NCERT SOLUTIONS

Quadratic Equation

Ex - 4.1

Q1. Check whether the following are quadratic equations :

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Sol. (i) $(x + 1)^2 = 2(x - 3)$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 2x - 2x + 1 + 6 = 0$

$\Rightarrow x^2 + 0x + 7 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3 - x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow -x - 2x - 2 + 3 = 0$

$\Rightarrow -3x + 1 = 0$ or $3x - 1 = 0$

It is not of the form $ax^2 + bx + c = 0$

Hence, the given equation is not a quadratic equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow 2x^2 - 5x - 3 = x^2 + 5x$

$\Rightarrow x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0$

Hence, the given equation is a quadratic equation.

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow x^2 - 11x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow$ $x^2 + 3x + 1 = x^2 + 4 - 4x$

$\Rightarrow$ $7x - 3 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(vii) $(x + 2)^3 = 2x (x^2 - 1)$

$\Rightarrow$ $x^3 + 3 \times x \times 2 (x + 2) + 2^3 = 2x (x^2 - 1)$

$\Rightarrow$ $x^3 + 6x (x + 2) + 8 = 2x^3 - 2x$

$\Rightarrow$ $x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

$\Rightarrow$ $- x^3 + 6x^2 + 14x + 8 = 0$

$\Rightarrow$ $x^3 - 6x^2 - 14x - 8 = 0$

It is a cubic equation and not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow$ $x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$\Rightarrow$ $2x^2 - 13x + 9 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

Q2. Represent the following situations in the form of quadratic equations :

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Sol. (i) Let breadth be = x meters

Then length = $(2x + 1)$ meters.

$x \times (2x + 1) = 528$ (Area of the plot)

or $2x^2 + x - 528 = 0$

(ii) Let the consecutive integers be x and x + 1. It is given that their product is 306.

$\therefore$ $x(x + 1) = 306$

$\Rightarrow$ $x^2 + x - 306 = 0$

(iii) Let Rohan's present age = x years

Then present age of Rohan's mother

= (x + 26) years

After 3 years, it is given that

$(x + 3) \times \{(x + 26) + 3\} = 360$

or $(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 32x + 87 = 360$

$\Rightarrow x^2 + 32x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

(iv) Let the speed of train be x km/h.

Time taken to travel 480 km $= \frac{480}{x}$ hrs

In second condition,

let the speed of train = (x – 8) km/h

It is also given that the train will take 3 hours more to cover the same distance.

Therefore, time taken to travel 480 km $= \left(\frac{480}{x} + 3\right)$ hrs

Speed × Time = Distance

$$(x - 8)\left(\frac{480}{x} + 3\right) = 480$$

$$\Rightarrow 480 + 3x - \frac{3840}{x} - 24 = 480$$

$$\Rightarrow 3x - \frac{3840}{x} = 24$$

$\Rightarrow 3x^2 - 24x + 3840 = 0$

$\Rightarrow x^2 - 8x + 1280 = 0$

Ex - 4.2

Q1. Find the roots of the following quadratic equations by factorisation :

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Sol. (i) $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x + 2)(x - 5) = 0$

$\Rightarrow x + 2 = 0$ or $x - 5 = 0$

$\Rightarrow x = -2$ or $x = 5$

Hence, the two roots are -2 and 5.

(ii) $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 3) = 0$

$\Rightarrow x + 2 = 0$ or $2x - 3 = 0$

$\Rightarrow x = -2$ or $x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$\Rightarrow x = -\sqrt{2}$ or $-\frac{5}{\sqrt{2}}$

Hence, the two roots are $-\sqrt{2}$ and $-\frac{5}{\sqrt{2}}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

or $16x^2 - 8x + 1 = 0$

or $(4x - 1)^2 = 0$

$\Rightarrow$ Both roots are given by $4x - 1 = 0$,

i.e., $x = \frac{1}{4}$. Hence, the roots are $\frac{1}{4}, \frac{1}{4}$.

(v) $100x^2 - 20x + 1 = 0$
$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$
$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$
$\Rightarrow (10x - 1)^2 = 0$
$\Rightarrow (10x - 1) = 0$ or $(10x - 1) = 0$
$\Rightarrow x = \frac{1}{10}$ or $x = \frac{1}{10}$

Q2. Represent the following situations mathematically.
(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.
(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

Sol. (i) Let the number of marbles John had be x.
Then the number of marbles Jivanti had
$= 45 - x$ (Why?).
The number of marbles left with John, when he lost 5 marbles $= x - 5$.
The number of marbles left with Jivanti, when she lost 5 marbles $= 45 - x - 5 = 40 - x$.
Therefore, the is product $= (x - 5)(40 - x)$
$= 40x - x^2 - 200 + 5x = -x^2 + 45x - 200$
So, $-x^2 + 45x - 200 = 124$
(Given that product = 124)
i.e., $-x^2 + 45x - 324 = 0$
i.e., $x^2 - 45x + 324 = 0$
Therefore, the number of marbles John had, satisfies the quadratic equation.
$x^2 - 45x + 324 = 0$
which is the required representation of the problem mathematically.

(ii) Let the number of toys produced be x.
$\therefore$ Cost of production of each toy = Rs $(55 - x)$
It is given that, total cost of production of the toys = Rs 750
$\therefore \quad x(55 - x) = 750$
Therefore, $x^2 - 55x + 750 = 0$
which is the required representation of the problem mathematically.

Q3. Find two numbers whose sum is 27 and product is 182.

Sol. Let one number be x, then second number $= 27 - x$
$x \times (27 - x) = 182$
$\Rightarrow 27x - x^2 = 182$
$\Rightarrow x^2 - 27x + 182 = 0$
$\Rightarrow x^2 - 14x - 13x + 182 = 0$

$\Rightarrow$ x (x – 14) – 13 (x – 14) = 0

$\Rightarrow$ (x – 13) (x – 14) = 0

$\Rightarrow$ x = 13 or 14

$\Rightarrow$ 27x = 14 or 13

Hence, the two marbles are 13 and 14.

Q4. Find two consecutive positive integers, sum of whose squares is 365.

Sol. Let the consecutive positive integers be x and x + 1.

Given that $x^2 + (x + 1)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x + 14) - 13(x + 14) = 0$

$\Rightarrow (x + 14)(x - 13) = 0$

Either x + 14 = 0 or x – 13 = 0,

i.e., x = –14 or x = 13

Since the integers are positive, x can only be 13.

$\therefore$ x + 1 = 13 + 1 = 14

Therefore, two consecutive positive integers will be 13 and 14.

Q5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

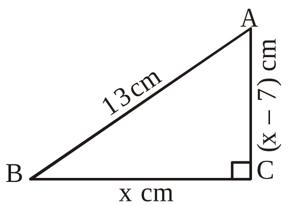
Sol. In ΔABC, base BC = x cm

and altitude AC = (x – 7) cm

$\angle ACB = 90°$

AB = 13 cm

By Pythagoras theorem, we have



$BC^2 + AC^2 = AB^2$

$\Rightarrow x^2 + (x - 7)^2 = 13^2$

$\Rightarrow x^2 + x^2 - 14x + 49 = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow$ $x^2 - 12x + 5x - 60 = 0$

$\Rightarrow$ $x(x - 12) + 5(x - 12) = 0$

$\Rightarrow$ $(x + 5)(x - 12) = 0$

$\Rightarrow$ $x = -5$ or $x = 12$

We reject $x = -5$

$\Rightarrow$ $x = 12$

Therefore, BC = 12 cm and AC = 5 cm.

Q6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production of that day was Rs. 90, find the number of articles produced and the cost of each article.

Sol. Let the number of articles produced be x.

Therefore, cost of production of each article = Rs $(2x + 3)$

It is given that the total production is Rs 90.

$\therefore$ $x(2x + 3) = 90$

$\Rightarrow$ $2x^2 + 3x - 90 = 0$

$\Rightarrow$ $2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow$ $x(2x + 15) - 6(2x + 15) = 0$

$\Rightarrow$ $(2x + 15)(x - 6) = 0$

Either $2x + 15 = 0$ or $x - 6 = 0$,

i.e., $x = \frac{-15}{2}$ or $x = 6$

As the number of articles produced can only be a positive integer,

therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = $2 \times 6 + 3$ = Rs. 15

Ex - 4.3

Q1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}\,x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Sol. (i) $2x^2 - 7x + 3 = 0$

$\Rightarrow 4x^2 - 14x + 6 = 0$

$\Rightarrow (2x)^2 - 7(2x) + 6 = 0$

$\Rightarrow y^2 - 7y + 6 = 0$ where $y = 2x$

$$\Rightarrow \left\{y^2 - \frac{7}{2}y - \frac{7}{2}y + \left(-\frac{7}{2}\right)^2\right\} - \left(-\frac{7}{2}\right)^2 + 6 = 0$$

$$\Rightarrow \left\{y - \frac{7}{2}\right\}^2 - \frac{49}{4} + 6 = 0 \Rightarrow \left\{y - \frac{7}{2}\right\}^2 - \frac{25}{4} = 0$$

$$\Rightarrow \left(2x - \frac{7}{2}\right)^2 = \frac{25}{4} \Rightarrow 2x - \frac{7}{2} = \pm\sqrt{\frac{25}{4}}$$

$$\Rightarrow 2x - \frac{7}{2} = \pm\frac{5}{2}$$

$$\Rightarrow 2x - \frac{7}{2} = \frac{5}{2} \text{ or } 2x - \frac{7}{2} = -\frac{5}{2}$$

$$\Rightarrow 2x = \frac{5}{2} + \frac{7}{2} \text{ or } 2x = -\frac{5}{2} + \frac{7}{2}$$

$\Rightarrow 2x = 6$ or $2x = 1$

Hence, the roots of the given quadratic equation are $\frac{1}{2}$ and 3.

(ii) $2x^2 + x - 4 = 0$

$\Rightarrow 2x^2 + x = 4$

On dividing both sides of the equation by 2,

we obtain $\Rightarrow x^2 + \frac{1}{2}x = 2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$$

$$\Rightarrow x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$$

$$\Rightarrow x = \frac{\pm\sqrt{33}}{4} - \frac{1}{4} \quad \Rightarrow x = \frac{\sqrt{33} - 1}{4} \text{ or } \frac{-\sqrt{33} - 1}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + \left(\sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x + \sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x + \sqrt{3}\right) = 0$ and $\left(2x + \sqrt{3}\right) = 0$

$\Rightarrow x = \frac{-\sqrt{3}}{2}$ and $x = \frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

$\Rightarrow 2x^2 + x = -4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = -2$

$\Rightarrow x^2 + 2 \times x \times \frac{1}{4} = -2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$

However, the square of a number cannot be negative. Therefore, there is no real root for the given equation.

Q2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Sol. (i) $2x^2 - 7x + 3 = 0$

$a = 2, b = -7, c = 3$

Discriminant $D = (-7)^2 - 4(2)(3) = 25$

$\Rightarrow \sqrt{D} = \sqrt{25} = 5$

Roots of the given quadratic equation are

$\frac{-b \pm \sqrt{D}}{2a}$,

i.e., $\frac{7 \pm 5}{2 \times 2}$, i.e., the roots are 3 and $\frac{1}{2}$.

(ii) $2x^2 + x - 4 = 0$

$a = 2, b = 1, c = -4$

$D = (1)^2 - 4(2)(-4) = 33 \Rightarrow \sqrt{D} = \sqrt{33}$

Roots are given by $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{33}}{4}$

Hence, the two roots of the quadratic equation are $\frac{-1 \pm \sqrt{33}}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 4$, $b = 4\sqrt{3}$, $c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm 0}{8}$$

$$\therefore \quad x = \frac{-\sqrt{3}}{2} \text{ or } \frac{-\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

On comparing this equation with

$ax^2 + bx + c = 0$,

we obtain $a = 2$, $b = 1$, $c = 4$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \quad x = \frac{-1 \pm \sqrt{1 - 32}}{4}$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{-31}}{4}$$

However, the square of a number cannot be negative. Therefore, there is no real root for the given equation.

Q3. Find the roots of the following equations :

(i) $x - \frac{1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Sol. (i) $x - \frac{1}{x} = 3 \quad \Rightarrow \quad x^2 - 3x - 1 = 0$

On comparing this equation with

$ax^2 + bx + c = 0$,

we obtain $a = 1$, $b = -3$, $c = -1$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \Rightarrow \quad x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$\Rightarrow \quad x = \frac{3 \pm \sqrt{13}}{2}$$

Therefore, $x = \frac{3 + \sqrt{13}}{2}$ or $\frac{3 - \sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$; $x \neq -4$ and $x \neq 7$

$\Rightarrow \frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30} \Rightarrow \frac{-11}{(x^2-3x-28)} = \frac{11}{30}$

$\Rightarrow -(x^2 - 3x - 28) = 30$

$\Rightarrow x^2 - 3x - 28 + 30 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$a = 1, b = -3, c = 2$

$D = (-3)^2 - 4(1)(2) = 1 \Rightarrow \sqrt{D} = 1$

The two roots are given by $\frac{-b \pm \sqrt{D}}{2a}$, i.e., $\frac{3 \pm 1}{2}$.

Hence, the two roots are 1 and 2.

Q4. The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

Sol. Let the present age of Rehman be x years.

We are given that $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow 3(2x + 2) = (x - 3)(x + 5)$

$\Rightarrow 6x + 6 = x^2 + 2x - 15$

$\Rightarrow x^2 - 4x - 21 = 0$

$a = 1, b = -4, c = -21$

$D = (-4)^2 - 4(1)(-21) = 16 + 84 = 100$

$\Rightarrow \sqrt{D} = \sqrt{100} = 10$

Then $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{4 \pm 10}{2}$, i.e., $x = 7, -3$

We reject $x = -3$ ($\because$ x cannot be negative)

$\Rightarrow x = 7$

Hence, Rehman's present age = 7 years.

Q5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Sol. Let the marks in Maths be x.

Then, the marks in English will be 30 – x.

According to the given question,

$(x + 2)(30 - x - 3) = 210$

$(x + 2)(27 - x) = 210$

$\Rightarrow$ $-x^2 + 25x + 54 = 210$

$\Rightarrow$ $x^2 - 25x + 156 = 0$

$\Rightarrow$ $x^2 - 12x - 13x + 156 = 0$

$\Rightarrow$ $x(x - 12) - 13(x - 12) = 0$

$\Rightarrow$ $(x - 12)(x - 13) = 0$

$\Rightarrow$ $x = 12, 13$

If the marks in Maths are 12, then marks in English will be 30 – 12 = 18

If the marks in Maths are 13, then marks in English will be 30 – 13 = 17.

Q6. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

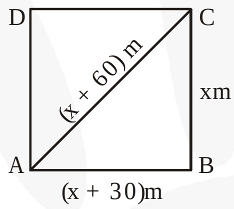
Sol. In rectangle ABCD, let the shorter side BC = x metres.

Then AB = (x + 30) metres and diagonal

AC = (x + 60) metres.

By Pythagoras Theorem we have

$AB^2 + BC^2 = AC^2$



$\Rightarrow$ $(x + 30)^2 + x^2 = (x + 60)^2$

$\Rightarrow$ $x^2 + 60x + 900 + x^2 = x^2 + 120x + 3600$

$\Rightarrow$ $x^2 - 60x - 2700 = 0$

$a = 1, b = -60, c = -2700$

$D = (-60)^2 - 4 \times 1 \times (-2700)$

$= 3600 + 10800 = 14400$

$\Rightarrow$ $\sqrt{D} = \sqrt{14400} = 120$

Then $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{60 \pm 120}{2} = 90, -30$

We reject $x = -30$ $(\because x \nless 0)$

$\Rightarrow$ $x = 90$

$\Rightarrow$ BC = 90 m and AB = (90 + 30) m = 120 m.

Hence, the sides of the rectangular field are 90 m and 120 m.

Q7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Sol. Let the larger and smaller number be x and y respectively.

According to the given question,

$x^2 - y^2 = 180$ and $y^2 = 8x$

$\Rightarrow$ $x^2 - 8x = 180$

$\Rightarrow$ $x^2 - 8x - 180 = 0$

$\Rightarrow$ $x^2 - 18x + 10x - 180 = 0$

$\Rightarrow$ $x(x - 18) + 10(x - 18) = 0$

$\Rightarrow$ $(x - 18)(x + 10) = 0$

$\Rightarrow$ $x = 18, -10$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible. Therefore, the larger number will be 18 only.

$x = 18$

$\therefore$ $y^2 = 8x = 8 \times 18 = 144$

$\Rightarrow$ $y = \pm\sqrt{144} = \pm 12$

$\therefore$ Smaller number $= \pm 12$

Therefore, the numbers are 18 and 12 or 18 and – 12.

Q8. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken I hour less for the same journey. Find the speed of the train.

Sol. Let the speed of the train be x km/hr.

We are given that 360 km distance is to be travelled at uniform speed of x km/hr.

Time taken to cover the distance of

360 km $= \dfrac{360}{x}$ hours.

In case, the speed is increased by 5 km/hr, the time required to cover 360 km $= \dfrac{360}{(x+5)}$ hours.

Now, $\dfrac{360}{x} - \dfrac{360}{x+5} = 1$

$\Rightarrow 360 \times \left\{\dfrac{1}{x} - \dfrac{1}{x+5}\right\} = 1$

$\Rightarrow \dfrac{360 \times 5}{x \times (x+5)} = 1$

$\Rightarrow$ $x \times (x + 5) = 360 \times 5$

$\Rightarrow$ $x^2 + 5x = 1800$ $\Rightarrow$ $x^2 + 5x - 1800 = 0$

$a = 1, b = 5, c = -1800$

$D = b^2 - 4ac = (5)^2 - 4 \times 1 \times (-1800)$

$= 25 + 7200 = 7225$

$\Rightarrow \sqrt{D} = \sqrt{7225} = 85$

Then x = $\frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm 85}{2}$

$\Rightarrow$ x = $\frac{-5 \pm 85}{2}$ or $\frac{-5 - 85}{2}$

$\Rightarrow$ x = 40 or – 45   We reject x = – 45

$\Rightarrow$ x = 40

Q9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol. Let the time taken by the smaller pipe to fill the tank be x hr.
Time taken by the larger pipe = (x - 10) hr

Part of tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together.

Therefore,

$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

$\frac{x-10+x}{x(x-10)} = \frac{8}{75}$

$\Rightarrow$ $75(2x - 10) = 8x^2 - 80x$

$\Rightarrow$ $150x - 750 = 8x^2 - 80x$

$\Rightarrow$ $8x^2 - 230x + 750 = 0$

$\Rightarrow$ $8x^2 - 200x - 30x + 750 = 0$

$\Rightarrow$ $8x(x - 25) - 30(x - 25) = 0$

$\Rightarrow$ $(x - 25)(8x - 30) = 0$

i.e., x = 25, $\frac{30}{8}$

Time taken by the smaller pipe cannot be $\frac{30}{8}$ = 3.75 hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible.
Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 – 10 = 15 hours respectively.

Q10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop in intermediate stations). If the average speed of the express train is 11 km/hr more than that of the passenger train, find the average speed of the two trains.

Sol. Let the average speed of the passenger train be x km/hr.
Then the average speed of express train = (x + 11) km/hr.
According to the given condition :

$$\begin{pmatrix}\text{Time taken by passenger train}\\ \text{for 132 km journey}\end{pmatrix}$$

$$\begin{pmatrix}\text{Time taken by the express train}\\ \text{for 132 km journey}\end{pmatrix} = 1 \text{ hour}$$

$$\Rightarrow \left(\frac{132}{x} - \frac{132}{x+11}\right) = 1$$

$$\Rightarrow 132 \times \left\{\frac{1}{x} - \frac{1}{x+11}\right\} = 1$$

$$\Rightarrow 132 \times \frac{11}{x \times (x+11)} = 1$$

$\Rightarrow 132 \times 11 = x \times (x + 11)$

$\Rightarrow 1452 = x^2 + 11x \Rightarrow x^2 + 11x - 1452 = 0$

$\Rightarrow a = 1, b = 11, c = -1452$

$D = b^2 - 4ac = 121 - 4 \times 1 \times (-1452)$

$= 121 + 5808 = 5929$

$$\Rightarrow \sqrt{D} = \sqrt{5929} = 77$$

Then $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-11 \pm 77}{2} = -44$ or 33

We reject $x = -44 \quad \Rightarrow \quad x = 33$

Hence, the speed of passenger train = 33 km/hr and the speed of express train = 44 km/hr

Q11. Sum of the area of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Sol. Let the sides of the two squares be x m and y m. Therefore, their perimeter will be 4x and 4y respectively and their areas will be x^2 and y^2 respectively.

It is given that $4x - 4y = 24$

$x - y = 6$

$x = y + 6$

Also, $x^2 + y^2 = 468$

$\Rightarrow (6 + y)^2 + y^2 = 468$

$\Rightarrow 36 + y^2 + 12y + y^2 = 468$

$\Rightarrow 2y^2 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y + 18)(y - 12) = 0$

$\Rightarrow y = -18$ or 12.

However, side of a square cannot be negative.

Hence, the sides of the squares are 12 m and

$(12 + 6)$ m = 18 m

Ex - 4.4

Q1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them :

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}\,x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Sol. (i) $2x^2 - 3x + 5 = 0$

$a = 2, b = -3, c = 5$

Discriminant $D = b^2 - 4ac = 9 - 4 \times 2 \times 5$

$= 9 - 40 = -31$

$\Rightarrow D < 0$

Hence, no real root.

(ii) $3x^2 - 4\sqrt{3}\,x + 4 = 0$

$a = 3, b = -4\sqrt{3}, c = 4$

Discriminant $D = b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$

$\Rightarrow D = 0$

$\Rightarrow$ Two roots are equal.

The roots are

$$= \frac{-b \pm \sqrt{D}}{2a} = \frac{4\sqrt{3} \pm 0}{2 \times 3} = \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$$

Hence, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) $2x^2 - 6x + 3 = 0$

$a = 2, b = -6, c = 3$

Discriminant $D = b^2 - 4ac = (-6)^2 - 4(2)(3)$

$= 36 - 24 = 12$

As $D > 0$,

Therefore, roots are distinct and real.

The roots are

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the roots are $\frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$.

Q2. Find the values of k for each of the following quadratic equations, so that they have two real equal roots.
(i) $2x^2 + kx + 3 = 0$
(ii) $kx(x - 2) + 6 = 0$

Sol. (i) $2x^2 + kx + 3 = 0$
$a = 2, b = k, c = 3$
$D = b^2 - 4ac = k^2 - 4 \times 2 \times 3 = k^2 - 24$
Two roots will be equal
if $D = 0$, i.e., if $k^2 - 24 = 0$
i.e., if $k^2 - 24$, i.e., if $k = \pm\sqrt{24}$
i.e., if $k = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$
or $kx^2 - 2kx + 6 = 0$
$a = k, b = -2k, c = 6$
Discriminant $D = b^2 - 4ac = (-2k)^2 - 4(k)(6)$
$= 4k^2 - 24k$
Two roots will be equal
if $D = 0$,
$4k^2 - 24k = 0$
$4k(k - 6) = 0$
Either $4k = 0$ or $k - 6 = 0$
$k = 0$ or $k = 6$
However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.
Hence $k = 6$.

Q3. Is it possible to design a rectangular mango grove whose length is twice its breadth,
and area is 800 m^2? If so, find its length and breadth.

Sol. Let x be the breadth and 2x be the length of the rectangle.
$x \times 2x = 800$
$\Rightarrow 2x^2 = 800$
$\Rightarrow x^2 = 400 = (20)^2$
$\Rightarrow x = 20$
Hence, the rectangle is possible and it has breadth = 20 m and length = 40 m.

Q4. Is the following situation possible? If so, determine their present ages.
The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in year was 48.

Sol. Let the age of one friend be x years.
Age of the other friend will be $(20 - x)$ years.
4 years ago, age of 1st friend $= (x - 4)$ years
And, age of 2nd friend $= (20 - x - 4)$
$= (16 - x)$ years
Given that,
$(x - 4)(16 - x) = 48$
$16x - 64 - x^2 + 4x = 48$
$-x^2 + 20x - 112 = 0$
$x^2 - 20x + 112 = 0$
$a = 1, b = -20, c = 112$
Discriminant $D = b^2 - 4ac = (-20)^2 - 4(1)(112)$
$= 400 - 448 = -48$
As $b^2 - 4ac < 0$,
Therefore, no real root is possible for this equation and hence, this situation is not possible.

Q5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

Sol. Perimeter of the rectangular park = 80 m
$\Rightarrow$ Length + Breath of the park $= \frac{80}{2}$ m = 40 m.
Let the breadth be x metres, then length
$= (40 - x)$ m.
Here, $x < 40$.
$x \times (40 - x) = 400$ [Each = area of the park]
i.e., $-x^2 + 40x - 400 = 0$
i.e., $x^2 - 40x + 400 = 0$
i.e., $(x - 20)^2 = 0$
$\Rightarrow x = 20$
Thus, we have length = breadth = 20 m

Therefore, the park is a square having 20 m side.