# NCERT SOLUTIONS

## Quadrilaterals

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# Ex - 8.1

**Q1.** The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

**Sol.** Let the four angles of the quadrilateral be
$3x, 5x, 9x$ and $13x$ .

$\therefore\ 3x + 5x + 9x + 13x = 360°$

$\therefore$ [Sum of all the angles of quadrilateral is 360°]

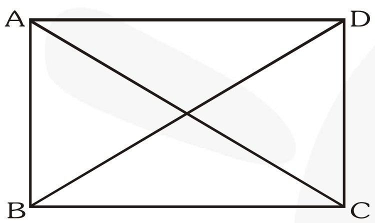
$\Rightarrow 30x = 360°$

$\Rightarrow x = 12°$

Hence, the angles of the quadrilateral are
$3 \times 12° = 36°$, $5 \times 12° = 60°$, $9 \times 12° = 108°$ and $13 \times 12° = 156°$.

**Q2.** If the diagonals of a parallelogram are equal, then show that it is a rectangle.

**Sol.** **Given :** ABCD is a parallelogram with diagonal
AC = diagonal BD




**To prove :** ABCD is a rectangle.

**Proof :** In triangle ABC and ABD,

| | | |
|---|---|---|
| | AB = AB | [Common] |
| | AC = BD | [Given] |
| | AD = BC | [Opp. Sides of a $\|$gm] |
| $\therefore$ | $\Delta ABC \cong BAD$ | [By SSS congruency] |
| $\Rightarrow$ | $\angle DAB = \angle CBA$ | [By C.P.C.T.] ....(i) |

[$\because$ AD$\|$BC and AB cuts them, the sum of the interior angle of the same side of transversal is 180°]

$\angle DAB + \angle CBA = 180°$ .....(ii)

From eq. (i) and (ii), $\angle DAB = \angle CBA = 90°$

Hence, ABCD is a rectangle

**Q3.** Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

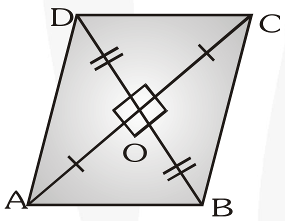
**Sol.** **Given :** ABCD is a quadrilateral where diagonals AC and BD meet at 0, such that AO = OC, OB = OD and AC $\perp$ BD

**To Prove :** Quadrilateral ABCD is a rhombus,

i.e., AB = BC = CD = DA

**Proof :** In $\Delta$AOB and $\Delta$AOD,

| | | |
|---|---|---|
| | OB = OD | [Common] |
| | AO = AO | [Given] |
| | $\angle$AOB = $\angle$AOD | [Each = 90°] |
| $\therefore$ | $\Delta$AOB $\cong$ $\Delta$AOD | [SAS Rule] |
| $\therefore$ | AB = AD | [C.P.C.T.] |




Similarly, we can prove that

AB = BC ...(i)

BC = CD ...(ii)

CD = AD ...(iii)

From (i), (ii), (iii) and (iv), we obtain

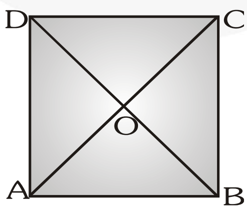
AB = BC = CD = DA

$\therefore$ Quadrilateral ABCD is a rhombus.

**Q4.** Show that the diagonals of a square are equal and bisect each other at right angles.

**Sol.** **Given:** ABCD is a square.

**To Prove :** (i) AC = BD (ii) AC and BD bisect each other at right angles.




**Proof:** In $\Delta$ABC and $\Delta$BAD,

| | |
|---|---|
| AB = BA | [Common] |
| BC = AD | [Opp. sides of square ABCD] |
| $\angle$ABC = $\angle$BAD | [Each = 90° ($\because$ ABCD is a square] |

$\therefore$ $\Delta ABC \cong \Delta BAD$ [SAS Rule]

$\therefore$ $AC = BD$ ... (i) [C.P.C.T.]

In $\Delta AOD$ and $\Delta BOC$

$AD = CB$ [Opp. sides of square ABCD]

$\angle OAD = \angle OCB$

[Alternate angles as $AD \parallel BC$ and transversal AC intersects them]

$\angle ODA = \angle OBC$

[Alternate angles as $AD \parallel BC$ and transversal BD intersects them]

$\Delta AOD \cong \Delta BOC$ [ASA Rule]

$\therefore$ $OA = OC$ and $OB = OD$ ...(ii) [C.P.C.T.]

So, O is the mid point of AC and BD.

Now, In $\Delta AOB$ and $\Delta COB$

$AB = BC$ [Given]

$OA = OC$ [from (ii)]

$OB = OB$ [Common]

$\therefore$ $\Delta AOB \cong \Delta COB$ [By SSS Rule]

$\therefore$ $\angle AOB = \angle BOC$ [C.P.C.T]

But $\angle AOB + \angle BOC = 180°$ [Linear pair]

$\angle AOB + \angle AOB = 180°$

[AOB = BOC proved earlier]

$\Rightarrow$ $2\angle AOB = 180°$

$\Rightarrow$ $\angle AOB = \frac{180°}{2} = 90°$

$\therefore$ $\angle AOB = \angle BOC = 90°$

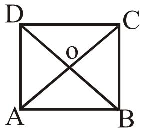
$\therefore$ AC and BD bisect each other at right angles.

**Q5.** Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

**Sol.** **Given :** The diagonals AC and BD of a quadrilateral ABCD are equal and bisect each other at right angles.

**To prove :** Quadrilateral ABCD is a square.

**Proof :**




In $\Delta AOD$ and $\Delta BOC$,

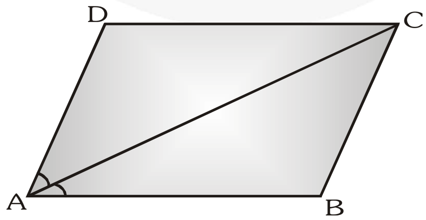
$OA = OC$ [Given]

$OD = OB$ [Given]

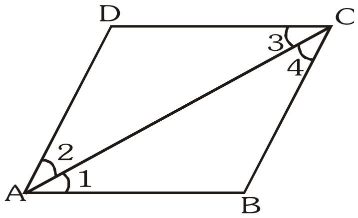
$\angle AOD = \angle COB$ [Vertically Opposite Angles]

| | | |
|---|---|---|
| $\therefore$ | $\Delta AOD \cong \Delta BOC$ | [SAS Rule] |
| $\therefore$ | $AD = BC$ | [C.P.C.T.] |
| | $\angle ODA = \angle OBC$ | [C.P.C.T.] |
| $\therefore$ | $AD \parallel BC$ | |
| | Now, $AD = CB$ and $AD \parallel CB$ | |
| $\therefore$ | Quadrilateral ABCD is a $\parallel$ gm. | |
| | In $\Delta AOB$ and $\Delta AOD$, | |
| | $AO = AO$ | [Common] |
| | $OB = OD$ | [Given] |
| | $\angle AOB = \angle AOD$ | [Each = 90° (Given)] |
| $\therefore$ | $\Delta AOB \cong \Delta AOD$ | [SAS Rule] |
| $\therefore$ | $AB = AD$ | |
| | Now, | |
| $\because$ | ABCD is a parallelogram and $AB = AD$ | |
| $\therefore$ | ABCD is a rhombus. | |
| | Again, in $\Delta ABC$ and $\Delta BAD$, | |
| | $AC = BD$ | [Given] |
| | $BC = AD$ | |
| | [$\because$ ABCD is a Rhombus] | |
| | $AB = BA$ | [Common] |
| $\therefore$ | $\Delta ABC \cong \Delta BAD$ | [SSS rule] |
| $\therefore$ | $\angle ABC = \angle BAD$ | [C.P.C.T.] |
| $\because$ | $AD \parallel BC$ | |
| | [Opposite sides of $\parallel$ gm ABCD ] | |
| | and transversal AB intersects them. | |
| $\therefore$ | $\angle ABC + \angle BAD = 180°$ [Sum of consecutive interior angles on the same side of the transversal is 180°] | |
| $\therefore$ | $\angle ABC = \angle BAD = 90°$ | |
| | Similarly, $\angle BCD = \angle ADC = 90°$ | |
| $\therefore$ | ABCD is a square. | |

**Q6.** In figure, ABCD is a parallelogram. Diagonal AC bisects $\angle A$. Show that
(i) it bisects $\angle C$ also (ii) ABCD is a rhombus.



**Sol.** **Given :**




Diagonal AC bisects $\angle A$ of the parallelogram ABCD.

**To prove :**

(i) AC bisects $\angle C$

(ii) ABCD is a rhombus

**Proof :**

(i) Since AB||DC and AC intersects them.

$\therefore \angle 1 = \angle 3$ [Alternate angles] ...(i)

Similarly $\angle 2 = \angle 4$ ...(ii)

But $\angle 1 = \angle 2$ [Given] ...(iii)

$\therefore \angle 3 = \angle 4$ [Using eq. (i), (ii) and (iii)]

Thus AC bisects $\angle C$.

(ii) $\angle 2 = \angle 3 = \angle 4 = \angle 1$

$\Rightarrow$ AD = CD [Sides opposite to equal angles]

$\therefore$ AB = CD = AD = BC

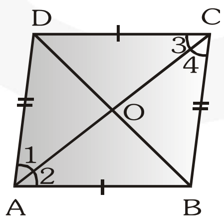
Hence, ABCD is a rhombus.

**Q7.** ABCD is a rhombus. Show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

**Sol.** **Given :** ABCD is a rhombus and AC and BD are its diagonal

**To prove :** (i) Diagonal AC bisects $\angle A$ as well as $\angle C$.

(ii) Diagonal BD bisect $\angle B$ as well as $\angle D$.




**Proof :**

(i) $\therefore$ In $\Delta ABC$

AB = BC (sides of Rhombus)

so, $\angle 2 = \angle 4$ (Angle opposite to equal sides are equal)

But $\angle 2 = \angle 3$ (Alternate angles as AB || CD)

so, $\angle 2 = \angle 3 = \angle 4$

But $\angle 1 = \angle 4$ (Alternate angles as AD || BC)

so, $\angle 1 = \angle 2 = \angle 3 = \angle 4$ ... (1)

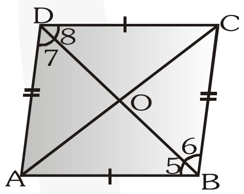
$\angle 1 = \angle 2$ by (1)

so, AC bisect $\angle A$

$\angle 3 = \angle 4$ by (1)

so, AC bisect $\angle C$

(ii) In $\Delta ABD$




AB = AD (Sides of Rhombus)

so, $\angle 5 = \angle 7$ (Angle opposite to equal sides are equal)

But $\angle 7 = \angle 6$ (Alternate angle as AD || BC)

so, $\angle 5 = \angle 6 = \angle 7$

$\angle 5 = \angle 8$ (Alternate angle as AB || CD)

so, $\angle 5 = \angle 6 = \angle 7 = \angle 8$ ... (2)

$\angle 5 = \angle 6$ by (2)

so, BD bisect $\angle B$

$\angle 7 = \angle 8$ by (2)

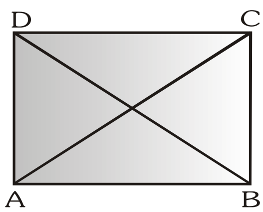
so, BD bisect $\angle D$

**Q8.** ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that

(i) ABCD is a square

(ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

**Sol.** **Given :** ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$.



**To prove :** (i) ABCD is a square

(ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

**Proof :**

(i) $\because$ AB $\|$ DC and transversal AC intersects them.

so, $\angle BAC = \angle DCA$ [ Alternate angles]

But $\angle BAC = \angle DAC$ [$\because$ AC bisects $\angle A$]

$\therefore$ $\angle DCA = \angle DAC$

$\Rightarrow$ DA = CD

[Sides opposite to equal angles of a triangle are equal]

But AB = CD and DA = BC [Opposite side of a rectangle]

$\therefore$ AB = BC = CD = DA

Also $\angle A = \angle B = \angle C = \angle D = 90°$

[$\because$ ABCD is a rectangle]

Hence, ABCD is a square

(ii) In $\Delta$ BAD and $\Delta$ BCD,

BA = BC [$\because$ ABCD is a square]

AD = CD [$\because$ ABCD is a square]

BD = BD [Common]

$\therefore$ $\Delta BAD \cong \Delta BCD$ [By SSS congruence rule]

$\therefore$ $\angle ABD = \angle CBD$ [By C.P.C.T.]

$\angle ADB = \angle CDB$ [By C.P.C.T.]

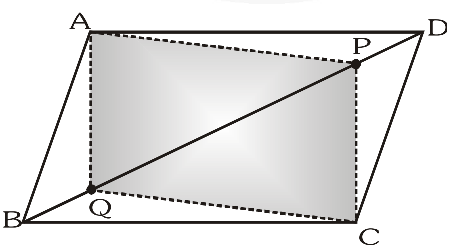
Hence, diagonal BD bisect $\angle B$ as well as $\angle D$

**Q9.** In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ. Show that :

(i) $\Delta APD \cong \Delta CQB$ (ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$ (iv) AQ = CP

(v) APCQ is a parallelogram



**Sol.** (i) In $\Delta APD$ and $\Delta CQB$, we have

$DP = BQ$ [Given]

$AD = CB$

[Opposite sides of parallelogram ABCD]

$\angle ADP = \angle CBQ$ [Pair of alternate angles]

$\Rightarrow$ $\Delta APD \cong \Delta CQB$ [SAS congruence criteria]

(ii) Then, by CPCT, we have $AP = CQ$

(iii) We can prove

$\Delta AQB \cong \Delta CPD$ [as we have done in (i)]

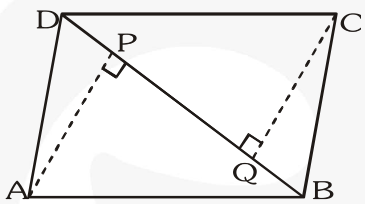
(iv) By CPCT, we have $AQ = CP$

(v) Now, we have $AP = CQ$ and $AQ = CP$

Hence, APCQ is a parallelogram.

**Q10.** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD. Show that

(i) $\Delta APB \cong \Delta CQD$ (ii) $AP = CQ$




**Sol.** **Given :** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively.

**To prove :** (i) $\Delta APB \cong \Delta CQD$ (ii) $AP = CQ$

**Proof :**

(i) In $\Delta APB$ and $\Delta CQD$,

$AB = CD$ [Opp. side of $\|$ gm ABCD]

$\angle ABP = \angle CDQ$ [$\therefore$ AB $\|$ DC and transversal BD intersect them]

$\angle APB = \angle CQD$ [Each = 90°]

$\therefore$ $\Delta APB \cong \Delta CQD$ [AAS Rule]

(ii) $\therefore$ $AP = CQ$ [C.P.C.T. ]

**Q11.** In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively. Show that :

(i) quadrilateral ABED is a parallelogram

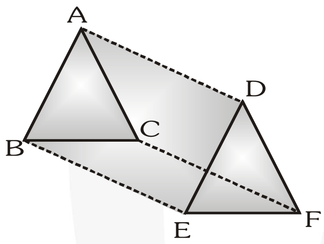
(ii) quadrilateral BEFC is a parallelogram

(iii) AD || CF and AD = CF

(iv) quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) ΔABC ≅ ΔDEF




**Sol.** **Given :** AB = DE, AB || DE, BC = EF & BC || EF

**To Prove**

(i) ABED is a parallelogram.

(ii) BEFC is parallelogram.

(iii) AD || CF and AD = CF

(iv) ACFD is a parallelogram

(v) AC = DF

(vi) ΔABC ≅ ΔDEF

**Proof**

(i) In Δ ABC and Δ DEF

AB = DE [Given]

and AB || DE [Given]

∴ ABED is a parallelogram

(ii) In Δ ABC and Δ DEF

BC = EF [Given]

and BC = EF [Given]

∴ BEFC is a parallelogram.

(iii) As ABED is a parallelogram.

∴ AD || BE and AD = BE ....(i)

Also, BEFC is a parallelogram

∴ CF || BE and CF = BE ....(ii)

From (i) and (ii), we get

∴ AD || CF and AD = CF

(iv) As AD || CF and AD = CF

⇒ ACFD is a parallelogram.

(v) As ACFD is a parallelogram.

∴ AC = DF

(vi) In Δ ABC and Δ DEF,

AB = DE [Given]

BC = EF [Given]

AC = DF [Proved]

∴ ΔABC ≅ ΔDEF [By SSS congruency]

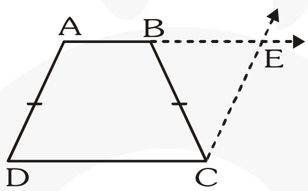
**Q12.** ABCD is a trapezium in which AB||CD and AD = BC. Show that (fig)

(i) ∠A = ∠B

(ii) ∠C = ∠D

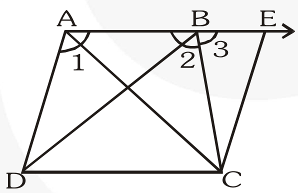
(iii) ΔABC ≅ ΔBAD

(iv) diagonal AC = diagonal BD




**Sol.** **Given :** ABCD is a trapezium.

AB || CD and AD = BC




**To Prove :**

(i) ∠A = ∠B

(ii) ∠C = ∠D

(iii) Δ ABC ≅ Δ BAD

(iv) Diagonal AC = Diagonal BD

**Construction :** Draw CE || AD and extend AB to intersect CE at E.

**Proof :**

(i) As AECD is a parallelogram.

[By construction]

∴ AD = EC

But AD = BC [Given]

∴ BC = EC

| | | |
|---|---|---|
| $\Rightarrow$ | $\angle 3 = \angle 4$ | [Angles opposite to equal sides are equal] |
| | Now, $\angle 1 + \angle 4 = 180°$ | [Interior angles] |
| | and $\angle 2 + \angle 3 = 180°$ | [Linear pair] |
| $\Rightarrow$ | $\angle 1 + \angle 4 = \angle 2 + \angle 3$ | |
| $\Rightarrow$ | $\angle 1 = \angle 2$ | [$\because \angle 3 = \angle 4$] |
| $\Rightarrow$ | $\angle A = \angle B$ | |
| (ii) | $\angle 3 = \angle BCD$ | [Alternate interior angles] |
| | $\angle D = \angle 4$ | [Opposite angles of a parallelogram] |
| | But $\angle 3 = \angle 4$ | [$\Delta$ BCE is an isosceles triangle] |
| $\therefore$ | $\angle BCD = \angle ADC$ | |
| $\therefore$ | $\angle C = \angle D$ | |
| (iii) | In $\Delta ABC$ and $\Delta BAD$, | |
| | $AB = AB$ | [Common] |
| | $\angle 1 = \angle 2$ | [Proved] |
| | $AD = BC$ | [Given] |
| $\therefore$ | $\Delta ABC \cong \Delta BAD$ | [By SAS congruency] |
| $\Rightarrow$ | $AC = BD$ | [By C.P.C.T.] |

# Ex - 8.2

**Q1.** ABCD is a quadrilateral in which P, Q, R and S are mid points of the sides AB, BC, CD and DA (fig.) AC is a diagonal. Show that

(i) SR||AC and SR = 1/2 C

(ii) PQ = SR

(iii) PQRS is a parallelogram.

**Sol.** **Given :** ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA. AC is a diagonal.

**To prove :** (i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

**Proof :** (i) In ΔDAC,

∵ S is the mid-point of DA and R is the mid-point of DC

∴ SR || AC and SR = $\frac{1}{2}$ AC [By Mid-point theorem]

(ii) In ΔBAC,

∵ P is the mid-point of AB and Q is the mid-point of BC

∴ PQ || AC and PQ = $\frac{1}{2}$ AC

[By Mid-point theorem]

But from (i) SR = $\frac{1}{2}$ AC & (ii) PQ = $\frac{1}{2}$ AC

⇒ PQ = SR

(iii) PQ || AC [From (ii)]

SR || AC [From (i)]

∴ PQ || SR

[Two lines parallel to the same line are parallel to each other]

Also, PQ = SR [From (ii)]

∴ PQRS is a parallelogram.

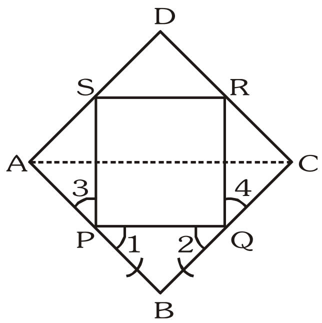
[A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

**Q2.** ABCD is a rhombus and P, Q, R and S are the mid points of sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

**Sol.** **Given :** P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.

**To prove :** PQRS is a rectangle.

**Construction :** Join A and C.




**Proof :** In $\Delta ABC$, P is the mid-point of AB and Q is the mid-point of BC.

$\therefore \quad PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ .....(i)

In $\Delta ADC$, R is the mid-point of CD and S is the mid-point of AD.

$\therefore \quad SR \parallel AC$ and $SR = \frac{1}{2}AC$ ....(ii)

From eq. (i) and (ii), $PQ \parallel SR$ and $PQ = SR$

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rhombus [Given]

$\therefore \quad AB = BC$

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}BC \Rightarrow PB = BQ$

$\therefore \quad \angle 1 = \angle 2$ [Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

$AP = CQ$ [P and Q are the mid-points of AB and BC and AB = BC]

Similarly, AS = CR and PS = QR

[Opposite sides of a parallelogram]

$\therefore \quad \Delta APS \cong \Delta CQR$ [By SSS congruency]

$\Rightarrow \quad \angle 3 = \angle 4$ [By C.P.C.T.]

Now, we have $\angle 1 + \angle SPQ + \angle 3 = 180°$

and $\quad \angle 2 + \angle PQR + \angle 4 = 180°$

$\therefore \quad \angle 1 + \angle SPQ + \angle 3 = \angle 2 + \angle PQR + \angle 4$

Since $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Proved above]

$\therefore \quad \angle SPQ = \angle PQR$ .....(iii)

Now PQRS is a parallelogram [Proved above]

$\therefore \quad \angle SPQ + \angle PQR = 180°$ .....(iv)

[Interior angles]

Using eq. (iii) and (iv),

$\angle SPQ + \angle SPQ = 180°$

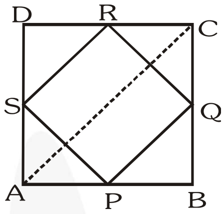
$\Rightarrow \quad 2\angle SPQ = 180° \Rightarrow \angle SPQ = 90°$

Hence, PQRS is a rectangle.

**Q3.** ABCD is a rectangle and P,Q,R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

**Sol.** **Given :** A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joinned.

**To prove :** PQRS is a rhombus.




**Construction :** Join AC.

**Proof :** In $\Delta$ ABC, P and Q are the mid-points of sides AB, BC respectively.

$\therefore$ PQ || AC and PQ $= \frac{1}{2}$ AC ....(i)

In $\Delta$ ADC, R and S are the mid-points of sides CD, AD respectively.

$\therefore$ SR || AC and SR $= \frac{1}{2}$ AC ...(ii)

From eq.(i) and (ii), PQ || SR and PQ=SR ...(iii)

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rectangle. [Given]

$\therefore$ AD = BC

$\Rightarrow$ $\frac{1}{2}$ AD $= \frac{1}{2}$ BC $\Rightarrow$ AS = BQ ...(iv)

In triangles APS and BPQ,

AP = BP [P is the mid-point of AB]

$\angle$PAS = $\angle$PBQ [Each 90°]

and AS = BQ [From eq. (iv)]

$\therefore$ $\Delta$APS $\cong$ $\Delta$BPQ [By SAS congruency]

$\Rightarrow$ PS = PQ [By C.P.C.T.] .....(v)

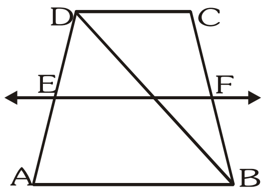
From eq.(iii) and (v), we get that PQRS is a parallelogram.

$\Rightarrow$ PS = PQ

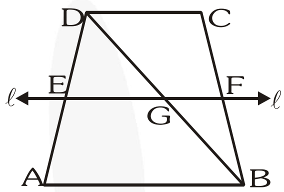
$\Rightarrow$ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

**Q4.** ABCD is a trapezium in which AB||DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (fig.). Show that F is the mid-point of BC.




**Sol.** Line $\ell$ ∥ AB and passes through E.




Line $\ell$ meets BC in F and BD in G.

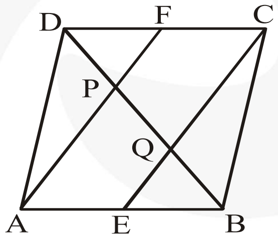
In ΔABD, E is mid-point of AD and EG || AB.

⇒ G is mid-point of BD.

Also, $\ell$ ∥ AB and AB || CD ⇒ $\ell$ ∥ CD

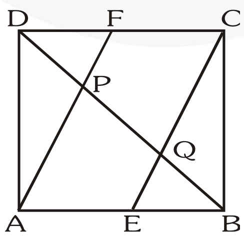
⇒ F is mid-point of BC. [∵ G is mid-point of BD]

**Q5.** In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (fig.). Show that the line segments AF and EC trisect the diagonal BD.




**Sol.** Since E and F are the mid-points of AB and CD respectively.

**Given :** ABCD is a parallelogram. E and F are midpoints of AB and AC respectively.




**To prove :** DP = PQ = QB

**Proof : -**

$\therefore \quad AE = \frac{1}{2}AB$ and $CF = \frac{1}{2}CD$ ....(i)

But ABCD is a parallelogram.

$\therefore \quad AB = CD$ and $AB \parallel DC$

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}CD$ and $AB \parallel DC$

$\Rightarrow \quad AE = FC$ and $AE \parallel FC$ [From eq. (i)]

$\therefore \quad$ AECF is a parallelogram.

$\Rightarrow \quad FA \parallel CE \quad \Rightarrow FP \parallel CQ$

[FP is a part of FA and CQ is a part of CE] ..... (ii)

Since the line segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In $\Delta DCQ$, F is the mid-point of CD and

$\Rightarrow \quad FP \parallel CQ$

$\therefore \quad$ P is the is mid-point of DQ.

$\Rightarrow \quad DP = PQ$ ....(iii)

Similarly, In $\Delta ABP$, E is the mid-point of AB and

$\Rightarrow \quad EQ \parallel AP$

$\therefore \quad$ Q is the mid-point of BP.

$\Rightarrow \quad BQ = PQ$ ....(iv)

From eq.(iii) and (iv),

$DP = PQ = BQ$ ....(v)

Now, $BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ$

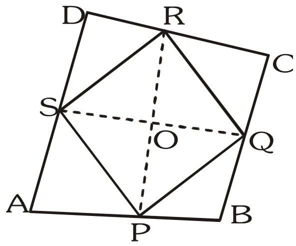
$\Rightarrow \quad BQ = \frac{1}{3}BD$ .....(vi)

From eq (v) and (vi), $DP = PQ = BQ = \frac{1}{3}BD$

$\Rightarrow \quad$ Points P and Q trisects BD. So AF and CE trisects BD.

**Q6.** Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

**Sol.** P,Q,R and S are the mid-points of the sides AB, BC, CD and AD of the quadrilateral ABCD.




We have to prove that, PR and QS bisects each other. Now, join PQ, QR, RS and PS.

Here, we can prove that PQRS is a parallelogram (as in solution).

Now, PR and QS are the diagonals of the parallelogram PQRS.

Hence, PR and QS bisect each other at O.

**Q7.** ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

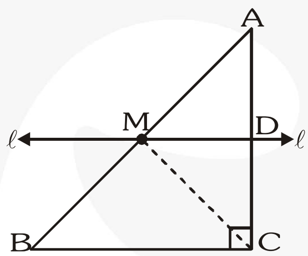
(i) D is the mid-point of AC

(ii) MD $\perp$ AC

(iii) CM = MA = 1/2 AB

**Sol.** (i) Through M, we draw line $\ell \parallel$ BC. $\ell$ intersects AC at D.

$\Rightarrow$ D is mid-point of AC.




(ii) $\angle ADM = \angle ACB = 90°$

[Corresponding angles]

$\Rightarrow$ $\angle ADM = 90°$ $\Rightarrow$ MD $\perp$ AC.

(iii) In $\Delta CMD$ and $\Delta AMD$;

CD = AD, MD = MD

and $\angle CDM = \angle ADM$ [Each = 90°]

Therefore, $\Delta CMD \cong \Delta AMD$

$\Rightarrow$ CM = AM; Also AM = 1/2 AB.