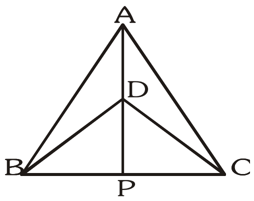
Ex - 7.3

Q1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see figure). If AD is extended to intersect BC at P, show that



(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$

(iv) AP is the perpendicular bisector of BC.

Sol. In ΔABD and ΔACD,

AB = AC (∵ ΔABC is isosceles)

DB = DC (∵ ΔDBC is isosceles)

AD = AD (Common side)

∴ $\Delta ABD \cong \Delta ACD$ (By SSS congruence rule)

(ii) Now, $\Delta ABD \cong \Delta ACD$

$\Rightarrow \angle BAD = \angle CAD$ (By CPCT) ...(1)

In ΔABP and ΔACP,

AB = AC (∵ ΔABC is isosceles)

$\Rightarrow \angle BAP = \angle CAP$ (By 1)

AP = AP (common side)

∴ $\Delta ABP \cong \Delta ACP$ (By SAS congruence rule)

(iii) $\Delta ABD \cong \Delta ADC$ (Proved above)

$\angle BAD = \angle CAD$ (by CPCT)

$\angle ADB = \angle ADC$ (by CPCT)

$180 - \angle ADB = 180 - \angle ADC$

$\Rightarrow \angle BDP = \angle CDP$

AP bisects $\angle A$ as well as $\angle D$

(iv) $\Delta ABP \cong \Delta ACP$

$\Rightarrow$ BP = CP (By CPCT)

$\Rightarrow$ AP bisects BC

$\angle APB = \angle APC$ (By CPCT)

$\angle APB + \angle APC = 180°$

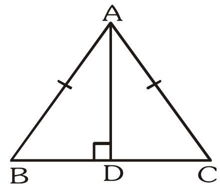
$2\angle APB = 180°$

$\angle APB = 90°$

AP is perpendicular bisector of BC

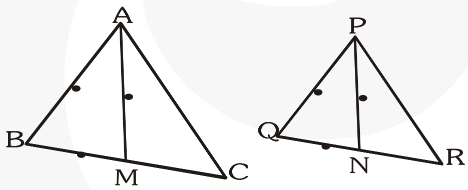
Q2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that
(i) AD bisects BC (ii) AD bisects ∠A

Sol. Given : AD is an altitude of an isosceles triangle ABC in which AB = AC.



To Prove : (i) AD bisect BC. (ii) AD bisect ∠A.
Proof : (i) In right ΔADB and right ΔADC.
Hyp.AB = Hyp. AC
∠ADB = ∠ADC (Each 90°)
Side AD = side AD (Common)
ΔADB ≅ ΔADC (RHS rule)
⇒ BD = CD (By CPCT)
⇒ AD bisect BC
(ii) ΔADB ≅ ΔADC
∠BAD =∠CAD (By CPCT)
⇒ AD bisect ∠A

Q3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to side PQ and QR and median PN of ΔPQR (see figure). Show that :
(i) ΔABM ≅ ΔPQN (ii) ΔABC ≅ ΔPQR



Sol. (i) BM = $\frac{1}{2}$BC (∵ M is mid-point of BC)

QN = $\frac{1}{2}$QR (∵ N is mid-point of QR)

⇒ BM = QN (∵ BC = QR is given)
Now, in ΔABM and ΔPQN, we have
AB = PQ (Given)
BM = QN (Proved)
AM = PN (Given)
∴ ΔABM ≅ ΔPQN (SSS congruence criteria)

(ii) $\Delta ABM \cong \Delta PQN$

$\Rightarrow \angle ABM = \angle PQN$

$\angle B = \angle Q$ (By CPCT)

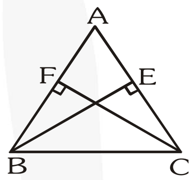
Now, in ΔABC and ΔPQN,

$AB = PQ$, $\angle B = \angle Q$ and $BC = QN$

$\Rightarrow \Delta ABC \cong \Delta PQN$ [by SAS cong.]

Q4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Sol. Given BE and CF are two altitude of ΔABC.



To prove : ΔABC is isosceles.

Proof : In right ΔBEC and right ΔCFB side

BE = side CF (Given)

Hyp.BC = Hyp CB (Common)

$\angle BEC = \angle BFC$ (Each 90°)

$\Delta BEC \cong \Delta CFB$ (RHS Rule)

$\therefore \angle BCE = \angle CBF$ (By CPCT)

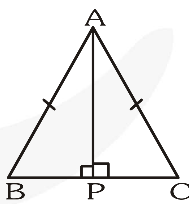
AB = AC

(Side opp. to equal angles are equal)

ΔABC is isosceles.

Q5. ABC is an isosceles triangle with AB = AC. Draw $AP \perp BC$ to show that $\angle B = \angle C$

Sol. In ΔAPB and ΔAPC



AB = AC (Given)

$\angle APB = \angle APC$ (Each = 90°)

AP = AP (common side)

Therefore, by RHS congruence criteria, we have

$\Delta APB \cong \Delta APC$

$\Rightarrow \angle ABP = \angle ACP$ (By CPCT)

$\Rightarrow \angle B = \angle C$