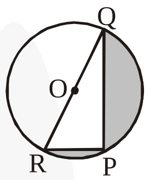
Ex - 12.3

NOTE: Unless stated otherwise, use $\pi = \frac{22}{7}$

Q1. Find the area of the shaded region in fig, if
PQ = 24, PR = 7 cm and O is the centre of the circle.



Sol. In the figure, $\angle RPQ = 90°$

(Angle subtended by a diameter on the circumference)

Therefore, ΔRPQ is right angled at P,

RP = 7 cm and PQ = 24 cm

Then by Pythagoras Theorem, we have

$QR^2 = RP^2 + PQ^2$

$= (7)^2 + (24)^2 = 625$

$\Rightarrow$ QR = 25 cm

$\therefore$ The radius of the circle

$= \frac{25}{2}$ cm

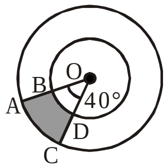
Now, the area of the shaded region (see figure)

$$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times RP \times PQ$$

$$= \left\{\frac{1}{2} \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2 - \frac{1}{2} \times 7 \times 24\right\} cm^2$$

$$= \left\{\frac{6875}{28} - 84\right\} cm^2 = \frac{4523}{28}\ cm^2$$

Q2. Find the area of the shaded region in fig., if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40°$.



Sol. Radius of the outer circle = 14 cm and $\theta = 40°$

$\therefore$ Area of the sector AOC

$= \frac{40°}{360°} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$= \frac{1}{9} \times 22 \times 2 \times 14 \text{ cm}^2 = \frac{616}{9} \text{ cm}^2$

Radius of the inner circle = 7 cm and $\theta = 40°$

$\therefore$ Area of the sector BOD

$= \frac{40°}{360°} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$

$= \frac{1}{9} \times 22 \times 7 \text{ cm}^2 = \frac{154}{9} \text{ cm}^2$

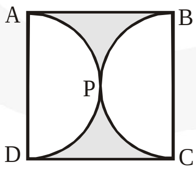
Now, area of the shaded region

= Area of sector AOC – Area of sector BOD

$= \frac{616}{9} - \frac{154}{9} \text{ cm}^2 = \frac{1}{9}(616 - 154) \text{ cm}^2$

$= \frac{1}{9} \times 462 \text{ cm}^2 = \frac{154}{3} \text{ cm}^2$

Q3. Find the area of the shaded region in fig., if ABCD is a square of side 14 cm and APD and BPC are semicircles.



Sol. The area of the square ABCD = $(14)^2 \text{ cm}^2 = 196 \text{ cm}^2$

($\because$ side of the square 14 cm)

The sum of the areas of the semicircles APD and BPC

= 2 × {area of semicircle APD}

($\because$ the areas of the two semicircles are equal)

$= 2 \times \left\{\frac{1}{2}\pi r^2\right\} = \pi \times \left(\frac{AD}{2}\right)^2 = \pi \times \left(\frac{14}{2}\right)^2$

(∵ AD is diameter of the semicircle APD)

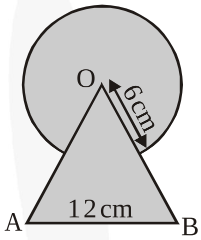
$= \frac{22}{7} \times 49 \text{ cm}^2 = 154 \text{ cm}^2$

The area of the shaded region

= The area of the square ABCD – The sum of the areas of the semicircles APD and BPC.

$= 196 \text{ cm}^2 - 154 \text{ cm}^2 = 42 \text{ cm}^2$

Q4. Find the area of the shaded region in fig., where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

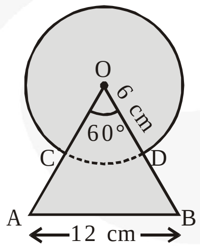


Sol. Area of the circle with radius 6 cm

$= \pi r^2 = \frac{22}{7} \times 6 \times 6 \text{ cm}^2 = \frac{792}{7} \text{ cm}^2$

Area of equilateral triangle, having side a = 12 cm, is given by

$\frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 12 \times 12 \text{ cm}^2 = 36\sqrt{3} \text{ cm}^2$



∵ Each angle of an equilateral triangle = 60°

∴ $\angle AOB = 60°$

∴ Area of sector COD

$= \frac{\theta}{360°} \times \pi r^2 = \frac{60°}{360°} \times \frac{22}{7} \times 6 \times 6 \text{ cm}^2$

$= \frac{22 \times 6}{7} \text{cm}^2 = \frac{132}{7} \text{cm}^2$

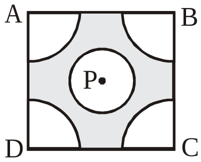
Now, area of the shaded region,

= [Area of the circle] + [Area of the equilateral triangle] – [Area of the sector COD]

$= \left[\frac{792}{7} + 36\sqrt{3} - \frac{132}{7}\right] \text{cm}^2$

$= \left[\frac{660}{7} + 36\sqrt{3}\right] \text{cm}^2$

Q5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in fig. Find the area of the remaining portion of the square.



Sol. Side of the square = 4 cm

$\therefore$ Area of the square ABCD = 4 × 4 cm^2

= 16 cm^2

$\because$ Each corner has a quadrant circle of radius 1cm.

$\therefore$ Area of all the 4 quadrant squares

$$4 \times \frac{1}{4}\pi r^2 = \pi r^2 = \frac{22}{7} \times 1 \times 1 \text{ cm}^2 = \frac{22}{7} \text{ cm}^2$$

Diameter of the middle circle = 2 cm

$\Rightarrow$ Radius of the middle circle = 1 cm

$\therefore$ Area of the middle circle

$$= \pi r^2 = \frac{22}{7} \times 1 \times 1 \text{ cm}^2 = \frac{22}{7} \text{ cm}^2$$

Now, area of the shaded region

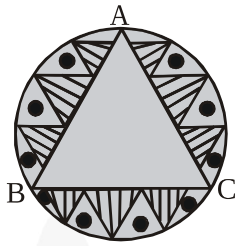
= [Area of the square ABCD] – [(Area of the 4 quadrant circles) + (Area of the middle circle)]

$$= [16 \text{ cm}^2] - \left[\left(\frac{22}{7} + \frac{22}{7}\right)\text{cm}^2\right]$$

$$= 16 \text{ cm}^2 - 2 \times \frac{22}{7} \text{ cm}^2$$

$$= 16 \text{ cm}^2 - \frac{44}{7} \text{ cm}^2 = \frac{112 - 44}{7} \text{ cm}^2 = \frac{68}{7} \text{ cm}^2.$$

Q6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC
in the middle as shown in fig. Find the area of the design.



Sol. O is the centre of the circular table cover and radius = 32 cm. ΔABC is equilateral. Join OA, OB, OC.

Now, ∠AOB = ∠BOC = ∠COA = 120°

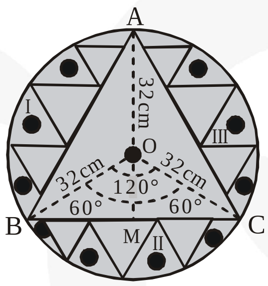
In ΔOBC, we have OB = OC

Draw OM ⊥ BC.

⇒ ∠BOM = ∠COM = 60°

(∵ ΔOMB ≅ ΔOMC by RHS congruence)

Now, $\frac{BM}{OB} = \sin 60°$



$\Rightarrow \frac{BM}{32} = \frac{\sqrt{3}}{2}$

$\Rightarrow$ BM = $16\sqrt{3}$ cm

Then, BC = 2 × BM

= $32\sqrt{3}$ cm

Thus, the side of the equilateral triangle ABC = $32\sqrt{3}$ cm

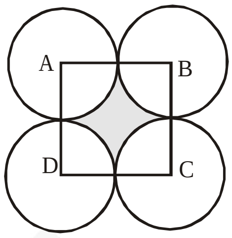
The area of the shaded region (designed)

= The area of circle – area of ΔABC

$= \{\pi \times (32)^2 - \frac{\sqrt{3}}{4} \times (32\sqrt{3})^2\}\text{cm}^2$

$= \{\frac{22}{7} \times 32 \times 32 - \frac{\sqrt{3}}{4} \times 32 \times 32 \times 3\}\text{cm}^2 = \{\frac{22528}{7} - 768\sqrt{3}\}\ \text{cm}^2$

Q7. In fig., ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.



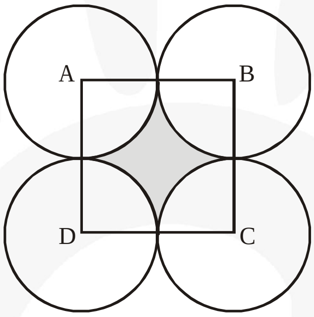
Sol. Side of the square ABCD = 14 cm

$\therefore$ Area of the sqaure ABCD = 14×14 cm^2

= 196 cm^2

$\because$ Circles touch each other

Radius of the circle = $\frac{14}{2}$ = 7 cm



Now, area of a sector of radius 7 cm and sector angle θ as 90°

$= \frac{90°}{360°} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = \frac{11 \times 7}{2} \text{ cm}^2$

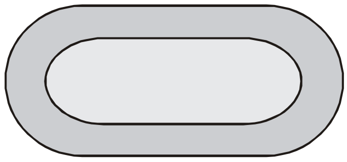
Area of 4 sectors

$= 4 \times \left[\frac{11 \times 7}{2}\right] = 2 \times 11 \times 7 \text{ cm}^2 = 154 \text{ cm}^2$

Area of the shaded region = [Area of the square ABCD] – [Area of the 4 sectors]

$= 196 \text{ cm}^2 - 154 \text{ cm}^2 = 42 \text{ cm}^2$

Q8. Fig. depicts a racing track whose left and right ends are semicircular.

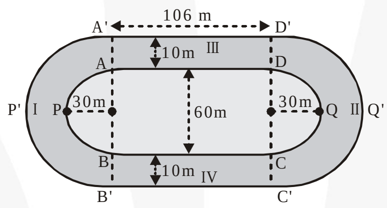


The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) the distance around the track along its inneredge

(ii) the area of the track.

Sol. (i) The distance around the track along the inner edge (as seen from figure)



= Perimeter of APB + BC

+ Perimeter CQD + AD

$= \{\pi \times 30 + 106 + \pi \times 30 + 106\}$ m

$= \{60\,\pi + 212\}$ m

$= \left\{60 \times \frac{22}{7} + 212\right\} = \frac{2804}{7}$ m

(ii) Area of region I $= \frac{1}{2}\pi \times (40)^2 - \frac{1}{2}\pi \times (30)^2$

$\{\because$ outer radius = 30 m + 10 m = 40 m$\}$

$= \frac{1}{2}\pi \times 700\ m^2 = \frac{1}{2} \times \frac{22}{7} \times 700\ m^2$

$= 1100\ m^2$

Similarly, area of the region II $= 1100\ m^2$

Area of the region III (106 m × 10 m rectangle)

$= 106 \times 10 = 1060\ m^2$

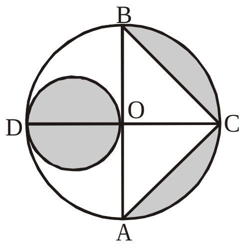
Similarly, the area of the region IV $= 1060\ m^2$

Then, the total area of the track

$= 2 \times 1100\ m^2 + 2 \times 1060\ m^2$

$= (2200 + 2120)\ m^2 = 4320\ m^2$

Q9. In fig., AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm. find the area of the shaded region.



Sol. O is the centre of the circle, OA = 7 cm

$\Rightarrow$ AB = 2(OA) = 2 × 7 = 14 cm

OC = OA = 7 cm

$\because$ AB and CD are perpendicular to each other

$\Rightarrow$ OC $\perp$ AB

$\therefore$ Area of ΔABC

$= \frac{1}{2} \times AB \times OC = \frac{1}{2} \times 14\text{ cm} \times 7\text{ cm} = 49\text{ cm}^2$

Again OD = OA = 7 cm

$\therefore$ Radius of the small circle

$= \frac{1}{2}(OD) = \frac{1}{2} \times 7 = \frac{7}{2}\text{ cm}$

$\therefore$ Area of the small circle $= \pi r^2$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\text{cm}^2 = \frac{11 \times 7}{2} = \frac{77}{2}\text{ cm}^2$

Radius of the big circle $= \frac{14}{2}\text{ cm} = 7\text{ cm}$

Area of the semi-circle OACB $= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\left(\frac{22}{7} \times 7 \times 7\right)\text{cm}^2 = 11 \times 7\text{ cm}^2 = 77\text{ cm}^2$

Now, Area of the shaded region

= [Area of the small circle] + [Area of the big semi-circle OABC] – [Area of ΔABC]

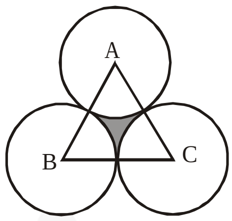
$= \frac{77}{2}\text{ cm}^2 + 77\text{ cm}^2 - 49\text{ cm}^2$

$= \frac{77 + 154 - 98}{2}\text{ cm}^2$

$= \frac{231 - 98}{2}\text{ cm}^2 = \frac{133}{2}\text{ cm}^2 = 66.5\text{ cm}^2$

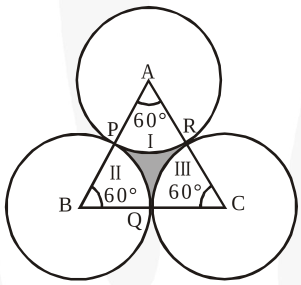
Q10. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle. Find the area of the

shaded region. (Use π = 3.14 and $\sqrt{3}$ = 1.73205)



Sol. Area of the ΔABC (equilateral) = 17320.5 cm^2

Let the side of the equilateral ΔABC be x cm.



Then, $\frac{\sqrt{3}}{4} \times x^2 = 17320.5$

$\Rightarrow \frac{1.73205}{4} \times x^2 = 17320.5$ ($\because \sqrt{3}$ = 1.73205)

$\Rightarrow x^2 = 40000 \Rightarrow x = 200$ cm

Then, radius of each circle = 100 cm.

Area of the sector APR

$= \frac{60}{360} \times \pi \times (100)^2\ cm^2 = \frac{\pi}{6} \times 10000\ cm^2$

Similarly, area of the sector BPQ = area of the sector CQR

$= \frac{\pi}{6} \times 10000\ cm^2$

Total area of regions I, II and III (i.e., non-shaded region of ΔABC)

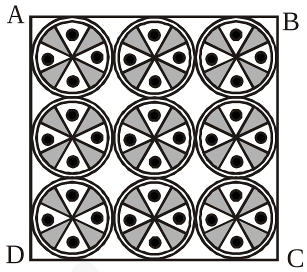
$= 3 \times \frac{\pi}{6} \times 10000\ cm^2 = \frac{1}{2} \times 3.14 \times 10000\ cm^2$

= 15700 cm^2

Then, the required area of the shaded region of ΔABC

= 17320.5 cm^2 – 15700 cm^2 = 1620.5 cm^2

Q11. On a square handkerchief, nine circular designs each of radius 7 cm are made. Find the area of the remaining portion of the handkerchief.



Sol. $\because$ The circles touch each other.

$\therefore$ The side of the square ABCD

= 3 × diameter of a circle

= 3 × (2 × radius of a circle) = 3 × (2 × 7 cm)

= 42 cm

$\Rightarrow$ Area of the square ABCD = 42 × 42 cm^2

= 1764 cm^2.

Now, area of one circle = πr^2

$$\Rightarrow \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = 154 \text{ cm}^2$$

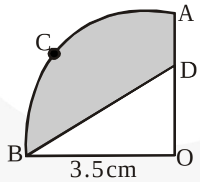
$\because$ There are 9 circles

$\therefore$ Total area of 9 circles = 154 × 9 = 1386 cm^2

$\therefore$ Area of the remaining portion of the handkerchief

= (1764 – 1386) cm^2 = 378 cm^2.

Q12. In fig., OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB, (ii) shaded region.



Sol. (i) Area of the quadrant OACB $\left(\text{radius} = \frac{7}{2}\text{ cm}\right)$

$$= \frac{1}{4} \times \pi \times r^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{ cm}^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{49}{4} \text{cm}^2 = \frac{11 \times 7}{8} \text{cm}^2$$

$$= \frac{77}{8} \text{ cm}^2$$

(ii) In right angled ΔOBD,

$OB = \frac{7}{2}$ cm, $OD = 2$ cm

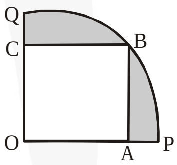
The area of $\Delta OBD = \frac{1}{2} \times OB \times OD$

$= \frac{1}{2} \times \frac{7}{2} \times 2 cm^2 = \frac{7}{2} cm^2$

Then area of the shaded region = The area of quadrant OACB – The area of ΔOBD

$= \frac{77}{8} cm^2 - \frac{7}{2} cm^2 = \frac{77-28}{8} cm^2 = \frac{49}{8} cm^2$

Q13. In fig., a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)



Sol. OABC is a square such that its side OA = 20 cm

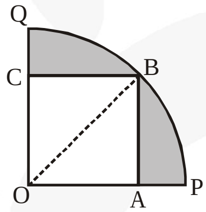
$\therefore$ OA = 20 cm

$\therefore OB^2 = OA^2 + AB^2$

$\therefore OB^2 = 20^2 + 20^2$

$= 400 + 400 = 800$

$OB = \sqrt{800} = 20\sqrt{2}$ cm



Now, area of the quadrant OPBQ $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{314}{100} \times 800 \text{ cm}^2 = 314 \times 2 = 628 \text{ cm}^2$

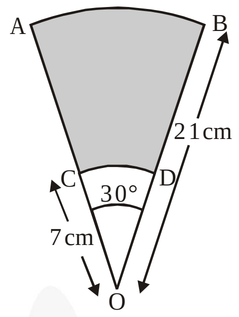
Area of the square OABC $= 20 \times 20 \text{ cm}^2$

$= 400 \text{ cm}^2$

$\therefore$ Area of the shaded region = Area of the quadrant OPBQ – Area of the square OABC

$= 628 \text{ cm}^2 - 400 \text{ cm}^2 = 228 \text{ cm}^2$

Q14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O. If $\angle AOB = 30°$, find the area of the shaded region.



Sol. Radius of bigger circle R = 21 cm and sector angle $\theta = 30°$

$\therefore$ Area of the sector OAB

$$= \frac{30°}{360°} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$$

$$= \frac{11 \times 21}{2} \text{cm}^2 = \frac{231}{2} \text{ cm}^2$$

Again, radius of the smaller circle, r = 7 cm

Also, the sector angle is 30°

$\therefore$ Area of the sector OCD

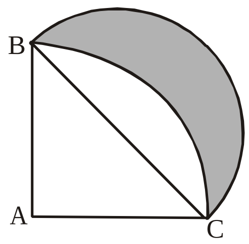
$$= \frac{30°}{360°} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = \frac{77}{6} \text{ cm}^2$$

$\therefore$ Area of the shaded region = Area of the sector OAB – Area of the sector OCD

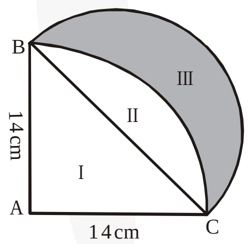
$$= \left[\frac{231}{2} - \frac{77}{6}\right] \text{cm}^2 = \frac{693 - 77}{6} \text{ cm}^2$$

$$= \frac{616}{6} \text{cm}^2 = \frac{308}{3} \text{cm}^2$$

Q15. In fig., ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Sol. BC = $\sqrt{(14)^2 + (14)^2} = 14\sqrt{2}$ cm



Area of region II = Area of sector ABC

– Area of ΔABC = $\left\{\frac{1}{4}\pi \times (14)^2 - \frac{1}{2} \times 14 \times 14\right\}$ cm^2

= $\left\{\frac{1}{4} \times \frac{22}{7} \times 196 - 98\right\}$ cm^2 = 56 cm^2

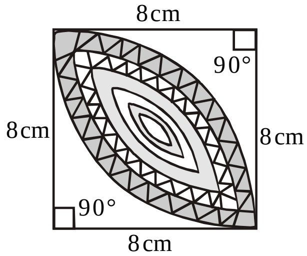
The area of the shaded region III = The area of the semicircle drawn on BC as diameter – The area of region II

= $\left\{\frac{1}{2}\pi \times \left(\frac{14\sqrt{2}}{2}\right)^2 - 56\right\}$ cm^2

= $\left\{\frac{1}{2} \times \frac{22}{7} \times 98 - 56\right\}$ cm^2

= {154 – 56} cm^2 = 98 cm^2

Q16. Calculate the area of the designed region in fig. common between the two quadrants of circles of radius 8 cm each.



Sol. Side of the square = 8 cm

$\therefore$ Area of the square (ABCD) = $8 \times 8\ cm^2$

$= 64\ cm^2$

Now, radius of the quadrant ADQB = 8 cm

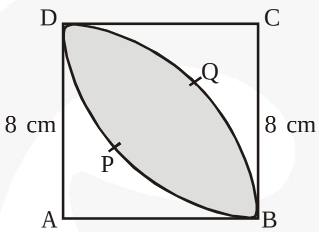
$\therefore$ Area of the quadrant ADQB

$= \frac{90°}{360°} \times \frac{22}{7} \times 8 \times 8\ cm^2$

$= \frac{1}{4} \times \frac{22}{7} \times 64\ cm^2 = \frac{22 \times 16}{7}\ cm^2$

Similarly, area of the quadrant

BPDC $= \frac{22 \times 16}{7}\ cm^2$



Sum of the two quadrant

$= 2\left[\frac{22 \times 16}{7}\right] cm^2 = \frac{704}{7}\ cm^2$

Now, area of design

= [Sum of the area of two quadrant] –

[Area of the square ABCD]

$= \frac{704}{7}\ cm^2 - 64\ cm^2 = \frac{704 - 448}{7}\ cm^2$

$= \frac{256}{7}\ cm^2$