Ex - 7.2

Q1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that : (i) OB = OC (ii) AO bisects ∠A.

Sol. (i) In ΔABC, OB and OC are bisectors of ∠B and ∠C.

$\therefore \angle OBC = \frac{1}{2}\angle B$ ...(1)

$\angle OCB = \frac{1}{2}\angle C$ ...(2)

Also, AB = AC (Given)

⇒ ∠B = ∠C ...(3)

From (1), (2), (3), we have

∠OBC = ∠OCB

Now, in ΔOBC, we have

∠OBC = ∠OCB

⇒ OB = OC

(Sides opposite to equal angles are equal)

(ii) $\angle OBA = \frac{1}{2}\angle B$ and $\angle OCA = \frac{1}{2}\angle C$

⇒ ∠OBA = ∠OCA (∵ ∠B = ∠C)

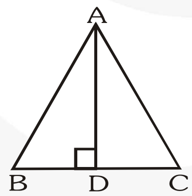
AB = AC and OB = OC

∴ ΔOAB ≅ ΔOAC (SAS congruence criteria)

⇒ ∠OAB = ∠OAC

⇒ AO bisects ∠A.

Q2. In ΔABC, AD is the perpendicular bisector of BC. Show that ΔABC is an isosceles triangle in which AB = AC.



Sol. Given : In ΔABC, AD is perpendicular bisector of BC.

To Prove : ΔABC is isosceles Δ with AB = AC

Proof : In ΔADB and ΔADC

∠ADB = ∠ADC (Each 90°)

DB = DC (AD is ⊥ bisector of BC)

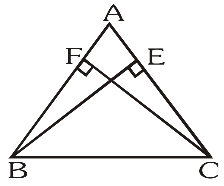
AD = AD (Common)

ΔADB ≅ ΔADC (By SAS rule)

AB = AC (By CPCT)

∴ ΔABC is an isosceles Δ with AB = AC

Q3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively. Show that these altitudes are equal.



Sol. In ΔABE and ΔACF, we have

∠BEA = ∠CFA (Each = 90°)

∠A = ∠A (Common angle)

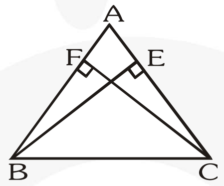
AB = AC (Given)

∴ ΔABE ≅ ΔACF (By AAS congruence criteria)

⇒ BE = CF (By CPCT)

Q4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that (i) ΔABE ≅ ΔACF

(ii) AB = AC, i.e., ABC is an isosceles triangle.



Sol. (i) In ΔABE and ΔACF, we have

∠A = ∠A (Common)

∠AEB = ∠AFC (Each = 90°)

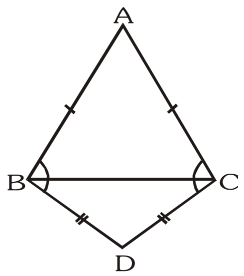
BE = CF (Given)

∴ ΔABE ≅ ΔACF(By ASA congruence)

(ii) ΔABE ≅ ΔACF

⇒ AB = AC (By CPCT)

Q5. ABC and DBC are two isosceles triangles on the same base BC (see figure). Show that ∠ABD = ∠ACD.



Sol. Given : ABC and BCD are two isosceles triangle on common base BC.

To prove : $\angle ABC = \angle ACD$

Proof : ABC is an isosceles

Triangle on base BC

$\therefore\ \angle ABC = \angle ACB$ ...(1)

$\because$ DBC is an isosceles Δ on base BC.

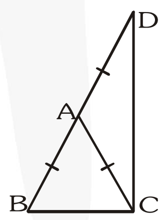
$\angle DBC = \angle DCB$ ...(2)

Adding (1) and (2)

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

$\Rightarrow \angle ABD = \angle ACD$

Q6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see figure). Show that $\angle BCD$ is a right angle.



Sol. In ΔABC, AB = AC

$\Rightarrow \angle ACB = \angle ABC$ ...(1)

In ΔACD,

AD = AB (By construction)

$\Rightarrow$ AD = AC

$\Rightarrow \angle ACD = \angle ADC$ ...(2)

Adding (1) and (2),

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$

$\Rightarrow \angle BCD = \angle ABC + \angle ADC$

In $\angle DBC + \angle ABC + \angle BCD + \angle CDB = 180°$

$\Rightarrow 2\ \angle BCD = 180°$

$\Rightarrow \angle BCD = 90°$

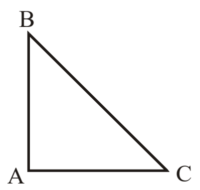
Q7. ABC is a right angled triangle in which $\angle A = 90°$ and AB = AC. Find $\angle B$ and $\angle C$.

Sol. In ΔABC

AB = AC

$\angle B = \angle C$ ...(1)

(angles opposite to equal sides are equal)



In ΔABC

$\angle A + \angle B + \angle C = 180°$

$90° + \angle B + \angle C = 180°$

$\angle B + \angle C = 90°$ ...(2)

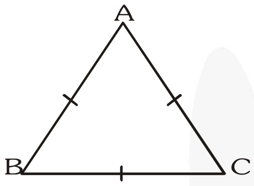
from (1) and (2)

$\angle B = \angle C = 45°$

Q8. Show that the angles of an equilateral triangle are 60° each.

Sol. ΔABC is equilateral triangle.

$\Rightarrow AB = BC = CA$



Now, $AB = BC$

$\Rightarrow BA = BC$

$\Rightarrow \angle C = \angle A$ ...(1)

Similarly, $\angle A = \angle B$ ...(2)

From (1) and (2),

$\angle A = \angle B = \angle C$ ...(3)

Also, $\angle A + \angle B + \angle C = 180°$ ...(4)

$\Rightarrow \angle A = \angle B = \angle C = \frac{1}{3} \times 180° = 60°$