NCERT SOLUTIONS

Triangles

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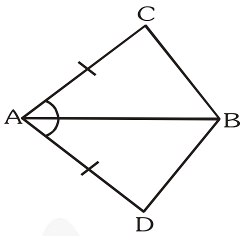
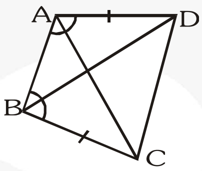
Ex - 7.1

Q1. In quadrilateral ACBD, AC = AD and AB bisects ∠A. Show that ΔABC ≅ ΔABD. What can you say about BC and BD?



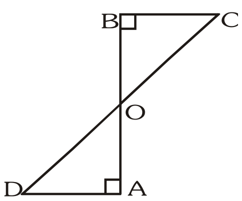
Sol. Given : In quadrilateral ACBD, AC = AD and AB bisect ∠A.
To prove : ΔABC ≅ ΔABD
Proof : In ΔABC and ΔABD
AC = AD (Given)
AB = AB (Common)
∠CAB = ∠DAB (AB bisect ∠A)
∴ ΔABC ≅ ΔABD (by SAS criteria)
BC = BD (by CPCT)

Q2. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA. Prove that
(i) ΔABD ≅ ΔBAC
(ii) BD = AC
(iii) ∠ABD = ∠BAC.



Sol. In ΔABD and ΔBAC,
AD = BC (Given)
∠DAB = ∠CBA (Given)
AB = AB (Common side)
∴ By SAS congruence rule, we have
ΔABD ≅ ΔBAC
Also, by CPCT, we have
BD = AC and ∠ABD = ∠BAC

Q3. AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.



Sol. Given : AD and BC are equal perpendiculars to line AB.

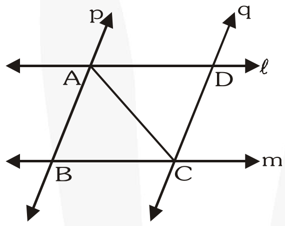
To prove : CD bisect AB

Proof : In ΔOAD and ΔOBC

$AD = BC$	(Given)
$\angle OAD = \angle OBC$	(Each 90°)
$\angle AOD = \angle BOC$	(Vertically opposite angles)
$\Delta OAD \cong \Delta OBC$	(AAS rule)
$OA = OB$	(by CPCT)

$\therefore$ CD bisect AB.

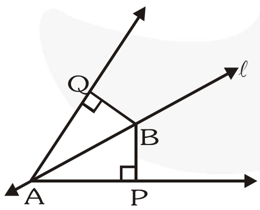
Q4. ℓ and m are two parallel lines intersected by another pair of parallel lines p and q. Show that $\Delta ABC \cong \Delta CDA$.



Sol. In ΔABC and ΔCDA

$\angle CAB = \angle ACD$	(Pair of alternate angle)
$\angle BCA = \angle DAC$	(Pair of alternate angle)
$AC = AC$	(Common side)
$\therefore \Delta ABC \cong \Delta CDA$	(ASA criteria)

Q5. Line ℓ is the bisector of an angle $\angle A$ and B is any point on ℓ. BP and BQ are perpendiculars from B to the arms of $\angle A$. Show that :



(i) $\Delta APB \cong \Delta AQB$

(ii) BP = BQ or B is equidistant from the arms of $\angle A$.

Sol. Given : line ℓ is bisector of angle A and B is any point on ℓ. BP and BQ are perpendicular from B to arms of $\angle A$.

To prove : (i) $\Delta APB \cong \Delta AQB$ (ii) $BP = BQ$.

Proof :

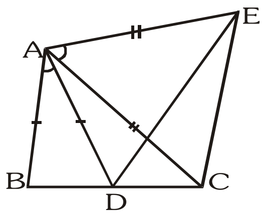
(i) In ΔAPB and ΔAQB

$\angle BAP = \angle BAQ$	(ℓ is bisector)
$AB = AB$	(common)
$\angle BPA = \angle BQA$	(Each 90°)
$\therefore \Delta APB \cong \Delta AQB$	(AAS rule)

(ii) $\Delta APB \cong \Delta AQB$

$BP = BQ$	(By CPCT)

Q6. In figure, AC = AE, AB = AD and $\angle BAD = \angle EAC$. Show that BC = DE.



Sol. Given : AC = AE

AB = AD,

$\angle BAD = \angle EAC$

To prove : BC = DE

Proof : In ΔABC and ΔADE

AB = AD (Given)

AC = AE (Given)

$\angle BAD = \angle EAC$

Add $\angle DAC$ to both

$\Rightarrow \angle BAD + \angle DAC = \angle DAC + \angle EAC$

$\angle BAC = \angle DAE$

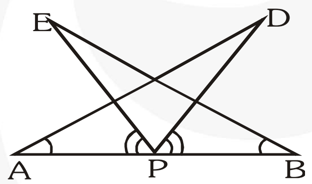
$\Delta ABC \cong \Delta ADE$ (SAS rule)

BC = DE (By CPCT)

Q7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that

$\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that

(i) $\Delta DAP \cong \Delta EBP$ (ii) AD = BE



Sol. $\angle EPA = \angle DPB$ (Given)

$\Rightarrow \angle EPA + \angle DPE = \angle DPB + \angle DPE$

$\Rightarrow \angle APD = \angle BPE$ ...(1)

Now, in ΔDAP and ΔEBP, we have

AP = PB ($\because$ P is mid point of AB)

$\angle PAD = \angle PBE$

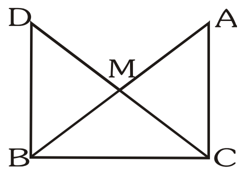
$$\left\{\begin{array}{l}\because \angle PAD = \angle BAD,\ \angle PBE = \angle ABE \\ \text{and we are given that } \angle BAD = \angle ABE\end{array}\right\}$$

Also, $\angle APD = \angle BPE$ (By 1)

$\therefore \Delta DAP \cong \Delta EBP$ (By ASA congruence)

$\Rightarrow$ AD = BE (By CPCT)

Q8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that :



(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

Sol. (i) In $\Delta AMC \cong \Delta BMD$,

$AM = BM$ ($\because$ M is mid point of AB)

$\angle AMC = \angle BMD$ (Vertically opposite angles)

$CM = DM$ (Given)

$\therefore \Delta AMC \cong \Delta BMD$ (By SAS congruence)

(ii) $\angle AMC = \angle BMD$,

$\Rightarrow \angle ACM = \angle BDM$ (By CPCT)

$\Rightarrow CA \parallel BD$

$\Rightarrow \angle BCA + \angle DBC = 180°$

$\Rightarrow \angle DBC = 90°$ ($\because \angle BCA = 90°$)

(iii) In ΔDBC and ΔACB,

$DB = AC$ ($\because \Delta BMD \cong \Delta AMC$)

$\angle DBC = \angle ACB$ (Each $= 90°$)

$BC = BC$ (Common side)

$\therefore \Delta DBC \cong \Delta ACB$ (By SAS congruence)

(iv) In $\Delta DBC \cong \Delta ACB \Rightarrow CD = AB$ ...(1)

Also, $\Delta AMC \cong \Delta BMD$

$\Rightarrow CM = DM$

$\Rightarrow CM = DM = \frac{1}{2} CD$

$\Rightarrow CD = 2\ CM$ ...(2)

From (1) and (2),

$2\ CM = AB$

$\Rightarrow CM = \frac{1}{2} AB$

Ex - 7.2

Q1. In an isosceles triangle ABC, with AB = AC, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that : (i) OB = OC (ii) AO bisects $\angle A$.

Sol. (i) In ΔABC, OB and OC are bisectors of $\angle B$ and $\angle C$.

$\therefore \angle OBC = \frac{1}{2}\angle B$ ...(1)

$\angle OCB = \frac{1}{2}\angle C$ ...(2)

Also, AB = AC (Given)

$\Rightarrow \angle B = \angle C$ ...(3)

From (1), (2), (3), we have

$\angle OBC = \angle OCB$

Now, in ΔOBC, we have

$\angle OBC = \angle OCB$

$\Rightarrow$ OB = OC

(Sides opposite to equal angles are equal)

(ii) $\angle OBA = \frac{1}{2}\angle B$ and $\angle OCA = \frac{1}{2}\angle C$

$\Rightarrow \angle OBA = \angle OCA$ ($\because \angle B = \angle C$)

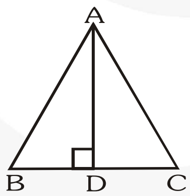
AB = AC and OB = OC

$\therefore \Delta OAB \cong \Delta OAC$ (SAS congruence criteria)

$\Rightarrow \angle OAB = \angle OAC$

$\Rightarrow$ AO bisects $\angle A$.

Q2. In ΔABC, AD is the perpendicular bisector of BC. Show that ΔABC is an isosceles triangle in which AB = AC.



Sol. Given : In ΔABC, AD is perpendicular bisector of BC.

To Prove : ΔABC is isosceles Δ with AB = AC

Proof : In ΔADB and ΔADC

$\angle ADB = \angle ADC$ (Each 90°)

DB = DC (AD is $\perp$ bisector of BC)

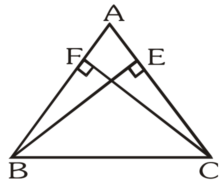
AD = AD (Common)

$\Delta ADB \cong \Delta ADC$ (By SAS rule)

AB = AC (By CPCT)

$\therefore \Delta ABC$ is an isosceles Δ with AB = AC

Q3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively. Show that these altitudes are equal.



Sol. In ΔABE and ΔACF, we have

$\angle BEA = \angle CFA$ (Each = 90°)

$\angle A = \angle A$ (Common angle)

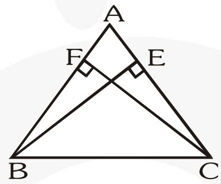
$AB = AC$ (Given)

$\therefore\ \Delta ABE \cong \Delta ACF$ (By AAS congruence criteria)

$\Rightarrow BE = CF$ (By CPCT)

Q4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that (i) $\Delta ABE \cong \Delta ACF$

(ii) AB = AC, i.e., ABC is an isosceles triangle.



Sol. (i) In ΔABE and ΔACF, we have

$\angle A = \angle A$ (Common)

$\angle AEB = \angle AFC$ (Each = 90°)

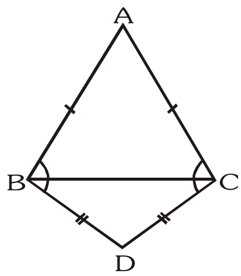
$BE = CF$ (Given)

$\therefore\ \Delta ABE \cong \Delta ACF$(By ASA congruence)

(ii) $\Delta ABE \cong \Delta ACF$

$\Rightarrow AB = AC$ (By CPCT)

Q5. ABC and DBC are two isosceles triangles on the same base BC (see figure). Show that $\angle ABD = \angle ACD$.



Sol. Given : ABC and BCD are two isosceles triangle on common base BC.

To prove : $\angle ABC = \angle ACD$

Proof : ABC is an isosceles

Triangle on base BC

$\therefore \angle ABC = \angle ACB$ ...(1)

$\because$ DBC is an isosceles Δ on base BC.

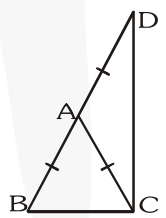
$\angle DBC = \angle DCB$ ...(2)

Adding (1) and (2)

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

$\Rightarrow \angle ABD = \angle ACD$

Q6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see figure). Show that $\angle BCD$ is a right angle.



Sol. In ΔABC, AB = AC

$\Rightarrow \angle ACB = \angle ABC$ ...(1)

In ΔACD,

AD = AB (By construction)

$\Rightarrow$ AD = AC

$\Rightarrow \angle ACD = \angle ADC$ ...(2)

Adding (1) and (2),

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$

$\Rightarrow \angle BCD = \angle ABC + \angle ADC$

In $\angle DBC + \angle ABC + \angle BCD + \angle CDB = 180°$

$\Rightarrow 2\ \angle BCD = 180°$

$\Rightarrow \angle BCD = 90°$

Q7. ABC is a right angled triangle in which $\angle A = 90°$ and AB = AC. Find $\angle B$ and $\angle C$.

Sol. In ΔABC

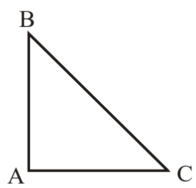
AB = AC

$\angle B = \angle C$ ...(1)

(angles opposite to equal sides are equal)

In ΔABC

$\angle A + \angle B + \angle C = 180°$



$90° + \angle B + \angle C = 180°$

$\angle B + \angle C = 90°$ ...(2)

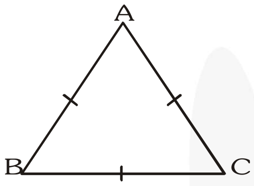
from (1) and (2)

$\angle B = \angle C = 45°$

Q8. Show that the angles of an equilateral triangle are 60° each.

Sol. ΔABC is equilateral triangle.

$\Rightarrow AB = BC = CA$



Now, $AB = BC$

$\Rightarrow BA = BC$

$\Rightarrow \angle C = \angle A$ ...(1)

Similarly, $\angle A = \angle B$ ...(2)

From (1) and (2),

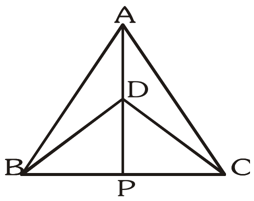
$\angle A = \angle B = \angle C$ ...(3)

Also, $\angle A + \angle B + \angle C = 180°$ ...(4)

$\Rightarrow \angle A = \angle B = \angle C = \frac{1}{3} \times 180° = 60°$

Ex - 7.3

Q1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see figure). If AD is extended to intersect BC at P, show that



(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$

(iv) AP is the perpendicular bisector of BC.

Sol. In ΔABD and ΔACD,

$AB = AC$ ($\because$ ΔABC is isosceles)

$DB = DC$ ($\because$ ΔDBC is isosceles)

$AD = AD$ (Common side)

$\therefore$ $\Delta ABD \cong \Delta ACD$ (By SSS congruence rule)

(ii) Now, $\Delta ABD \cong \Delta ACD$

$\Rightarrow \angle BAD = \angle CAD$ (By CPCT) ...(1)

In ΔABP and ΔACP,

$AB = AC$ ($\because$ ΔABC is isosceles)

$\Rightarrow \angle BAP = \angle CAP$ (By 1)

$AP = AP$ (common side)

$\therefore$ $\Delta ABP \cong \Delta ACP$ (By SAS congruence rule)

(iii) $\Delta ABD \cong \Delta ADC$ (Proved above)

$\angle BAD = \angle CAD$ (by CPCT)

$\angle ADB = \angle ADC$ (by CPCT)

$180 - \angle ADB = 180 - \angle ADC$

$\Rightarrow \angle BDP = \angle CDP$

AP bisects $\angle A$ as well as $\angle D$

(iv) $\Delta ABP \cong \Delta ACP$

$\Rightarrow BP = CP$ (By CPCT)

$\Rightarrow$ AP bisects BC

$\angle APB = \angle APC$ (By CPCT)

$\angle APB + \angle APC = 180°$

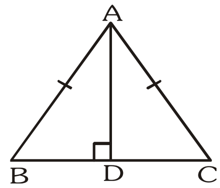
$2\angle APB = 180°$

$\angle APB = 90°$

AP is perpendicular bisector of BC

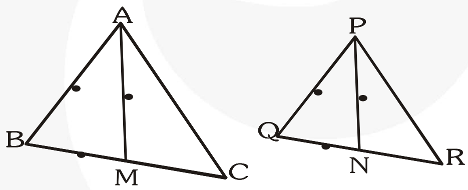
Q2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that
(i) AD bisects BC (ii) AD bisects $\angle A$

Sol. Given : AD is an altitude of an isosceles triangle ABC in which AB = AC.



To Prove : (i) AD bisect BC. (ii) AD bisect $\angle A$.
Proof : (i) In right ΔADB and right ΔADC.
Hyp.AB = Hyp. AC
$\angle ADB = \angle ADC$ (Each 90°)
Side AD = side AD (Common)
$\Delta ADB \cong \Delta ADC$ (RHS rule)
$\Rightarrow$ BD = CD (By CPCT)
$\Rightarrow$ AD bisect BC
(ii) $\Delta ADB \cong \Delta ADC$
$\angle BAD = \angle CAD$ (By CPCT)
$\Rightarrow$ AD bisect $\angle A$

Q3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to side PQ and QR and median PN of ΔPQR (see figure). Show that :
(i) $\Delta ABM \cong \Delta PQN$ (ii) $\Delta ABC \cong \Delta PQR$



Sol. (i) $BM = \frac{1}{2}BC$ ($\because$ M is mid-point of BC)

$QN = \frac{1}{2}QR$ ($\because$ N is mid-point of QR)

$\Rightarrow$ BM = QN ($\because$ BC = QR is given)
Now, in ΔABM and ΔPQN, we have
AB = PQ (Given)
BM = QN (Proved)
AM = PN (Given)
$\therefore\ \Delta ABM \cong \Delta PQN$ (SSS congruence criteria)

(ii) $\Delta ABM \cong \Delta PQN$

$\Rightarrow \angle ABM = \angle PQN$

$\angle B = \angle Q$ (By CPCT)

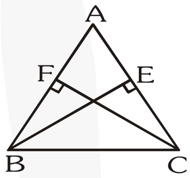
Now, in ΔABC and ΔPQN,

$AB = PQ$, $\angle B = \angle Q$ and $BC = QN$

$\Rightarrow \Delta ABC \cong \Delta PQN$ [by SAS cong.]

Q4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Sol. Given BE and CF are two altitude of ΔABC.



To prove : ΔABC is isosceles.

Proof : In right ΔBEC and right ΔCFB side

BE = side CF (Given)

Hyp.BC = Hyp CB (Common)

$\angle BEC = \angle BFC$ (Each $90°$)

$\Delta BEC \cong \Delta CFB$ (RHS Rule)

$\therefore \angle BCE = \angle CBF$ (By CPCT)

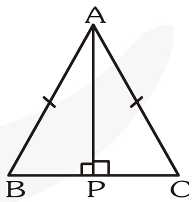
AB = AC

(Side opp. to equal angles are equal)

ΔABC is isosceles.

Q5. ABC is an isosceles triangle with AB = AC. Draw $AP \perp BC$ to show that $\angle B = \angle C$

Sol. In ΔAPB and ΔAPC



AB = AC (Given)

$\angle APB = \angle APC$ (Each = $90°$)

AP = AP (common side)

Therefore, by RHS congruence criteria, we have

$\Delta APB \cong \Delta APC$

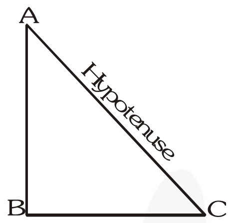
$\Rightarrow \angle ABP = \angle ACP$ (By CPCT)

$\Rightarrow \angle B = \angle C$

Ex - 7.4

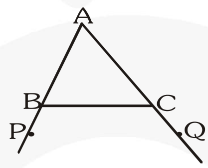
Q1. Show that in a right angled triangle, the hypotenuse is the longest side.

Sol. ΔABC is right angled at B. AC is hypotenuse.



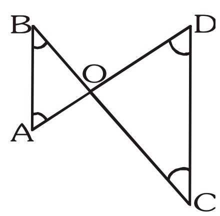
Now, $\angle B = 90°$
and $\angle A + \angle C = 90°$
$\Rightarrow \angle A < 90°$
and $\angle C < 90°$
$\Rightarrow \angle B > \angle A$
and $\angle B > \angle C$
$\Rightarrow AC > BC$
and $AC > AB$.
$\therefore$ Hypotenuse AC is the longest side of the right angled ΔABC.

Q2. In figure, sides AB and AC of ΔABC are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$



Sol. Sides AB and AC of ΔABC are extended to points P and Q
To prove : $AC > AB$
Proof : $\angle PBC < \angle QCB$ (Given)
$180 - \angle PBC > 180 - \angle QCB$
$\angle ABC > ACB$
$\Rightarrow AC > AB$
(sides opposite to greater angle is longer)

Q3. In fig, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.



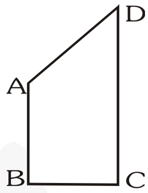
Sol. $\angle B < \angle A$ in $\Delta OAB \Rightarrow OA < OB$ ...(1)
Also, $\angle C < \angle D$ in $\Delta OCD \Rightarrow OD < OC$ ...(2)

Adding (1) and (2),

OA + OD < OB + OC

$\Rightarrow$ AD < BC

Q4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see figure). Show that $\angle A > \angle C$ and $\angle B > \angle D$.



Sol. In quadrilateral ABCD, AB is the smallest side and CD is the longest side. Join AC and BD.

In ΔABC, BC > AB (∵ AB is smallest side)

$\Rightarrow \angle 1 > \angle 3$ ...(1)

In ΔACD, CD > AD (∵ CD is longest side)

$\Rightarrow \angle 2 > \angle 4$ ...(2)

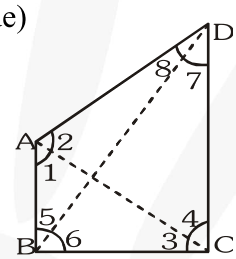
Ading (1) and (2), we have

$\angle 1 + \angle 2 > \angle 3 + \angle 4$

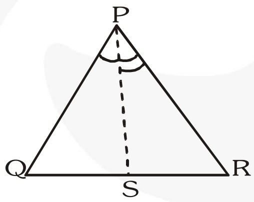
$\Rightarrow \angle A > \angle C$

Similarly, we can prove that

$\angle B > \angle D$



Q5. In figure, PR > PQ and PS bisects $\angle$QPR. Prove that $\angle PSR > \angle PSQ$.



Sol. Given : PR > PQ, PS bisect $\angle$QPR.

To prove : $\angle PSR > \angle PSQ$

Proof : In ΔPQR

PR > PQ (Given)

$\angle PQR > \angle PRQ$ ...(1)

(angle opposite to longer side is greater)

PS bisects $\angle$QPR

$\Rightarrow \angle QPS = \angle RPS$

In ΔPQS

$\angle PQR + \angle QPS + \angle PSQ = 180°$ ...(2)

In ΔPRS

$\angle PSR + \angle SPR + \angle SRP = 180°$ ...(3)

From (2) and (3)

$\angle PQR + \angle QPS + \angle PSQ$

$= \angle PSR + \angle SPR + \angle SRP$

$\angle PQR + \angle PSQ = \angle PSR + \angle PRS$

$\angle PRS + \angle PSR = \angle PQR + \angle PSQ$

$\angle PRS + \angle PSR > \angle PRQ + \angle PSQ$

$\angle PRQ + \angle PSR > \angle PRQ + \angle PSQ$

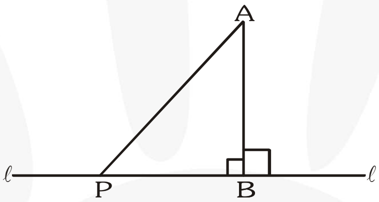
$\angle PSR > \angle PSQ$

Q6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Sol. Let us have AB as the perpendicular line segment and AP is any other line segment. Now ΔABP is right angled and AP is hypotenuse.

Here, $\angle B > \angle P$ $\quad$ ($\because \angle B = 90°$)

$\Rightarrow AP > AB$



Thus perpendicular line segment is smallest.