ky - 20k$

Sol. (i) Volume $= 3x^2 - 12x$

$= 3x(x - 4) = 3 \times x(x - 4)$

$\therefore$ Dimensions are 3 units, x-units and $(x - 4)$ units

(ii) $12ky^2 + 8ky - 20k$

$= 4k(3y^2 + 2y - 5) = 4k(3y^2 + 5y - 3y - 5)$

$= 4k\{y(3y + 5) - 1(3y + 5)\}$

$= 4k(3y + 5)(y - 1)$

$\therefore$ Dimensions of cuboid are $4k$, $3y + 5$, $y - 1$