NCERT SOLUTIONS

Surface Areas and Volume

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Ex - 13.1

Q1. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is opened at the top. Ignoring the thickness of the plastic sheet, determine:

(i) The area of the sheet required for making the box.

(ii) The cost of sheet for it, if a sheet measuring 1 m^2 costs Rs 20.

Sol. (i) $\ell = 1.5$ m, b = 1.25 m, h = 0.65

Area of sheet required for making box which is open at the top

$= \ell \times b + 2(\ell + b)h$

$= 1.5 \times 1.25 + 2(1.5 + 1.25) \times 0.65 \text{ m}^2$

$= (1.875 + 3.575) \text{ m}^2 = 5.45 \text{ m}^2$

(ii) Cost = Rs. 20 × 5.45 = Rs. 109.

Q2. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs 7.50 per m^2.

Sol. Length = 5 m, breadth = 4 m, height = 3m.

$\therefore$ Area for white washing

= [Lateral surface area] + [Area of the ceiling]

$= [2(\ell + b)h] + [\ell \times b]$

$= [2(5 + 4) \times 3] + [5 \times 4]\text{m}^2$

$= 54 \text{ m}^2 + 20 \text{ m}^2 = 74 \text{ m}^2$

Cost of white washing for 1 m^2 = Rs. 7.50

$\therefore$ Cost of white washing for 74 m^2 = Rs. 7.50 × 74

$$= \frac{750}{100} \times 74 = \text{Rs. } 555$$

$\therefore$ The required cost of white washing = Rs. 555

Q3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs. 10 per m^2 is Rs. 15000, find the height of the hall.

Sol. Let the height of the hall be h m.

Area of 4 walls $= 2(\ell + b)h$ = perimeter × h

Then, $250 \times h \times 10 = 15000 \Rightarrow h = 6$ m.

Q4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Sol. For a brick length, $\ell = 22.5$ cm, breadth, b = 10 cm, height, h = 7.5 cm

$\therefore$ Total surface area of a brick $= 2(\ell b + bh + h\ell)$

$= 2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5)$

$= 2(225 + 75 + 168.75)$

$= 2(468.75) = 937.5 \text{ cm}^2 = .09375 \text{ m}^2$

$\therefore$ Number of bricks that can be painted out

$= \dfrac{9.375}{.09375} = 100$

Hence, 100 bricks can be painted out of the given container.

Q5. A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

(i) Which box has the greater lateral surface area and by how much?

(ii) Which box has the smaller total surface area and by how much?

Sol. For cubical box with edge = 10 cm

Lateral surface area $= 4a^2 \Rightarrow 4 \times 10 \Rightarrow 400 \text{ cm}^2$

Total surface area $= 6a^2 \Rightarrow 6 \times 100 \Rightarrow 600 \text{ cm}^2$

For the cubodial box with $\ell = 12.5$ cm, b = 10 cm, h = 8 cm

$\therefore$ Lateral surface area $= 2[\ell + b] \times h$

$= 2[12.5 + 10] \times 8$

$= 360 \text{ cm}^2$

$\therefore$ Total surface area $= 2[\ell b + bh + h\ell]$

$= 2[(12.5 \times 10) + (10 \times 8) + (8 \times 12.5)]$

$= 2[125 + 80 + 100] = 610 \text{ cm}^2$

(i) Cubical box has the greater lateral surface area by 40 cm^2

(ii) Total surface area of cubical box is smaller than the cubodial box by 10 cm^2.

Q6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

(i) What is the area of the glass?

(ii) How much of tape is needed for all the 12 edges?

Sol. The herbarium is like a cuboid.

$\ell = 30$ cm, b = 25 cm, h = 25 cm

(i) Area of a cuboid $= 2[\ell b + bh + h\ell]$

$\therefore$ Surface area of the herbarium (glass)

$= 2[(30 \times 25) + (25 \times 25) + (25 \times 30)] \text{ cm}^2$

$= [750 + 625 + 750] \text{ cm}^2$

$= 2[2125] \text{ cm}^2 = 4250 \text{ cm}^2$

Thus, the required area of glass is 4250 cm^2

(ii) Total length of 12 edges $= 4\ell + 4b + 4h$

$= 4(\ell + b + h)$

$= 4 \times 80 = 320$ cm.

Thus, the length of tape needed = 320 cm.

Q7. Shanti Sweets Stall was placing an order for making carboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the card board is Rs. 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

Sol. Surface area of one box of size 25 cm × 20 cm × 5 cm

$= 2\,[25 \times 20 + 20 \times 5 + 5 \times 25)\ cm^2 = 1450\ cm^2$

Surface area of 250 such boxes

$= 250 \times 1450\ cm^2 = 362500\ cm^2$

Surface area of one box of size 15 cm × 12 cm × 5 cm

$= 2\,[15 \times 12 + 12 \times 5 + 5 \times 15)\ cm^2 = 630\ cm^2$

Surface area of 250 such boxes

$= 250 \times 630\ cm^2 = 157500\ cm^2$

Total surface area of the boxes of two types

$= 362500\ cm^2 + 157500\ cm^2 = 520000\ cm^2$

Area of sheet required for making 250 boxes of each including extra required area of 5% for overlaps etc.

$$= \left(520000 + 520000 \times \frac{5}{100}\right) cm^2 = 546000\ cm^2$$

Total cost of sheet at the rate of Rs. 4 for 1000 cm^2 = Rs. $\frac{4}{1000} \times 546000$ = Rs. 2184

Q8. Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m ?

Sol. For shelter

length, $\ell = 4$ m, breadth, $b = 3$ m, height, $h = 2.5$ m

∴ Total surface area of the shelter

$= 2\,(\ell + b)\,h + \ell b$

$= 2\,(4 + 3)\,(2.5) + (4)\,(5)$

$= 2\,(7)\,(2.5) + 12 = 47\ m^2$

Hence, 47 m^2 of tarpaulin will be required.

Ex - 13.2

Q1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.

Sol. Let the radius of the base of the cylinder be r cm.

height, h = 14 cm

Curved surface area = 88 cm^2

$\Rightarrow \; 2\pi rh = 88$

$\Rightarrow 2 \times \frac{22}{7} \times r \times 14 = 88$

$\Rightarrow \; r = \frac{88 \times 7}{2 \times 22 \times 14}$

$\Rightarrow \; r = 1$

$\Rightarrow 2r = 2$

Hence, the diameter of the base of the cylinder is 2 cm.

Q2. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?

Sol. Here, height = 1m

Diameter of the base = 140 cm = 1.40 m

$\therefore \;$ Radius (r) = $\frac{1.40}{2} = 0.70$ m

Total surface area of the cylinder = $2\pi r\,(h + r)$

$= 2 \times \frac{22}{7} \times 0.70\,(1 + 0.70)\ m^2$

$= 2 \times 22 \times 0.10\,(1.70)\ m^2$

$= 44 \times \frac{17}{100}\ m^2 = \frac{748}{100}\ m^2 = 7.48\ m^2$

Hence, the required sheet is 7.48 m^2

Q3. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm. Find its :

(i) inner curved surface area,

(ii) outer curved surface area,

(iii) total surface area.

Sol. Lenght of the metal pipe = 77 cm

$\therefore$ Height of the metal pipe = 77 cm

Inner diameter = 4 cm

Inner radius = $\frac{4}{2}$ = 2 cm

Outer radius = $\frac{4.4}{2}$ = 2.2 cm

(i) Inner curved surface area = $2\pi Rh$

$= 2 \times \frac{22}{7} \times 2 \times 77 \text{ cm}^2$

$= 968 \text{ cm}^2$

(ii) Outer curved surface area = $2\pi Rh$

$= 2 \times \frac{22}{7} \times 2.2 \times 77 \text{ cm}^2$

$= 1064.8 \text{ cm}^2$

(iii) Total surface area = [Inner curved surface area] + [Outer curved surface area] + [Area of ends of two circular ends]

$= [2\pi rh] + [2\pi Rh] + [2\pi(R^2 - r^2)]$

$= 968 + 1064.8 + 2 \times \frac{22}{7} \times [(2.2)^2 - (2)^2]$

$= 968 + 1064.8 + 2 \times \frac{22}{7}(4.84 - 4)$

$= 2032.8 + \frac{2 \times 22 \times 0.84}{7}$

$= 2038.08 \text{ cm}^2$

Q4. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2.

Sol. Radius of the roller (r) = $\frac{84}{2}$ cm = 42 cm

length of the roller (h) = 120 cm

$\therefore$ Area of the playground levelled in taking 1 complete revolution = $2\pi rh$

$= 2 \times \frac{22}{7} \times 42 \times 120 = 31680 \text{ cm}^2$

$\therefore$ Area of the playground

$= 31680 \times 500 = 15840000 \text{ cm}^2$

$= \frac{15840000}{100 \times 100} m^2 = 1584 \text{ m}^2.$

Hence, the area of the playground is 1584 m^2.

Q5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs 12.50 per m^2.

Sol. Diameter of the pillar = 50 cm

Radius (r) = $\frac{50}{2}$ = 25 cm and Height = 3.50 m

$\therefore$ Curved surface area of a cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{1}{4} \times 3.50 \text{ m}^2$

$= 11 \times \frac{1}{2} \text{ m}^2$

Cost of painting of 1 m^2 = Rs. 12.50

Cost of painting of $\frac{11}{2}$ m^2 = Rs. $\frac{11}{2} \times 12.50$ = Rs. 68.75

Q6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.

Sol. r = 0.7 m

$2\pi r \times h = 4.4$

$\Rightarrow 2 \times \frac{22}{7} \times \frac{7}{10} \times h = \frac{44}{10} \Rightarrow h = 1 \text{ m}$

Q7. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find

(i) its inner curved surface area,

(ii) the cost of plastering this curved surface at the rate of Rs 40 per m^2.

Sol. (i) $r = \frac{35}{20} \text{m} = \frac{7}{4} \text{m}$, h = 10 m

Inner curved surface area of the well $= 2 \times \frac{22}{7} \times \frac{7}{4} \times 10 \text{m}^2 = 110 \text{ m}^2$

(ii) Cost of plastering = Rs. 40 × 110 = Rs. 4400

Q8. In a hot water heating system, there is a cylindrical pipe of length 28 m,and diameter 5 cm. Find the total radiating surface in the system.

Sol. Here the length, h of the cylindrical pipe = 28 m and radius,

$r = \frac{5}{2} \text{ cm} = \frac{5}{2 \times 100} \text{m} = \frac{5}{200} \text{m} = \frac{1}{40} \text{m}$

$\therefore$ Total radiating surface in the system $= 2\pi rh = 2 \times \frac{22}{7} \times \frac{1}{40} \times 28 = 4.4 \text{ m}^2$.

Q9. Find

(i) The lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

(ii) How much steel was actually used, if 1/12 of the steel actually used was wasted in making the tank.

Sol. (i) $r = \frac{42}{20}m = \frac{21}{10}m,\ h = \frac{45}{10}m$

and lateral surface area $= 2 \times \frac{22}{7} \times \frac{21}{10} \times \frac{45}{10}\ m^2 = 59.4m^2$

(ii) Total surface area of the tank $= 2\pi r(r + h)$

$= 2 \times \frac{22}{7} \times 2.1\ (2.1 + 4.5)m^2$

$= 87.12\ m^2$

Let actual area of the steel used = x m^2.

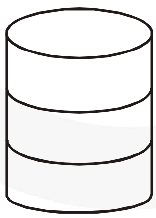
$\therefore$ Area of steel that wasted $= \frac{1}{12} \times x = \frac{x}{12} m^2$

$\Rightarrow$ Area of steel used $= x - \frac{x}{12} = \frac{11x}{12} m^2$

$\Rightarrow \frac{11x}{12} = 87.12 \Rightarrow x = \frac{8712 \times 12}{100 \times 11} m^2$

$x = \frac{104544}{1100} m^2 = 95.04\ m^2$

Q10. In figure, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.



Sol. r = 10 cm, h = (30 + 2.5 + 2.5) cm = 35 cm] Ans. 2200 cm^2.

Q11. The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with carboard. If there were 35 competitors, how much cardboard was required to be brought for the competition?

Sol. Radius of a cylinder = 3 cm

Height of a cylinder = 10.5 cm

Since, surface area of penholder (cylinder)

$= [\text{Lateral surface area}] + [\text{Base area}]$

$= [2\pi rh] + \pi r^2$

$= (44 \times 3 \times 1.5) + \frac{198}{7}\text{cm}^2$

$= 198 + \frac{198}{7}$

$= \frac{1386 + 198}{7}\text{cm}^2 = \frac{1584}{7}\text{cm}^2$

$\Rightarrow$ Surface area of 35 penholders $= 35 \times \frac{1584}{7}\text{cm}^2$

$\Rightarrow$ 7920 cm^2

Thus, 7920 cm^2 of cardboard was required to be bought.

Ex - 13.3

Q1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Sol. $\because$ Diameter of the base = 10.5 cm

$\therefore$ Radius of the base (r) = $\frac{10.5}{2}$ cm = 5.25 cm

Slant height (ℓ) = 10 cm

$\therefore$ Curved surface area of the cone

$= \pi r\ell = \frac{22}{7} \times 5.25 \times 10 = 165 \text{ cm}^2$.

Q2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Sol. ℓ = 21m, r = 12 m

Total surface area $= \pi r\,(r + \ell) = \frac{22}{7} \times 12 \times 33 \text{ m}^2$

$= 1244.57 \text{ m}^2$

Q3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find
(i) radius of the base and (ii) total surface area of the cone.

Sol. (i) Slant height (ℓ) = 14 cm

Curved surface area = 308 cm^2

$\Rightarrow \pi r\ell = 308 \qquad \Rightarrow \frac{22}{7} \times r \times 14 = 308$

$\Rightarrow r = \frac{308 \times 7}{22 \times 14} \qquad \Rightarrow r = 7 \text{ cm}$

Hence, the radius of the base is 7 cm.

(ii) Total surface area of the cone $= \pi r(\ell + r) = \frac{22}{7} \times 7 \times (14 + 7) = \frac{22}{7} \times 7 \times 21 = 462 \text{ cm}^2$

Hence, the total surface area of the cone is 462 cm^2.

Q4. A conical tent is 10 m high and the radius of its base is 24 m. Find
(i) Slant height of the tent.
(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs. 70.

Sol. Height of the tent (h) = 10 m

Radius of the base (r) = 24 m

(i) The slant height, $\ell = \sqrt{h^2 + r^2}$

$\ell = \sqrt{(24)^2 + (10)^2}$ m = $\sqrt{576 + 100}$ m

ℓ = 26 m

Thus, the required slant height of the tent is 26 m.

(ii) Curved surface area of the cone = $\pi r \ell$

$\therefore$ Area of the canvas required = $\frac{13728}{7}$ m^2

$\therefore$ Cost of $\frac{13728}{7}$ m^2 canvas

= Rs 70 × $\frac{13728}{7}$ = Rs 137280

Q5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m ? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use π = 3.14)

Sol. Area of Tarpaulin required = Curved surface of the conical tent

$l = \sqrt{8^2 + 6^2} = 10m$

Area of tarpaulin = 3.14 × 6× 10

Acc. to quest = 188.4

3m × length = 188.4

length = 62.8m

wastage = 20cm = 0.2 m

Total length required = 62.8 + 0.2 = 63m

i.e., $\ell \times b = \pi r \ell$]

Ans. 63 m.

Q6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white washing its curved surface at the rate of Rs. 210 per 100 m^2.

Sol. ℓ = 25 m, r = 7 m

Curved surface = $\frac{22}{7} \times 7 \times 25m^2 = 550\ m^2$

Cost of white washing = Rs. $\frac{210}{100} \times 550$ = Rs. 1155

Q7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Sol. $r = 7$ cm, $h = 24$ cm $\ell^2 = h^2 + r^2$

$= 576 + 49 = 625 \Rightarrow \ell = 25$ cm

Sheet required for one cap

$= \frac{22}{7} \times 7 \times 25 \text{ cm}^2 = 550 \text{ cm}^2$

Sheet required for 10 caps $= 10 \times 550 \text{ cm}^2 = 5500 \text{ cm}^2$

Q8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs 12 per m^2, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

Sol. Radius (r) $= \frac{40}{2}$ cm $= \frac{20}{100}$ m $= 0.2$ m

Height (h) = 1 m

Slant height (ℓ) $= \sqrt{r^2 + h^2} = \sqrt{(0.2)^2 + (1)^2}$

$= 1.02$ m.

Now, curved surface area $= \pi r \ell$

$\therefore$ Curved surface area of 1 cone

$= 3.14 \times 0.2 \times 1.02 \text{ m}^2$

$= \frac{314}{100} \times \frac{2}{10} \times \frac{102}{100} \text{ m}^2$

Curved surface area of 50 cones

$= 50 \times \left[\frac{314}{100} \times \frac{2}{10} \times \frac{102}{100}\right] \text{m}^2$

$= \frac{314 \times 102}{10 \times 100} \text{ m}^2$

Cost of painting per m^2 = Rs 12

$\therefore$ Cost of painting $\left[\frac{314 \times 102}{1000}\right] \text{m}^2$

$= \frac{12 \times 314 \times 102}{1000} =$ Rs 384.34 (approx)

Ex - 13.4

Q1. Find the surface area of a sphere of radius :
(i) 10.5cm (ii) 5.6 cm (iii) 14 cm

Sol. (i) Surface area $= 4 \times \frac{22}{7} \times (10.5)^2 \text{ cm}^2$
$= 1386 \text{ cm}^2$

(ii) Surface area $= 4 \times \frac{22}{7} \times 5.6 \times 5.6 \text{ cm}^2$
$= 394.24 \text{ cm}^2$

(iii) Surface area $= 4 \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$
$= 2464 \text{ cm}^2$

Q2. (i) Find the surface area of a sphere of diameter 14cm.

Sol. Diameter = 14 cm

$\therefore$ Radius (r) $= \frac{14}{2}$ cm = 7 cm

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times (7)^2 = 616 \text{ cm}^2$.

Q3. Find the total surface area of a hemisphere of radius 10 cm. (Use $\pi = 3.14$)

Sol. r = 10 cm.
$\therefore$ Total surface area of the hemisphere $= 3\pi r^2 = 3 \times 3.14 \times (10)^2 = 942 \text{ cm}^2$.

Q4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Sol. $r_1 = 7$ cm & $r_2 = 14$cm and let S_1 and S_2 be the surface areas of respective spheres.

$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

Ans. 1 : 4

Q5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2.

Sol. Inner diameter = 10.5 cm, Radius = $\frac{105}{20}$ cm

Curved surface area of a hemisphere = $2\pi r^2$

$\therefore$ Inner curved surface area of hemispherical bowl

$= 2 \times \frac{22}{7} \times \frac{105}{20} \times \frac{105}{20} \text{ cm}^2 = \frac{17325}{100} \text{ cm}^2$

Cost of tinplating for 100 cm^2 = Rs 16

$\therefore$ Cost of tinplating for $\frac{17325}{100} \text{ cm}^2$

$= \text{Rs } \frac{16}{100} \times \frac{17325}{100}$

$= \text{Rs } \frac{277200}{100 \times 100} = \text{Rs } 27.72$

Q6. Find the radius of a sphere whose surface area is 154 cm^2

Sol. $4\pi r^2 = 154 \Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{7 \times 7}{4} \Rightarrow r = \frac{7}{2}$ cm, i.e., r = 3.5 cm

Q7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Sol. Let d_1 and d_2 be the diameters of the moon and the earth respectively and S_1 and S_2 be their respective surface areas.

$d_1 = \frac{1}{4} d_2 \Rightarrow \frac{d_1}{d_2} = \frac{1}{4} \Rightarrow \frac{2r_1}{2r_2} = \frac{1}{4} \Rightarrow \frac{r_1}{r_2} = \frac{1}{4}$]

Ans. 1 : 16.

Q8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. find the outer curved surface area of the bowl.

Sol. r = 5 cm, thickness of steel sheet = 0.25 cm

$\Rightarrow$ R = 5 cm + 0.25 cm = 5.25 cm

Outer curved surface area of the bowl = $2\pi R^2$

$= 2 \times \frac{22}{7} \times \frac{525}{100} \times \frac{525}{100} \text{ cm}^2$

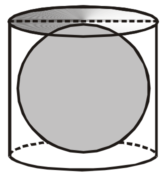
$= 173.25 \text{ cm}^2$

Q9. A right circular cylinder just encloses a sphere of radius r. Find

(i) Surface area of the sphere,

(ii) Curved surface area of the cylinder,

(iii) Ratio of the areas obtained in (i) and (ii).



Sol. Radius of cylinder = radius of sphere = r

Height of cylinder = 2 × radius of sphere = 2r]

Ans. (i) $4\ \pi r^2$ (ii) $4\ \pi r^2$ (iii)1 : 1.

Ex - 13.5

Q1. A match box measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?

Sol. Volume of a matchbox

$= 4 \times 2.5 \times 1.5\ cm^3 = 15\ cm^3$

$\therefore$ Volume of a packet containing 12 such boxes

$= 15 \times 12\ cm^3 = 180\ cm^3$.

Q2. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? $(1\ m^3 = 1000\ \ell)$

Sol. $\ell = 6m,\ b = 5m,\ h = 4.5\ m$

$\therefore$ Capacity $= \ell \times b \times h = 6 \times 5 \times 4.5\ m^3 = 135\ m^3$

$1\ m^3 = 1000\ \ell \Rightarrow 135\ m^3 = 135000\ \ell$

Q3. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

Sol. Height, $h = \frac{380}{10 \times 8} m = 4.75\ m$

Q4. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs. 30 per m^3.

Sol. Internal space of the pit $= 8 \times 6 \times 3 m^3$

Cost = Rs. $30 \times 8 \times 6 \times 3$ = Rs. 4320

Q5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Sol. Capacity of tank = 50,000 litres

$$= \frac{50000}{1000} m^3 = 50\ m^3$$

Breadth of the tank $= \frac{50}{2.5 \times 10} m$

i.e., Breadth = 2 m.

Q6. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15m × 6m. For how many days will the water of this tank last?

Sol. $\ell = 20$ m, b = 15 m, h = 6 m

$\therefore$ Volume of the tank $= \ell \times b \times h$

$= 20 \times 15 \times 6 \text{ m}^3$

$= 1800 \text{ m}^3$

Since $1 \text{ m}^3 = 1000\ \ell$

$\therefore$ Capacity of the tank $= 1800 \times 1000\ \ell$

$= 1800000\ \ell$

Village population = 4000.

Amount of water required per day

$\Rightarrow\ 150 \times 4000\ \ell$

Let the required number of days = x.

$\therefore\ 4000 \times 150 \times x = 1800000$

$x = 3$

Q7. A godown measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.

Sol. Volume of the godown $= 40 \times 25 \times 10 \text{ m}^3$

Volume of a crate $= 1.5 \times 1.25 \times 0.5 \text{ m}^3$

$$= \frac{15}{10} \times \frac{125}{100} \times \frac{5}{10} \text{ m}^3$$

$$= \frac{3}{2} \times \frac{5}{4} \times \frac{1}{2} = \frac{15}{16} \text{ m}^3$$

Let the required number of crates is "n".

$$\Rightarrow n \times \left[\frac{3}{2} \times \frac{5}{4} \times \frac{1}{2}\right] = 40 \times 25 \times 10$$

$$\Rightarrow n = \frac{32000}{3} = 10666.66$$

Q8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Sol. Let the side of new cube be a cm.

Volume of bigger cube = 8 × volume of a smaller cube]

$$\frac{123}{8} = a^3$$

a = 6 ratio of surface area $= \frac{6 \times 12^2}{6.6^2} = \frac{4}{1}$

Ans. 6 cm ; 4 : 1

Q9. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Sol. Water flowing in one minute $= \frac{2000}{60} \times 40 \times 3m^3$

$= 4000\ m^3$

Ex - 13.6

Q1. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? ($1000\ cm^3 = 1\ell$)

Sol. Let the base radius of the cylindrical vessel be r cm.

Then, circumference of the base of the cylindrical vessel = $2\pi r$ cm.

According to the question, $2\pi r = 132$

$$\Rightarrow\ 2 \times \frac{22}{7} \times r = 132 \Rightarrow r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Height of the cylindrical vessel, h = 25 cm

$\therefore$ Capacity of the cylindrical vessel

$$= \pi r^2 h = \frac{22}{7}(21)^2(25)\ cm^3$$

$$= 34650\ cm^3 = \frac{34650}{1000}\ \ell = 34.65\ \ell$$

Hence, the cylindrical vessel can hold 34.65 ℓ of water.

Q2. The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.

Sol. $\because$ Inner diameter = 24 cm

$\therefore$ Inner radius (r) = $\frac{24}{2}$ cm = 12 cm

$\because$ Outer diameter = 28 cm

$\therefore$ Outer radius (R) = $\frac{28}{2}$ cm = 14 cm

Length of the pipe (h) = 35 cm

Outer volume = $\pi R^2 h = \frac{22}{7} \times (14)^2 \times 35$

$= 21560\ cm^3$

Inner volume = $\pi r^2 h = \frac{22}{7} \times (12)^2 \times 35 = 15840\ cm^3$

$\therefore$ Volume of the wood used = Outer volume – Inner volume = $21560\ cm^3 - 15840\ cm^3$

$= 5720\ cm^3$

$\therefore$ Mass of the pipe $= 5720 \times 0.6$ g

$= 3432$ g $= 3.432$ kg

Hence, the mass of the pipe is 3.432 kg.

Q3. A soft drink is available in two packs - (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

Sol. $V_1 = 5 \times 4 \times 15 \text{ cm}^3 = 300 \text{ cm}^3$

$V_2 = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 10 \text{ cm}^3 = 385 \text{ cm3}$

$V_2 > V_1$, i.e., the cylinder has 85 cm^3 greater capacity.

Q4. If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find (i) radius of its base (ii) its volume. (Use π = 3.14)

Sol. (i) $2\pi r \times h = 94.2 \Rightarrow 2 \times 3.14 \times r \times 5 = 94.2 \Rightarrow r = \frac{94.2}{31.4} = 3\text{cm} \Rightarrow r = 3 \text{ cm}$

(ii) Volume $= 3.14 \times (3)^2 \times 5 \text{ cm}^3$

$= 15.7 \times 9 \text{ cm}^3 = 141.3 \text{ cm}^3$

Q5. It costs Rs. 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of Rs. 20 per m^2, find

(i) Inner curved surface area of the vessel,

(ii) Radius of the base,

(iii) Capacity of the vessel.

Sol. Inner curved surface area × Rs. 20 = Rs. 2200]

Ans. (i) 110 m^2 ; (ii) 1.75 m ; (iii) 96.25 m^3 (or 96.25 kl).

(i) Inner curved surface area = $\frac{2200}{20} = 110\text{m}^2$

(ii) $2 \times \frac{22}{7} \times r \times 10 = 110$

$r = \frac{7}{4} = 1.75$

(iii) Vol. of vessel = $\frac{22}{7} \times 1.75 \times 10$ = 96.25 m^3

Q6. The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?

Sol. Let the radius of the vessel be r m.

Volume of vessel = 15.4 ℓ = 0.0154 m^3

$\Rightarrow \frac{22}{7} \times r^2 \times h = 0.0154$]

Ans. 0.07 m^2.

Q7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

Sol. We have, 10 mm = 1 cm, $\therefore$ $1\text{mm} = \frac{1}{10}\text{cm}$

For graphite cylinder

$\text{diameter} = 1\text{mm} = \frac{1}{10}\text{cm}$

$\text{Radius} = \frac{1}{10} \times \frac{1}{2}\ \text{cm} = \frac{1}{20}\ \text{cm}$

Length (h) = 14 cm

$\therefore$ $\text{Volume} = \pi r^2 h = \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 14\ \text{cm}^3$

$= 0.11\ \text{cm}^2$

Now, diameter of pencil = 7 mm = $\frac{7}{10}$ cm

$\therefore$ Radius of the pencil (R) = $\frac{7}{20}$ cm

Height of the pencil h = 14 cm

$\text{Volume} = \frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 14\ \text{cm}^3$

Volume = 5.39 cm^3

Volume of the wood = Volume of pencil – Volume of graphite

$\Rightarrow 5.39\ \text{cm}^3 - 0.11\ \text{cm}^3 = 5.28\ \text{cm}^3$

Q8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

Sol. r = 7/2 cm, h = 4 cm

$\text{Capacity of one bowl} = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 4\text{cm}^3 = 154\ \text{cm}^3$

Soup required for 250 patients

$= 250 \times 154\ \text{cm}^3 = 38500\ \text{cm}^3$

$= \frac{38500}{1000}\text{lit.} = 38.5\ \text{lit.}$

Ex - 13.7

Q1. Find the volume of the right circular cone with
(i) radius 6 cm, height 7 cm
(ii) radius 3.5 cm, height 12 cm

Sol. (i) $r = 6$ cm, $h = 7$ cm

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7 \text{ cm}^3 = 264 \text{ cm}^3$$

(ii) $r = \frac{7}{2}$ cm, $h = 12$ cm

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 12 \text{ cm}^3 = 154 \text{ cm}^3$$

Q2. Find the capacity in litres of a conical vessel with
(i) radius 7 cm, slant height 25 cm.
(ii) height 12 cm, slant height 13 cm.

Sol. (i) $r = 7$ cm, $\ell = 25$ cm

$r^2 + h^2 = \ell^2$

$\Rightarrow (7)^2 + h^2 = (25)^2 \Rightarrow h^2 = (25)^2 - (7)^2$

$\Rightarrow h^2 = 625 - 49 \Rightarrow h^2 = 576$

$\Rightarrow h = \sqrt{576} \Rightarrow h = 24$ cm

$$\therefore \text{Capacity} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24$$

$$= 1232 \text{ cm}^3 = 1.232\ \ell.$$

(ii) $h = 12$ cm, $\ell = 13$ cm

$r^2 + h^2 = \ell^2$

$\Rightarrow r^2 + (12)^2 = (13)^2 \Rightarrow r^2 + 144 = 169$

$\Rightarrow r^2 = 169 - 144 \Rightarrow r^2 = 25$

$\Rightarrow r = \sqrt{25} \Rightarrow r = 5$ cm

$$\therefore \text{Capacity} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12$$

$$= \frac{2200}{7} \text{ cm}^3 = \frac{2200}{7000}\ \ell = \frac{11}{35}\ \ell.$$

Q3. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use $\pi = 3.14$)

Sol. h = 15 cm, volume = 1570 cm^3

$$\Rightarrow \frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$$

$$\Rightarrow r^2 = \frac{1570}{15.70} = 100$$

$$\Rightarrow r = 10 \text{ cm}$$

Q4. If the volume of a right circular cone of height 9 cm is 48 π cm^3, find the diameter of its base.

Sol. h = 9 cm, volume = 48 πcm^3

$$\frac{1}{3}\pi r^2 \times h = 48\pi$$

$$\Rightarrow \frac{1}{3} r^2 \times 9 = 48$$

$$\Rightarrow r^2 = 16 \Rightarrow r = 4 \text{ cm}$$

Q5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

Sol. For conical pit

Diameter = 3.5 m

$\therefore$ Radius (r) = $\frac{3.5}{2}$ m = 1.75 m

Depth (h) = 12 m

$\therefore$ Capacity of the conical pit

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12 \text{ m}^3$$

$$= 38.5 \text{ m}^3 = 38.5 \times 1000\ \ell = 38.5 \text{ kl.}$$

Q6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find (i) height of the cone (ii) slant height of the cone (iii) curved surface area of the cone

Sol. (i) Volume = 9856 cm^3, r = 14 cm

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times (14)^2 \times 4 = 9856$$

$$\Rightarrow h = \frac{9856 \times 3}{22 \times 28} \text{cm} \Rightarrow h = 48 \text{ cm}$$

(ii) $\ell^2 = h^2 + r^2 = (48)^2 + (14)^2 = 2500$

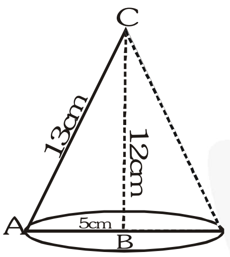
$\Rightarrow \ell = 50$ cm

(iii) Curved surface area $= \frac{22}{7} \times 14 \times 50$ cm^2

$= 2200$ cm^2

Q7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

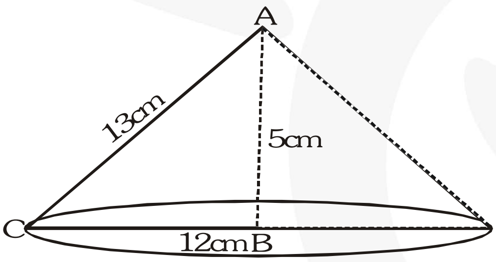
Sol.



Radius, r = 5 cm ; height, h = 12 cm & slant height, ℓ = 13 cm] $= \frac{1}{2}\pi\, 5^2 \times 12 = 100\pi$

Q8. If the triangle ABC in the question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Question 7 and 8.

Sol.



Radius, r = 12 cm ; height, h = 5 cm & slant height, ℓ = 13 cm]

Vol. $= \frac{1}{3}\pi\, 12^2 \times 5 = 240\pi$

Ans. 240 π cm^3 ; 5 : 12.

Q9. A heap of wheat is in the form of a cone whose diameter is 1 0.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Sol. Diameter = 10.5 m

$\therefore$ Base Radius (r) $= \frac{10.5}{2}$ m $= \frac{105}{20}$ m

Height (h) = 3m

$\therefore$ Volume of the heap $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{105}{20}\right)^2 \times 3$$

$= 86.625\ m^3$

$\therefore$ Area of the canvas $= \pi r \ell$

where, $\ell = \sqrt{r^2 + h^2}$

$$= \sqrt{\left(\frac{10.5}{2}\right)^2 + (3)^2} = \sqrt{\frac{110.25}{4} + 9}$$

$$= \sqrt{\frac{146.25}{4}} = 6.046 \text{ m (approx)}$$

Now, $\pi r \ell = \frac{22}{7} \times \frac{10.5}{2} \times 6.05\ m^2$

$= 11 \times 1.5 \times 6.05\ m^2$

$= 99.825\ m^2$

Thus, the required area of the canvas is $99.825\ m^2$

Ex - 13.8

Q1. Find the volume of a sphere whose radius is
(i) 7 cm (ii) 0.63 m

Sol. (i) r = 7 cm

$$\text{Volume} = \frac{4}{3}\times\frac{22}{7}\times(7)^3\text{ cm}^3 = 1437\frac{1}{3}\text{ cm}^3$$

(ii) r = 0.63 m

$$\text{Volume} = \frac{4}{3}\times\frac{22}{7}\times(0.63)^3\text{ m}^3 = 1.047816\text{ m}^3 = 1.05\text{ m}^3\text{ (approx)}$$

Q2. (i) Find the amount of water displaced by a solid spherical ball of diameter 28 cm.

Sol. Diameter = 28 cm

$\therefore$ Radius (r) = $\frac{28}{2}$ cm = 14 cm

$\therefore$ Amount of water displaced

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times(14)^3 = \frac{34496}{3}\text{ cm}^3$$

$$= 11498\frac{2}{3}\text{ cm}^3.$$

Q3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3 ?

Sol. $\text{Density} = \frac{\text{mass}}{\text{volume}}$

Volume of metallic ball = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times\frac{22}{7}\times(4.2)^3 = 310.46\text{cm}^3$$

mass = density × Volume

$8.9\text{g/cm}^3 \times 310.46\text{cm}^3$

= 2763.12gm=2.7kg

Q4. The diameter of the moon is approximately one-fourth the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Sol. Let d_1 and d_2 be the diameters of the moon and the earth respectively. Then, $d_1 = \frac{1}{4} d_2$

$$\Rightarrow \frac{r_1}{r_2} = \frac{1}{4} \;;\; \frac{\text{Volume of moon}}{\text{Volume of earth}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3] \text{ Ans. } \frac{1}{64}$$

Q5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

Sol. $r = \frac{10.5}{2} = \frac{21}{4}$ cm

Capacity of the bowl $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{21}{4} \times \frac{21}{4} \times \frac{21}{4} cm^3 = \frac{4851}{16} cm^3$

$= 303.2\ cm^3$ (approx.)

$= \frac{303.2}{1000}$ lit. $= 0.303$ lit. (approx.)

Q6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol. Inner radius (r) = 1 m

Thickness of iron sheet = 1 cm = 0.01 m

$\therefore$ Outer radius (R) = Inner radius (r) + Thickness of iron sheet = 1 m + 0.01 m = 1.01 m

$\therefore$ Volume of the iron used to make the tank

$= \frac{2}{3}\pi(R^3 - r^3) = \frac{2}{3} \times \frac{22}{7} \times \{(1.01)^3 - 1^3\}$

$= 0.06348\ m^3$ (Approx).

Q7. Find the volume of a sphere whose surface area is 154 cm^2.

Sol. $4\pi r^2 = 154 \Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{49}{4} \Rightarrow r = \frac{7}{2}$ cm

Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} cm^3 = \frac{539}{3} cm^3$

$= 179\frac{2}{3} cm^3$

Q8. A dome of a building is in the form of a hemisphere. From inside, it was white washed at the cost of Rs. 498.96. if the cost of white washing is Rs. 2.00 per square metre, find the
(i) Inside surface area of the dome,
(ii) Volume of the air inside the dome.

Sol. (i) Total cost of white washing = Rs 498.96
Cost of 1 m^2 of white washing = Rs 2
$\therefore$ Inside surface Area = 498.96 = 249.48 m^2
$\therefore$ Inside surface area = $2\pi r^2$
$\Rightarrow 2\pi r^2 = 249.48$

$$\Rightarrow 2 \times \frac{22}{7} \times r^2 = \frac{24948}{100}\ ;\ r^2 = \frac{3969}{100}$$

$$\Rightarrow r = \left(\frac{63}{10}\right)^2 \text{m} \Rightarrow r = \frac{63}{10} = 6.3 \text{ m}$$

(ii) The volume of air in the dome

$$\text{Volume} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times (6.3)^3 \text{ m}^3$$

$$= \frac{523908}{1000} \text{m}^3 = 523.9 \text{ m}^3 \text{ (approx)}$$

Q9. Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the (i) radius r' of the new sphere, (ii) ratio of S and S'.

Sol. Volume of 27 solid iron sphere each of radius r = volume of new sphere of radius R.

$$27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$\Rightarrow$ R = 3r
S = $4\pi r^2$
S' = $4\pi(3r)^2]$
Ans. 3r ; 1 : 9

Q10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Sol. $r = \frac{3.5}{2}$mm

Capacity of the capsule = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times \frac{3.5}{2} \text{mm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4} \text{mm}^3 = \frac{11}{24} \times 49 \text{ mm}^3$$

$$= \frac{539}{24} \text{mm}^3 = 22.346 \text{ mm}^3$$