NCERT SOLUTIONS

Similar Triangle

Ex - 6.1

Q1. Fill in the blanks using the correct word given in brackets :

(i) All circles are _____. (congruent, similar)

(ii) All squares are _____. (similar, congruent)

(iii) All _____ triangles are similar.
(isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____ and (b) their corresponding sides are _____. (equal, proportional)

Sol. (i) All circles are similar.

(ii) All squares are similar.

(iii) All equilateral triangles are similar.

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are equal and (b) their corresponding sides are proportional.

Q2. Give two different examples of pair of

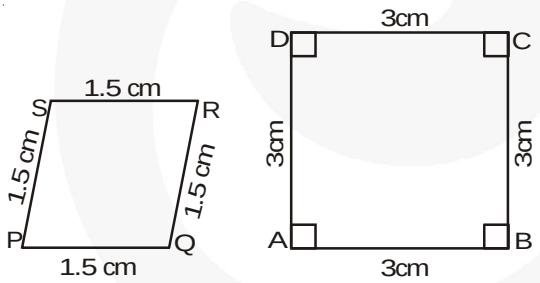
(i) Similar figures.

(ii) Non-similar figures.

Sol. (i) 1. Pair of equilateral triangles are similar figures.
2. Pair of squares are similar figures.

(ii) 1. One equilateral triangle and one isosceles triangle are non-similar.
2. Square and rectangle are non-similar.

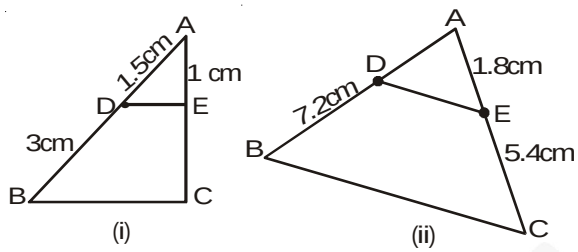
Q3. State whether the following quadrilaterals are similar or not :



Sol. The two quadrilateral in figure are not similar because their corresponding angles are not equal.

Ex - 6.2

Q1. In figure, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



Sol. (i) In figure, (i) DE || BC (Given)

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$ (By Basic Proportionality Theorem)

$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$

{∵ AD = 1.5 cm, DB = 3 cm and AE = 1 cm}

$\Rightarrow$ EC = $\frac{3}{1.5}$ = 2 cm

(ii) In fig. (ii) $DE \| BC$ (given)

So, $\frac{AD}{BD} = \frac{AE}{CE} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$

{∵ BD = 7.2, AE = 1.8 cm and CE = 5.4 cm}

AD = 2.4 cm

Q2. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, State whether EF || QR :

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm.

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm.

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.

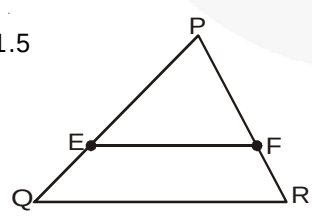
Sol. (i) In figure,

$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3,$

$\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = 1.5$

$\Rightarrow \frac{PE}{EQ} \neq \frac{PF}{FR}$

$\Rightarrow$ EF is not || QR



(ii) In figure,

$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$ and $\frac{PF}{FR} = \frac{8}{9}$

$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR} \Rightarrow$ EF || QR

(iii) In figure,

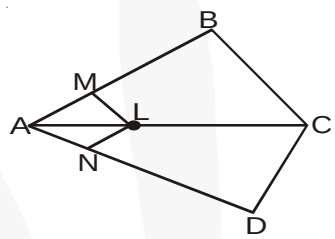
$$\frac{PE}{QE}=\frac{0.18}{PQ-PE}=\frac{0.18}{1.28-0.18}=\frac{0.18}{1.10}$$

$$=\frac{18}{110}=\frac{9}{55}=\frac{PF}{FR}=\frac{0.36}{PR-PF}$$

$$=\frac{0.36}{2.56-0.36}=\frac{0.36}{2.20}=\frac{9}{55}=\frac{PE}{QE}=\frac{PF}{FR}$$

$\therefore\ EF \| QR$ (By converse of Basic Proportionality Theorem)

Q3. In figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB}=\frac{AN}{AD}$.



Sol. In ΔACB (see figure), LM || CB (Given)

$\Rightarrow \frac{AM}{MB}=\frac{AL}{LC}$ ...(1)

(Basic Proportionality Theorem)

In ΔACD (see figure), LN || CD(Given)

$\Rightarrow \frac{AN}{ND}=\frac{AL}{LC}$ ...(2)

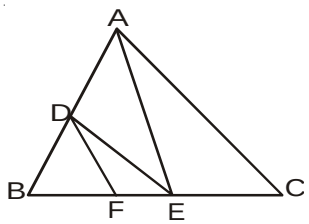
(Basic Proportionality Theorem)

From (1) and (2), we get

$$\frac{AM}{MB}=\frac{AN}{ND}$$

$$\Rightarrow \frac{AM}{AM+MB}=\frac{AN}{AN+ND} \Rightarrow \frac{AM}{AB}=\frac{AN}{AD}$$

Q4. In figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE}=\frac{BE}{EC}$.



Sol. In ΔABE,

$DF \parallel AE$ (Given)

$\frac{BD}{DA} = \frac{BF}{FE}$ ...(i) (Basic Proportionality Theorem)

In ΔABC,

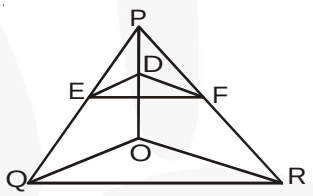
$DE \parallel AC$ (Given)

$\frac{BD}{DA} = \frac{BE}{EC}$ .....(ii) (Basic Proportionality Theorem)

From (i) and (ii), we get

$\frac{BF}{FE} = \frac{BE}{EC}$ Hence proved.

Q5. In figure, DE || OQ and DF || OR. Show that EF || QR.



Sol. In figure, DE || OQ and DF || OR, then by Basic Proportionality Theorem,

We have $\frac{PE}{EQ} = \frac{PD}{DO}$ ...(1)

and $\frac{PF}{FR} = \frac{PD}{DO}$ ...(2)

From (1) and (2), $\frac{PE}{EQ} = \frac{PF}{FR}$

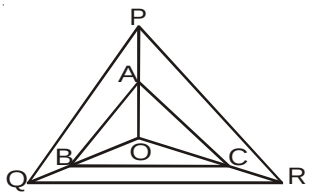
Now, in ΔPQR, we have proved that

$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$

EF || QR

(By converse of Basic Proportionality Theorem)

Q6. In figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Sol. In ΔPOQ,

$AB \parallel PQ$ (given)

$\frac{OB}{BQ} = \frac{OA}{AP}$ ...(i) (Basic Proportionality Theorem)

In ΔPOR,

$AC \parallel PR$ (given)

$\frac{OA}{AP} = \frac{OC}{CR}$ ...(ii) (Basic Proportionality Theorem)

From (i) and (ii), we get

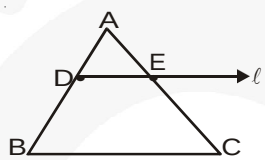
$\frac{OB}{BQ} = \frac{OC}{CR}$

$\therefore$ By converse of Basic Proportionality Theorem,

$BC \parallel QR$

Q7. Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

Sol. In ΔABC, D is mid point of AB (see figure)



i.e., $\frac{AD}{DB} = 1$ ...(1)

Straight line $\ell \parallel BC$.

Line ℓ is drawn through D and it meets AC at E.

By Basic Proportionality Theorem

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AE}{EC} = 1$ [From (1)]

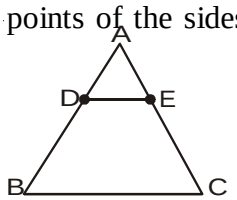
$\Rightarrow$ AE = EC $\Rightarrow$ E is mid point of AC.

Q8. Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

Sol. In ΔABC, D and E are mid points of the sides AB and AC respectively.

$\Rightarrow \frac{AD}{DB} = 1$

and $\frac{AE}{EC} = 1$ (see figure)



$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} \quad \Rightarrow$ DE || BC

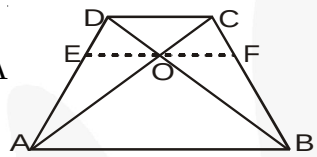
(By Converse of Basic Proportionality Theorem)

Q9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Sol. We draw EOF || AB(also || CD) (see figure)

In ΔACD, OE || CD

$\Rightarrow \frac{AE}{ED} = \frac{AO}{OC}$ ...(1)



In ΔABD, OE || BA

$\Rightarrow \frac{DE}{EA} = \frac{DO}{OB}$

$\Rightarrow \frac{AE}{ED} = \frac{OB}{OD}$ ...(2)

From (1) and (2)

$\frac{AO}{OC} = \frac{OB}{OD}$,

i.e., $\frac{AO}{BO} = \frac{CO}{DO}$.

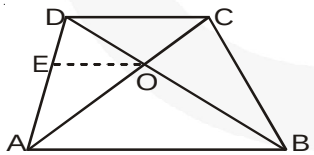
Q10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Sol. In figure $\frac{AO}{BO} = \frac{CO}{DO}$

$\Rightarrow \frac{AO}{OC} = \frac{BO}{OD}$ ...(1) (given)

Through O, we draw

OE || BA



OE meets AD at E.

From ΔDAB,

EO || AB

$\Rightarrow \frac{DE}{EA} = \frac{DO}{OB}$ (by Basic Proportionality Theorem)

$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD}$ ...(2)

From (1) and (2),

$\frac{AO}{OC} = \frac{AE}{ED} \quad \Rightarrow$ OE || CD

(by converse of basic proportionality theorem)

Now, we have BA || OE

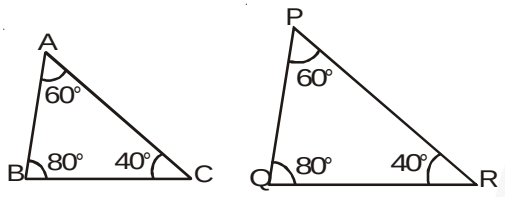
and OE || CD

$\Rightarrow$ AB || CD

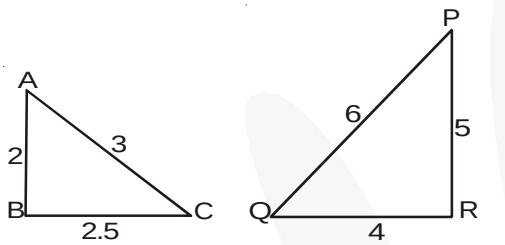
$\Rightarrow$ Quadrilateral ABCD is a trapezium.

Ex - 6.3

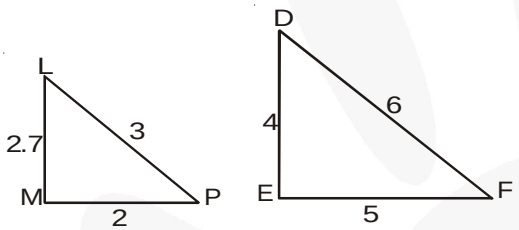
Q1. State which pairs of triangles in figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :



(i)



(ii)



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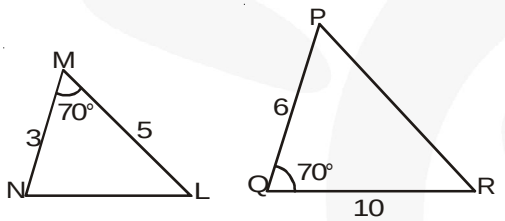
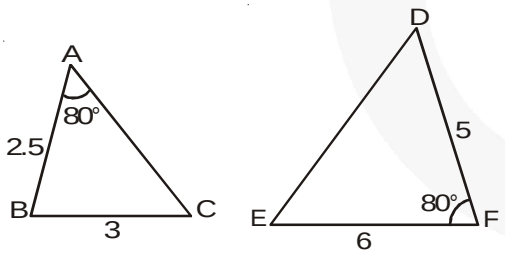
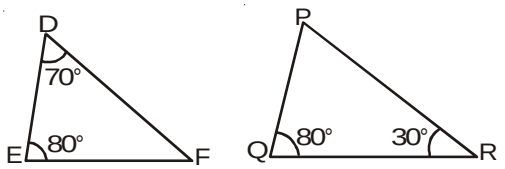


(iv)



(v)



(vi)

Sol. (i) Yes. $\angle A = \angle P = 60°$, $\angle B = \angle Q = 80°$,

$\angle C = \angle R = 40°$

Therefore, $\Delta ABC \sim \Delta PQR$.

By AAA similarity criterion

(ii) Yes.

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}, \frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2}$$

Therefore, $\Delta ABC \sim \Delta QRP$.

By SSS similarity criterion.

(iii) No.

$$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}, \frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}, \frac{LM}{EF} = \frac{2.7}{5} \neq \frac{1}{2}$$

i.e., $\frac{MP}{DE} = \frac{LP}{DF} \neq \frac{LM}{EF}$

Thus, the two triangles are not similar.

(iv) Yes,

$$\frac{MN}{QP} = \frac{ML}{QR} = \frac{1}{2}$$

and $\angle NML = \angle PQR = 70°$

By SAS similarity criterion

$\Delta NML \sim \Delta PQR$

(v) No,

$$\frac{AB}{FD} \neq \frac{AC}{FE}$$

Thus, the two triangles are not similar

(vi) In triangle DEF $\angle D + \angle E + \angle F = 180°$

$70° + 80° + \angle F = 180°$

$\angle F = 30°$

In triangle PQR

$\angle P + 80° + 30° = 180°$

$\angle P = 70°$

$\angle E = \angle Q = 80°$

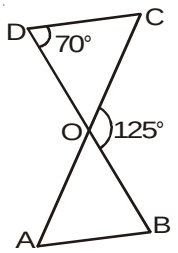
$\angle D = \angle P = 70°$

$\angle F = \angle R = 30°$

By AAA similarity criterion,

$\Delta DEF \sim \Delta PQR$.

Q2. In figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Sol. From figure,

$\angle DOC + 125° = 180°$

$\Rightarrow \angle DOC = 180° - 125° = 55°$

$\angle DCO + \angle CDO + \angle DOC = 180°$

(Sum of three angles of ΔODC)

$\Rightarrow \angle DCO + 70° + 55° = 180°$

$\Rightarrow \angle DCO + 125° = 180°$

$\Rightarrow \angle DCO = 180° - 125° = 55°$

Now, we are given that $\Delta ODC \sim \Delta OBA$

$\Rightarrow \angle OCD = \angle OAB$

$\Rightarrow \angle OAB = \angle OCD = \angle DCO = 55°$

i.e., $\angle OAB = 55°$

Hence, we have

$\angle DOC = 55°$, $\angle DCO = 55°$, $\angle OAB = 55°$

Q3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

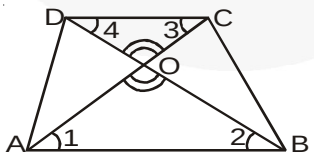
Sol. In figure, AB || DC

$\Rightarrow \angle 1 = \angle 3$, $\angle 2 = \angle 4$

(Alternate interior angles)

Also $\angle DOC = \angle BOA$

(Vertically opposite angles)

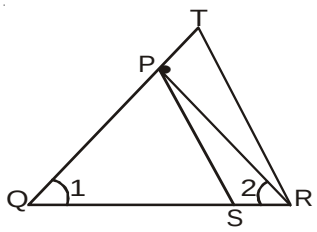


$\Rightarrow \Delta OCD \sim \Delta OAB \quad \Rightarrow \frac{OC}{OA} = \frac{OD}{OB}$

(Ratios of the corresponding sides of the similar triangle)

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$ (Taking reciprocals)

Q4. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Sol. In figure, $\angle 1 = \angle 2$ (Given)

$\Rightarrow$ PQ = PR

(Sides opposite to equal angles of ΔPQR)

We are given that

$$\frac{QR}{QS} = \frac{QT}{PR}$$

$$\Rightarrow \frac{QR}{QS} = \frac{QT}{PQ} \quad (\because \text{ PQ = PR proved})$$

$$\Rightarrow \frac{QS}{QR} = \frac{PQ}{QT} \quad \text{(Taking reciprocals) ...(1)}$$

Now, in ΔPQS and ΔTQR, we have

$\angle PQS = \angle TQR$ (Each = $\angle 1$)

and $\frac{QS}{QR} = \frac{PQ}{QT}$ (By (1))

Therefore, by SAS similarity criterion, we have

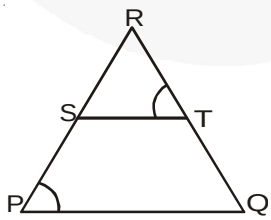
$\Delta PQS \sim \Delta TQR$.

Q5. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Sol. In figure, We have ΔRPQ and ΔRTS in which

$\angle RPQ = \angle RTS$ (Given)

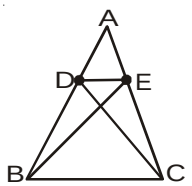
$\angle PRQ = \angle SRT$ (Each = $\angle R$)



Then by AA similarity criterion, we have

$\Delta RPQ \sim \Delta RTS$

Q6. In figure, if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.



Sol. In figure,

$\Delta ABE \cong \Delta ACD$ (Given)

$\Rightarrow$ AB = AC and AE = AD (CPCT)

$\Rightarrow \frac{AB}{AC} = 1$ and $\frac{AD}{AE} = 1$

$\Rightarrow \frac{AB}{AC} = \frac{AD}{AE}$ (Each = 1)

Now, in ΔADE and ΔABC, we have

$\frac{AD}{AE} = \frac{AB}{AC}$ (proved)

i.e., $\frac{AD}{AB} = \frac{AE}{AC}$

and also $\angle DAE = \angle BAC$ (Each = $\angle A$)

$\Rightarrow$ $\Delta ADE \sim \Delta ABC$(By SAS similarity criterion)

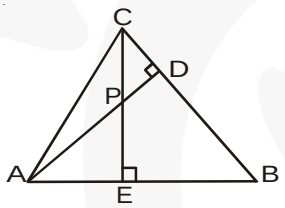
Q7. In figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that :

(i) $\Delta AEP \sim \Delta CDP$

(ii) $\Delta ABD \sim \Delta CBE$

(iii) $\Delta AEP \sim \Delta ADB$

(iv) $\Delta PDC \sim \Delta BEC$



Sol. (i) In ΔAEP and ΔCDP,

$\angle APE = \angle CPD$ (vertically opposite angles)

$\angle AEP = \angle CDP = 90°$

$\therefore$ By AA similarity

$\Delta AEP \sim \Delta CDP$

(ii) In ΔABD and ΔCBE,

$\angle ABD = \angle CBE$ (common)

$\angle ADB = \angle CEB = 90°$

$\therefore$ By AA similarity

$\Delta ABD \sim \Delta CBE$

(iii) In ΔAEP and ΔADB,

$\angle PAE = \angle DAB$ (common)

$\angle AEP = \angle ADB = 90°$

$\therefore$ By AA similarity

$\Delta AEP \sim \Delta ADB$

(iv) In ΔPDC and ΔBEC,

$\angle PCD = \angle BCE$ (common)

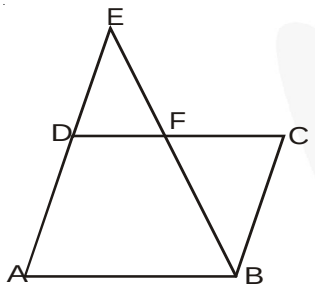
$\angle PDC = \angle BEC = 90°$

$\therefore$ By AA similarity

$\Delta PDC \sim \Delta BEC$

Q8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$.

Sol.



In ΔABE and ΔCFB,

$\angle EAB = \angle BCF$ (opp. angles of parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles, As $AE \parallel BC$)

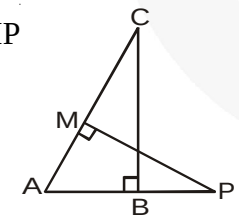
$\therefore$ By AA similarity

$\Delta ABE \sim \Delta CFB$

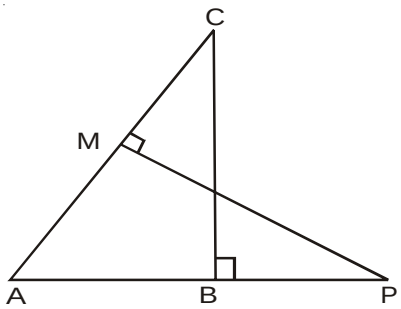
Q9. In figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$



Sol.



(i) In ΔABC and ΔAMP

$\angle CAB = \angle PAM$ (common)

$\angle ABC = \angle AMP = 90°$

$\therefore$ By AA similarity

$\Delta ABC \sim \Delta AMP$

(ii) As $\Delta ABC \sim \Delta AMP$ (Proved above)

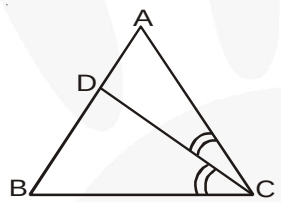
$$\therefore \quad \frac{CA}{PA} = \frac{BC}{MP}$$

Q10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, show that :

(i) $\frac{CD}{GH} = \frac{AC}{FG}$ (ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

Sol. $\Delta ABC \sim \Delta FEG$



$\Rightarrow \angle ACB = \angle EGF$

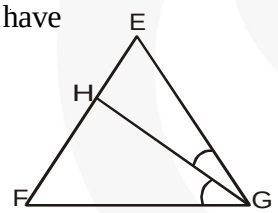
$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle EGF$

$\Rightarrow \angle DCB = \angle HGE$ ...(1)

Also, $\angle B = \angle E$

$\Rightarrow \angle DBC = \angle HEG$ ...(2)

From (1) and (2), we have



$\Rightarrow \Delta DCB \sim \Delta HGE$

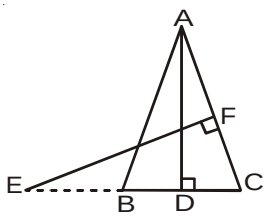
Similarly, we have

$\Delta DCA \sim \Delta HGF$

Now, $\Delta DCA \sim \Delta HGF$

$\Rightarrow \frac{DC}{HG} = \frac{CA}{GF} \Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

Q11. In figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD $\perp$ BC and EF $\perp$ AC, prove that Δ ABD $\sim$ ΔECF.



Sol. In figure,

We are given that ΔABC is isosceles.

and $AB = AC$

$\Rightarrow$ $\angle B = \angle C$ ...(1)

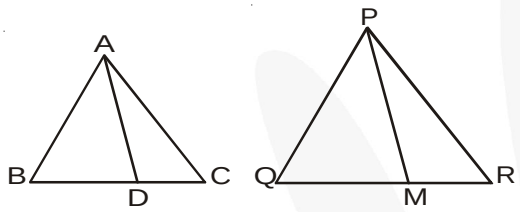
For triangles ABD and ECF,

$\angle ABD = \angle ECF$ {from (1)}

and $\angle ADB = \angle EFC$ {each = 90°}

$\Rightarrow$ $\Delta ABD \sim \Delta ECF$ (AA similarity)

Q12. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

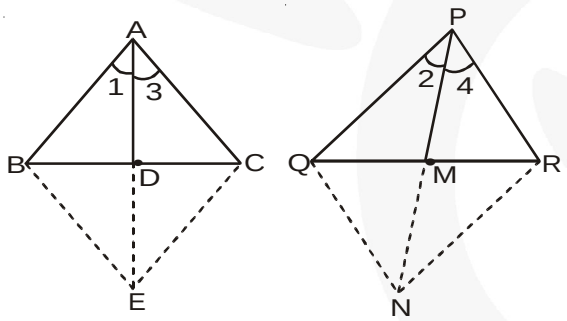


Sol. Given. ΔABC and ΔPQR. AD and PM are their medians respectively.

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad ...(1)$$

To prove. $\Delta ABC \sim \Delta PQR$.

Construction : Produce AD to E such that AD = DE and produce PM to N such that PM = MN. Join BE, CE, QN, RN.



Proof : Quadrilaterals ABEC and PQNR are parallelograms because their diagonals bisect each other at D and M respectively.

$\Rightarrow$ BE = AC and QN = PR.

$\Rightarrow \frac{BE}{QN} = \frac{AC}{PR} \Rightarrow \frac{BE}{QN} = \frac{AB}{PQ}$ (By 1)

i.e., $\frac{AB}{PQ} = \frac{BE}{QN}$ ...(2)

From (1), $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$

i.e., $\frac{AB}{PQ} = \frac{AE}{PN}$ ...(3)

From (2) and (3), we have

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$\Rightarrow$ $\Delta ABE \sim \Delta PQN \Rightarrow$ $\angle 1 = \angle 2$ ...(4)

Similarly, we can prove

$\Rightarrow$ $\Delta ACE \sim \Delta PRN \Rightarrow$ $\angle 3 = \angle 4$ ...(5)

Adding (4) and (5), we have

$\Rightarrow$ $\angle 1 + \angle 3 = \angle 2 + \angle 4$ $\Rightarrow$ $\angle A = \angle P$

$\Rightarrow$ $\Delta ABC \sim \Delta PQR$ (SAS similarity criterion)

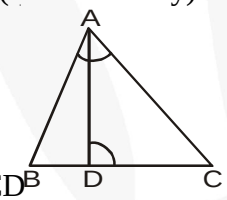
Q13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.\ CD$.

Sol. For ΔABC and ΔDAC, We have

$\angle BAC = \angle ADC$ (Given)

and $\angle ACB = \angle DCA$ (Each $= \angle C$)

$\Rightarrow$ $\Delta ABC \sim \Delta DAC$ (AA similarity)



$\Rightarrow$ $\frac{AC}{DC} = \frac{CB}{CA}$

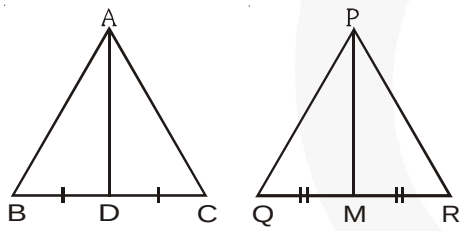
$\Rightarrow$ $\frac{CA}{CD} = \frac{CB}{CA}$

$\Rightarrow$ $CA \times CA = CB \times CD$

$\Rightarrow$ $CA^2 = CB \times CD$

Q14. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see figure). Show that $\Delta ABC \sim \Delta PQR$.

Sol.



As, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ (Given)

So, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$$\left\{\because \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QM}\right\}$$

$\therefore$ By SSS similarity,

$\Delta ABD \sim \Delta PQM$.

As, $\Delta ABD \sim \Delta PQM$.

$\therefore \quad \angle ABD = \angle PQM$

Now, In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{BC}{QR} \text{ (Given)}$$

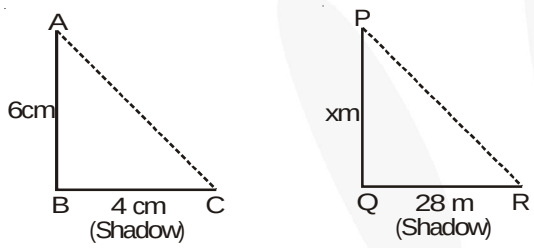
$\angle ABC = \angle PQR$ (Proved above)

$\therefore$ By SAS similarity

$\Delta ABC \sim \Delta PQR$.

Q15. A vertical stick of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol.



$\Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{6}{x} = \frac{4}{28}$$

$\Rightarrow \quad x = 42$ m

Q16. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Sol. $\Delta ABC \sim \Delta PQR$ (Given)

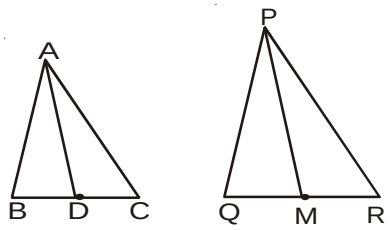
$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR};$$

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ ...(1)

Now, $BD = CD = \frac{1}{2}BC$

and $QM = RM = \frac{1}{2}QR$ ...(2)

($\because$ D is mid-point of BC and M is mid-point of QR)



From (1), $\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$ (By (2))

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$

Thus, we have $\frac{AB}{PQ} = \frac{BD}{QM}$

and $\angle ABD = \angle PQM$ $(\because \angle B = \angle Q)$

$\Rightarrow \Delta ABD \sim \Delta PQM$ (By SAS similarity criterion)

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$

Ex - 6.4

Q1. Let $\Delta ABC \sim \Delta DEF$ and their areas be 64 cm^2 and 121 cm^2 respectively. If EF = 15.4 cm, find BC.

Sol. $\Delta ABC \sim \Delta DEF$ (Given)

$$\Rightarrow \frac{ar(ABC)}{ar(DEF)} = \frac{BC^2}{EF^2} \quad \text{(By theorem 6.7)}$$

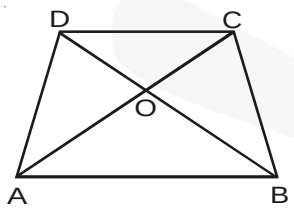
$$\Rightarrow \frac{64}{121} = \frac{BC^2}{EF^2} \qquad \Rightarrow \left\{\frac{BC}{EF}\right\}^2 = \left\{\frac{8}{11}\right\}^2$$

$$\Rightarrow \frac{BC}{EF} = \frac{8}{11} \qquad \Rightarrow BC = \frac{8}{11} \times EF$$

$$\Rightarrow BC = \frac{8}{11} \times 15.4 \text{ cm} = 11.2 \text{ cm}$$

Q2. Diagonals of trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

Sol.



In ΔAOB and ΔCOD,

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\angle OBA = \angle ODC$ (Alternate interior angles)

$\therefore$ By AA, similarity

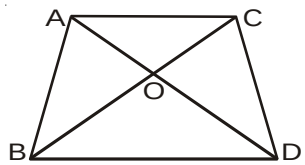
$\Delta AOB \sim \Delta COD$

So, $\frac{ar.\Delta AOB}{ar.\Delta COD} = \left(\frac{AB}{CD}\right)^2$

$= \left(\frac{2}{1}\right)^2$ $\{\because AB = 2CD\}$

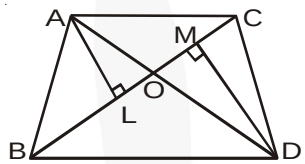
$= 4 : 1$

Q3. In figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$.



Sol. Draw AL ⊥ BC and DM ⊥ BC (see figure)

ΔOLA ~ ΔOMD (AA similarity criterion)



$\Rightarrow \frac{AL}{DM} = \frac{AO}{DO}$ ...(1)

Now, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}\times(BC)\times(AL)}{\frac{1}{2}\times(BC)\times(DM)}$

$= \frac{AL}{DM} = \frac{AO}{DO}$ (By 1)

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Q4. If the areas of two similar triangles are equal, prove that they are congruent.

Sol. Let ΔABC ~ ΔPQR and

area (ΔABC) = area (ΔPQR) (Given)

i.e., $\frac{area(\Delta ABC)}{area(\Delta PQR)} = 1$

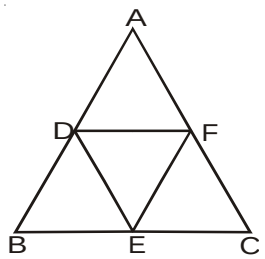
$\Rightarrow \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{PR^2} = 1$

⇒ AB = PQ, BC = QR and CA = PR

⇒ ΔABC ≅ ΔPQR

Q5. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

Sol.



$DF = \frac{1}{2}BC$, $DE = \frac{1}{2}AC$, $EF = \frac{1}{2}AB$

[By midpoint theorem]

So, $\frac{DF}{BC} = \frac{DE}{AC} = \frac{EF}{AB} = \frac{1}{2}$

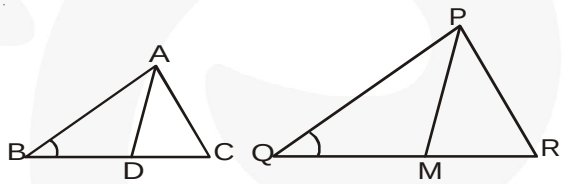
$\therefore$ ΔDEF ~ ΔCAB

So, $\frac{ar\,\Delta DEF}{ar\,\Delta ABC} = \left(\frac{DE}{AC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ or 1 : 4

Q6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Sol. In figure, AD is a median of ΔABC and PM is a median of ΔPQR. Here, D is mid-point of BC and M is mid-point of QR.

Now, we have ΔABC ~ ΔPQR.



$\Rightarrow \angle B = \angle Q$ ...(1)

(Corresponding angles are equal)

Also $\frac{AB}{PQ} = \frac{BC}{QR}$

(Ratio of corresponding sides are equal)

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$

($\because$ D is mid-point of BC and M is mid-point of QR)

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$ ...(2)

In ΔABD and ΔPQM

$\angle ABD = \angle PQM$ (By 1)

and $\frac{AB}{PQ} = \frac{BD}{QM}$ (By 2)

$\Rightarrow$ $\Delta ABD \sim \Delta PQM$ (SAS similarity)

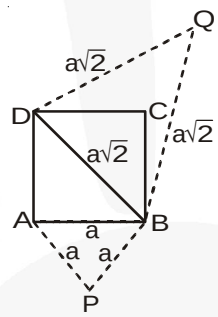
$\Rightarrow$ $\frac{AB}{PQ} = \frac{AD}{PM}$ ...(3)

Now, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$ (By theorem 6.7)

$\Rightarrow$ $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$ $\left(\because \frac{AB}{PQ} = \frac{AD}{PM}\right)$

Q7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Sol. ABCD is a square having sides of length = a.



Then the diagonal BD = $a\sqrt{2}$.

We construct equilateral Δs PAB and QBD

$\Rightarrow$ $\Delta PAB \sim \Delta QBD$ (Equilateral triangles are similar)

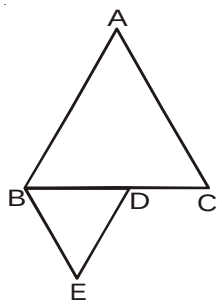
$\Rightarrow$ $\frac{ar(\Delta PAB)}{ar(\Delta QBD)} = \frac{AB^2}{BD^2} = \frac{a^2}{(a\sqrt{2})^2} = \frac{1}{2}$

$\Rightarrow$ ar (ΔPAB) = $\frac{1}{2}$ ar (ΔQBD).

Tick the correct answer and justify

Q8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

(1) 2 : 1 (2) 1 : 2 (3) 4 : 1 (4) 1 : 4

Sol. 

($\because$ BC = 2BD)

Since, both are equilateral triangles.

$\Delta ABC \sim \Delta EBD$

$$\frac{ar\,\Delta ABC}{ar\,\Delta BDE} = \left(\frac{BC}{BD}\right)^2 = \left(\frac{2}{1}\right)^2 = 4 : 1$$

Q9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(1) 2 : 3 (2) 4 : 9 (3) 81 : 16 (4) 16 : 81

Sol. $$\frac{\text{area of } 1^{st}\,\Delta}{\text{area of } 2^{nd}\,\Delta} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

Ex - 6.5

Q1. Sides of some triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Sol. (i) $(7)^2 + (24)^2 = 49 + 576 = 625 = (25)^2$

Therefore, given sides 7 cm, 24 cm, 25 cm make a right triangle.

(ii) $(6)^2 + (3)^2 = 36 + 9 = 45$

$(8)^2 = 64$

$(6)^2 + (3)^2 \neq (8)^2$

Therefore, given sides 3cm, 8 cm, 6 cm does not make a right triangle.

(iii) $(50)^2 + (80)^2 = 2500 + 6400 = 8900$

$(100)^2 = 10000$

$(50)^2 + (80)^2 \neq 100^2$

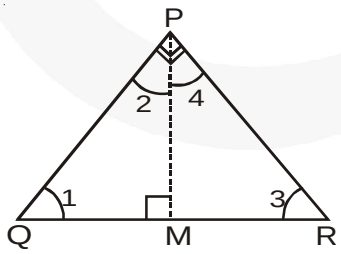
Therefore, given sides 50cm, 80 cm, 100 cm does not make a right triangle.

(iv) $(12)^2 + (5)^2 = 144 + 25 = 169 = (13)^2$

Therefore, given sides 13cm, 12 cm, 5 cm make a right triangle.

Q2. PQR is a triangle right angled at P and M is a point on QR such that PM $\perp$ QR. Show that $PM^2 = QM \times MR$.

Sol. $\angle 1 + \angle 2 = \angle 2 + \angle 4$ (Each = 90°)



$\Rightarrow \angle 1 = \angle 4$

Similarly, $\angle 2 = \angle 3$. Now, this gives

$\Delta QPM \sim \Delta PRM$ (AA similarity)

$\Rightarrow \dfrac{ar(\Delta QPM)}{ar(\Delta PRM)} = \dfrac{PM^2}{RM^2}$ (By theorem 6.7)

$$\Rightarrow \frac{\frac{1}{2}(QM)\times(PM)}{\frac{1}{2}(RM)\times(PM)} = \frac{PM^2}{RM^2} \quad \left(\begin{array}{l}\text{Area of a triangle}\\ = \frac{1}{2}\times \text{Base}\times\text{Height}\end{array}\right)$$

$$\Rightarrow \frac{QM}{RM} = \frac{PM^2}{RM^2}$$

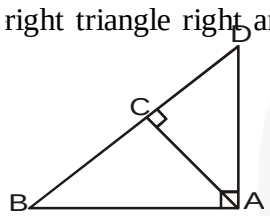
$\Rightarrow$ $PM^2 = QM \times RM$ or $PM^2 = QM \times MR$

Q3. In figure, ABD is a right triangle right angled at A and AC ⊥ BD. Show that

(i) $AB^2 = BC.BD$

(ii) $AC^2 = BC.DC$

(iii) $AD^2 = BD.CD$



Sol. In the given figure, we have

$\Delta ABC \sim \Delta DAC \sim \Delta DBA$

(i) $\Delta ABC \sim \Delta DBA$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBA)} = \frac{AB^2}{DB^2} \Rightarrow \frac{\frac{1}{2}(BC)\times(AC)}{\frac{1}{2}(BD)\times(AC)} = \frac{AB^2}{DB^2} \Rightarrow \quad AB^2 = BC.BD$$

(ii) $\Delta ABC \sim \Delta DAC$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DAC)} = \frac{AC^2}{DC^2} \Rightarrow \frac{\frac{1}{2}(BC)\times(AC)}{\frac{1}{2}(DC)\times(AC)} = \frac{AC^2}{DC^2} \Rightarrow \quad AC^2 = BC.DC$$

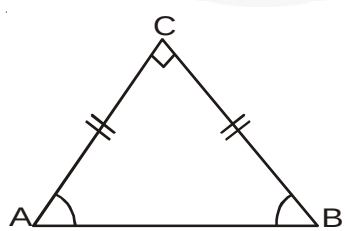
(iii) $\Delta DAC \sim \Delta DBA$

$$\Rightarrow \frac{ar(\Delta DAC)}{ar(\Delta DBA)} = \frac{DA^2}{DB^2} \Rightarrow \frac{\frac{1}{2}(CD)\times(AC)}{\frac{1}{2}(BD)\times(AC)} = \frac{AD^2}{BD^2} \Rightarrow \quad AD^2 = BD.CD$$

Q4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Sol. In ΔABC, $\angle ACB = 90°$. We are given that

ΔABC is an isosceles triangle.



$\Rightarrow \angle A = \angle B = 45°$

$\Rightarrow AC = BC$ ...(1)

By pythagoras theorem, we have

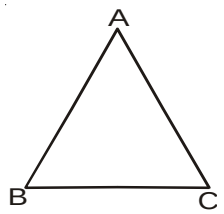
$AB^2 = AC^2 + BC^2$

$= AC^2 + AC^2$ $\{\because BC = AC$ by (1)]

$= 2\ AC^2$

Q5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2\ AC^2$, prove that ABC is a right triangle.

Sol.



As, $AB^2 = 2AC^2$

$AB^2 = AC^2 + AC^2$

$= AC^2 + BC^2$ $[\because AC = BC]$

As it satisfy the pythagoran triplet

So, ΔABC is right triangle, right angled at $\angle C$.

Q6. ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Sol. Altitude of equilateral triangle

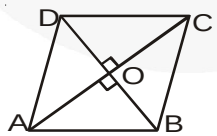
$= \frac{\sqrt{3}}{2} \times \text{Side} = \frac{\sqrt{3}}{2} \times 2a = \sqrt{3}\,a$

Q7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Sol. ABCD is a rhombus in which AB = BC = CD = DA = a (say). Its diagonals AC and BD are right bisectors of each other at O.

In ΔOAB, $\angle AOB = 90°$,

$OA = \frac{1}{2}AC$ and $OB = \frac{1}{2}BD$



By pythagoras theorem, we have

$OA^2 + OB^2 = AB^2$

$\Rightarrow \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 = AB^2$

$\Rightarrow AC^2 + BD^2 = 4\ AB^2$

or $4\ AB^2 = AC^2 + BD^2$

$\Rightarrow$ $AB^2 + BC^2 + CD^2 + DA^2$

$= AC^2 + BD^2$.

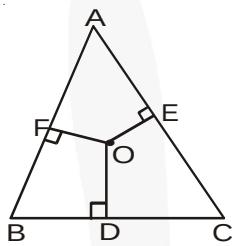
Hence proved.

Q8. In figure, O is a point in the interior of a triangle ABC, OD $\perp$ BC, OE $\perp$ AC and OF $\perp$ AB. Show that

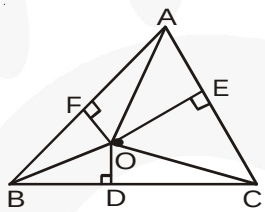
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

$= AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.



Sol. (i) In right angled ΔOFA,



$OA^2 = OF^2 + AF^2$ (Pythagoras theorem)

$\Rightarrow$ $OA^2 - OF^2 = AF^2$ ...(1)

Similarly, $OB^2 - OD^2 = BD^2$ ...(2)

and $OC^2 - OE^2 = CE^2$ ...(3)

Adding (1), (2) and (3), we get

$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

$= AF^2 + BD^2 + CE^2$.

(ii) We have proved that

$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

$= AF^2 + BD^2 + CE^2$. ...(4)

Similarly, we can prove that

$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

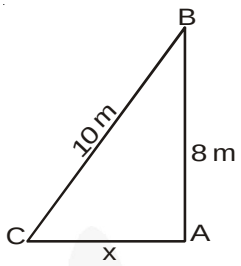
$= BF^2 + CD^2 + AE^2$. ...(5)

From (4) and (5), we have

$AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.

Q9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Sol. Let AC = x metres be the distance of the foot of the ladder from the base of the wall.



AB = 8 m (Height of window)

BC = 10 m (length of ladder)

Now, $x^2 + (8)^2 = (10)^2$

$\Rightarrow x^2 = 100 - 64 = 36 \Rightarrow x = 6$, i.e., AC = 6 m

Q10. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Sol. Let AB be the vertical pole of 18 m and AC be the wire of 24 m.

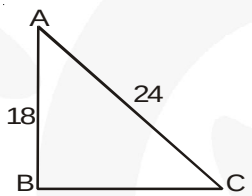
The ΔABC, by pythagoras theorem

$AC^2 = AB^2 + BC^2$

$24^2 = 18^2 + BC^2$

$BC^2 = 252$

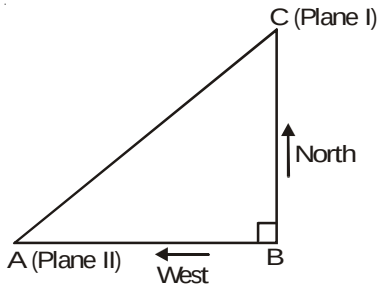
$BC = 6\sqrt{7}$ m



Q11. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Sol. The first plane travels distance BC in the direction of north in $1\frac{1}{2}$ hours at a speed of 1000 km/hr.

$\therefore$ BC = $1000 \times \frac{3}{2}$ km = 1500 km.



The second plane travels distance BA in the direction of west in $1\frac{1}{2}$ hours at a speed of 1200 km/hr.

$\therefore$ BA = 1200 × $\frac{3}{2}$ km = 1800 km.

From right angled ΔABC,

$AC^2 = AB^2 + BC^2$

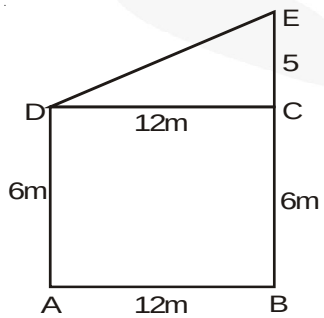
$= (1800)^2 + (1500)^2$

$= 3240000 + 2250000 = 5490000$

$\Rightarrow$ AC = $\sqrt{5490000}$ m $\Rightarrow$ AC = $300\sqrt{61}$ m

Q12. Two poles of height 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Sol.



Let AD and BE be two poles of height 6 m and 11 m and AB = 12 m

In ΔDEC, by pythagoras theorem

$DE^2 = CD^2 + CE^2$

$DE^2 = 12^2 + 5^2$ (DC = AB = 12 m)

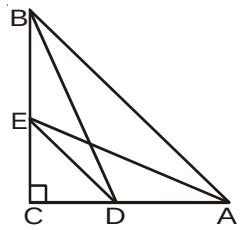
DE = $\sqrt{144+25}$ = $\sqrt{169}$ = 13 m

Thus, distance between their tops is 13 m.

Q13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Sol. In right angled ΔACE,

$AE^2 = CA^2 + CE^2$ ...(1)



and in right angled ΔBCD,

$BD^2 = BC^2 + CD^2$ ...(2)

Adding (1) and (2), we get

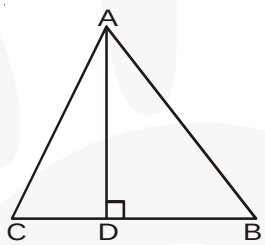
$AE^2 + BD^2 = (CA^2 + CE^2) + (BC^2 + CD^2)$

$= (BC^2 + CA^2) + (CD^2 + CE^2)$

$= BA^2 + DE^2$

$\therefore$ $AE^2 + BD^2 = AB^2 + DE^2$

Q14. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3 CD (see figure). Prove that $2\ AB^2 = 2\ AC^2 + BC^2$.



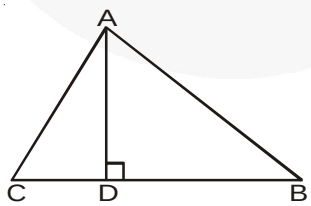
Sol. DB = 3 CD

$\Rightarrow$ $CD = \frac{1}{4}\ BC$ ...(1)

and $DB = \frac{3}{4}\ BC$

In ΔABD, $AB^2 = DB^2 + AD^2$ ...(2)

In ΔACD, $AC^2 = CD^2 + AD^2$ ...(3)



Subtracting (3) from (2), we get

$AB^2 - AC^2 = DB^2 - CD^2$

$$= \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \frac{9}{16}BC^2 - \frac{1}{16}BC^2$$

$= \frac{1}{2}BC^2 \Rightarrow 2AB^2 - 2AC^2 = BC^2$

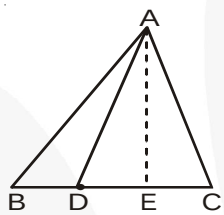
$\Rightarrow 2AB^2 = 2AC^2 + BC^2$

Hence proved.

Q15. In an equailateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7\ AB^2$.

Sol. AB = BC = CA = a (say)

$BD = \frac{1}{3}BC = \frac{1}{3}a$



$\Rightarrow CD = \frac{2}{3}BC = \frac{2}{3}a$

$AE \perp BC$

$\Rightarrow BE = EC = \frac{1}{2}a$

$DE = \frac{1}{2}a - \frac{1}{3}a = \frac{1}{6}a$

$AD^2 = AE^2 + DE^2 = AB^2 - BE^2 + DE^2$

$= a^2 - \left(\frac{1}{2}a\right)^2 + \left(\frac{1}{6}a\right)^2$

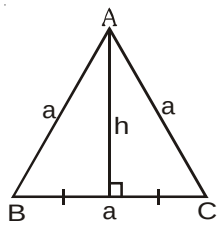
$= a^2 - \frac{1}{4}a^2 + \frac{1}{36}a^2$

$= \frac{(36-9+1)a^2}{36} = \frac{28}{36}a^2 = \frac{7}{9}AB^2$

$\Rightarrow 9AD^2 = 7\ AB^2$

Q16. In an equailateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Sol.



Altitude of equilateral $\Delta = \frac{\sqrt{3}}{2}$ side

$h = \frac{\sqrt{3}}{2} a$

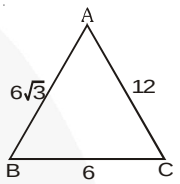
$h^2 = \frac{3}{4} a^2$

$\boxed{4h^2 = 3a^2}$

Q17. Tick the correct answer and justify : In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. The angle B is :

(1) 120° (2) 60° (3) 90° (4) 45°

Sol. $AB^2 = (6\sqrt{3})^2 = 108$



$BC^2 = 6^2 = 36$

$AC^2 = 12^2 = 144$

So, $AB^2 + BC^2 = AC^2$

ΔABC is right Δ, right angled at B

∠B = 90°.