Miscellaneous Solutions

Question 1:

Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of *x*-axis.

Answer

If $\vec{r}$ is a unit vector in the XY-plane, then $\vec{r} = \cos\theta\hat{i} + \sin\theta\hat{j}.$

Here, θ is the angle made by the unit vector with the positive direction of the *x*-axis.

Therefore, for θ = 30°:

$$\vec{r} = \cos 30°\hat{i} + \sin 30°\hat{j} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$$

Hence, the required unit vector is $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

Question 2:

Find the scalar components and magnitude of the vector joining the points $\mathrm{P}(x_1, y_1, z_1)$ and $\mathrm{Q}(x_2, y_2, z_2)$.

Answer

The vector joining the points $\mathrm{P}(x_1, y_1, z_1)$ and $\mathrm{Q}(x_2, y_2, z_2)$ can be obtained by,

$$\overrightarrow{\mathrm{PQ}} = \text{Position vector of Q} - \text{Position vector of P}$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\left|\overrightarrow{\mathrm{PQ}}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Hence, the scalar components and the magnitude of the vector joining the given points are respectively $\{(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)\}$ and $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

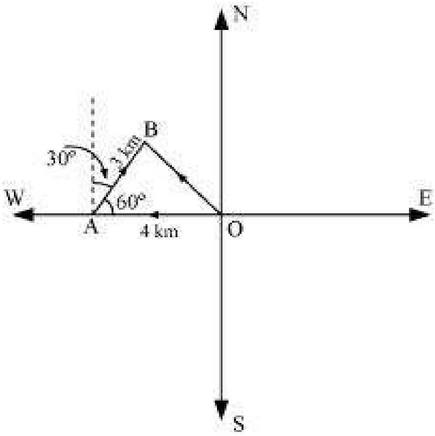
Question 3:

A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

Answer

Let O and B be the initial and final positions of the girl respectively.

Then, the girl's position can be shown as:



Now, we have:

$$\overrightarrow{OA} = -4\hat{i}$$

$$\overrightarrow{AB} = \hat{i}\left|\overrightarrow{AB}\right|\cos 60° + \hat{j}\left|\overrightarrow{AB}\right|\sin 60°$$

$$= \hat{i}3\times\frac{1}{2} + \hat{j}3\times\frac{\sqrt{3}}{2}$$

$$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

By the triangle law of vector addition, we have:

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

$$= \left(-4\hat{i}\right) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$$

$$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

Hence, the girl's displacement from her initial point of departure is

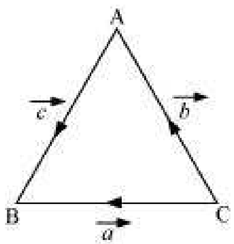
$$\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}.$$

Question 4:

If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

Answer

In ΔABC, let $\overrightarrow{CB} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, and $\overrightarrow{AB} = \vec{c}$ (as shown in the following figure).



Now, by the triangle law of vector addition, we have $\vec{a} = \vec{b} + \vec{c}$.

It is clearly known that $|\vec{a}|$, $|\vec{b}|$, and $|\vec{c}|$ represent the sides of ΔABC.

Also, it is known that the sum of the lengths of any two sides of a triangle is greater than the third side.

$\therefore |\vec{a}| < |\vec{b}| + |\vec{c}|$

Hence, it is not true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$.

Question 5:

Find the value of *x* for which $x\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector.

Answer

$x\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector if $\left|x\left(\hat{i} + \hat{j} + \hat{k}\right)\right| = 1$.

Now,

$$\left|x\left(\hat{i}+\hat{j}+\hat{k}\right)\right|=1$$

$$\Rightarrow \sqrt{x^2+x^2+x^2}=1$$

$$\Rightarrow \sqrt{3x^2}=1$$

$$\Rightarrow \sqrt{3}\,x=1$$

$$\Rightarrow x=\pm\frac{1}{\sqrt{3}}$$

Hence, the required value of *x* is $\pm\frac{1}{\sqrt{3}}$.

Question 6:

Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$.

Answer

We have,

$\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$

Let $\vec{c}$ be the resultant of $\vec{a}$ and $\vec{b}$.

Then,

$$\vec{c}=\vec{a}+\vec{b}=(2+1)\hat{i}+(3-2)\hat{j}+(-1+1)\hat{k}=3\hat{i}+\hat{j}$$

$$\therefore |\vec{c}|=\sqrt{3^2+1^2}=\sqrt{9+1}=\sqrt{10}$$

$$\therefore \hat{c}=\frac{\vec{c}}{|\vec{c}|}=\frac{\left(3\hat{i}+\hat{j}\right)}{\sqrt{10}}$$

Hence, the vector of magnitude 5 units and parallel to the resultant of vectors $\vec{a}$ and $\vec{b}$ is

$$\pm 5\cdot\hat{c}=\pm 5\cdot\frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{j}\right)=\pm\frac{3\sqrt{10}\hat{i}}{2}\pm\frac{\sqrt{10}}{2}\hat{j}.$$

Question 7:

If $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$, find a unit vector parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

Answer

We have,

$\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$

$$\begin{aligned}2\vec{a}-\vec{b}+3\vec{c}&=2\left(\hat{i}+\hat{j}+\hat{k}\right)-\left(2\hat{i}-\hat{j}+3\hat{k}\right)+3\left(\hat{i}-2\hat{j}+\hat{k}\right)\\&=2\hat{i}+2\hat{j}+2\hat{k}-2\hat{i}+\hat{j}-3\hat{k}+3\hat{i}-6\hat{j}+3\hat{k}\\&=3\hat{i}-3\hat{j}+2\hat{k}\end{aligned}$$

$$\left|2\vec{a}-\vec{b}+3\vec{c}\right|=\sqrt{3^2+(-3)^2+2^2}=\sqrt{9+9+4}=\sqrt{22}$$

Hence, the unit vector along $2\vec{a}-\vec{b}+3\vec{c}$ is

$$\frac{2\vec{a}-\vec{b}+3\vec{c}}{\left|2\vec{a}-\vec{b}+3\vec{c}\right|}=\frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}}=\frac{3}{\sqrt{22}}\hat{i}-\frac{3}{\sqrt{22}}\hat{j}+\frac{2}{\sqrt{22}}\hat{k}.$$

Question 8:

Show that the points A (1, –2, –8), B (5, 0, –2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.

Answer

The given points are A (1, –2, –8), B (5, 0, –2), and C (11, 3, 7).

$$\therefore \overrightarrow{AB}=(5-1)\hat{i}+(0+2)\hat{j}+(-2+8)\hat{k}=4\hat{i}+2\hat{j}+6\hat{k}$$

$$\overrightarrow{BC}=(11-5)\hat{i}+(3-0)\hat{j}+(7+2)\hat{k}=6\hat{i}+3\hat{j}+9\hat{k}$$

$$\overrightarrow{AC}=(11-1)\hat{i}+(3+2)\hat{j}+(7+8)\hat{k}=10\hat{i}+5\hat{j}+15\hat{k}$$

$$\left|\overrightarrow{AB}\right|=\sqrt{4^2+2^2+6^2}=\sqrt{16+4+36}=\sqrt{56}=2\sqrt{14}$$

$$\left|\overrightarrow{BC}\right|=\sqrt{6^2+3^2+9^2}=\sqrt{36+9+81}=\sqrt{126}=3\sqrt{14}$$

$$\left|\overrightarrow{AC}\right|=\sqrt{10^2+5^2+15^2}=\sqrt{100+25+225}=\sqrt{350}=5\sqrt{14}$$

$$\therefore \left|\overrightarrow{AC}\right|=\left|\overrightarrow{AB}\right|+\left|\overrightarrow{BC}\right|$$

Thus, the given points A, B, and C are collinear.

Now, let point B divide AC in the ratio $\lambda:1$. Then, we have:

$$\overrightarrow{OB}=\frac{\lambda\overrightarrow{OC}+\overrightarrow{OA}}{(\lambda+1)}$$

$$\Rightarrow 5\hat{i}-2\hat{k}=\frac{\lambda\left(11\hat{i}+3\hat{j}+7\hat{k}\right)+\left(\hat{i}-2\hat{j}-8\hat{k}\right)}{\lambda+1}$$

$$\Rightarrow (\lambda+1)\left(5\hat{i}-2\hat{k}\right)=11\lambda\hat{i}+3\lambda\hat{j}+7\lambda\hat{k}+\hat{i}-2\hat{j}-8\hat{k}$$

$$\Rightarrow 5(\lambda+1)\hat{i}-2(\lambda+1)\hat{k}=(11\lambda+1)\hat{i}+(3\lambda-2)\hat{j}+(7\lambda-8)\hat{k}$$

On equating the corresponding components, we get:

$$5(\lambda+1)=11\lambda+1$$

$$\Rightarrow 5\lambda+5=11\lambda+1$$

$$\Rightarrow 6\lambda=4$$

$$\Rightarrow \lambda=\frac{4}{6}=\frac{2}{3}$$

Hence, point B divides AC in the ratio $2:3.$

Question 9:

Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a}+\vec{b}\right)\text{and}\left(\vec{a}-3\vec{b}\right)$ externally in the ratio 1: 2. Also, show that P is the mid point of the line segment RQ.

Answer

It is given that $\overrightarrow{OP}=2\vec{a}+\vec{b},\ \overrightarrow{OQ}=\vec{a}-3\vec{b}$.

It is given that point R divides a line segment joining two points P and Q externally in the ratio 1: 2. Then, on using the section formula, we get:

$$\overrightarrow{OR}=\frac{2\left(2\vec{a}+\vec{b}\right)-\left(\vec{a}-3\vec{b}\right)}{2-1}=\frac{4\vec{a}+2\vec{b}-\vec{a}+3\vec{b}}{1}=3\vec{a}+5\vec{b}$$

Therefore, the position vector of point R is $3\vec{a}+5\vec{b}$.

Position vector of the mid-point of RQ = $\dfrac{\overrightarrow{OQ}+\overrightarrow{OR}}{2}$

$$=\frac{\left(\vec{a}-3\vec{b}\right)+\left(3\vec{a}+5\vec{b}\right)}{2}$$

$$=2\vec{a}+\vec{b}$$

$$=\overrightarrow{\mathrm{OP}}$$

Hence, P is the mid-point of the line segment RQ.

Question 10:

The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$.

Find the unit vector parallel to its diagonal. Also, find its area.

Answer

Adjacent sides of a parallelogram are given as: $\vec{a}=2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}-3\hat{k}$

Then, the diagonal of a parallelogram is given by $\vec{a}+\vec{b}$.

$$\vec{a}+\vec{b}=(2+1)\hat{i}+(-4-2)\hat{j}+(5-3)\hat{k}=3\hat{i}-6\hat{j}+2\hat{k}$$

Thus, the unit vector parallel to the diagonal is

$$\frac{\vec{a}+\vec{b}}{\left|\vec{a}+\vec{b}\right|}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{3^2+(-6)^2+2^2}}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}=\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}.$$

$\therefore$ Area of parallelogram ABCD = $\left|\vec{a}\times\vec{b}\right|$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}$$

$$=\hat{i}(12+10)-\hat{j}(-6-5)+\hat{k}(-4+4)$$

$$=22\hat{i}+11\hat{j}$$

$$=11\left(2\hat{i}+\hat{j}\right)$$

$$\therefore\left|\vec{a}\times\vec{b}\right|=11\sqrt{2^2+1^2}=11\sqrt{5}$$

Hence, the area of the parallelogram is $11\sqrt{5}$ square units.

Question 11:

Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Answer

Let a vector be equally inclined to axes OX, OY, and OZ at angle *α*.

Then, the direction cosines of the vector are cos *α*, cos *α*, and cos *α*.

Now,

$$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

$$\Rightarrow 3\cos^2\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

Hence, the direction cosines of the vector which are equally inclined to the axes are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Question 12:

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k},\ \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 15$.

Answer

Let $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$.

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:

$$\vec{d} \cdot \vec{a} = 0$$

$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \qquad \text{...(i)}$$

And,

$$\vec{d} \cdot \vec{b} = 0$$

$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \qquad \text{...(ii)}$$

Also, it is given that:

$$\vec{c} \cdot \vec{d} = 15$$

$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \qquad \text{...(iii)}$$

On solving (i), (ii), and (iii), we get:

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3}$$

$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$.

Question 13:

The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

Answer

$$\left(2\hat{i} + 4\hat{j} - 5\hat{k}\right) + \left(\lambda\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

Therefore, unit vector along $\left(2\hat{i} + 4\hat{j} - 5\hat{k}\right) + \left(\lambda\hat{i} + 2\hat{j} + 3\hat{k}\right)$ is given as:

$$\frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 6^2 + (-2)^2}} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{4 + 4\lambda + \lambda^2 + 36 + 4}} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}}$$

Scalar product of $\left(\hat{i} + \hat{j} + \hat{k}\right)$ with this unit vector is 1.

$$\Rightarrow \left(\hat{i} + \hat{j} + \hat{k}\right) \cdot \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$
$$\Rightarrow \frac{(2+\lambda) + 6 - 2}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$
$$\Rightarrow \sqrt{\lambda^2 + 4\lambda + 44} = \lambda + 6$$
$$\Rightarrow \lambda^2 + 4\lambda + 44 = (\lambda + 6)^2$$
$$\Rightarrow \lambda^2 + 4\lambda + 44 = \lambda^2 + 12\lambda + 36$$
$$\Rightarrow 8\lambda = 8$$
$$\Rightarrow \lambda = 1$$

Hence, the value of λ is 1.

Question 14:

If $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$.

Answer

Since $\vec{a},\vec{b},$ and $\vec{c}$ are mutually perpendicular vectors, we have

$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0.$

It is given that:

$|\vec{a}|=|\vec{b}|=|\vec{c}|$

Let vector $\vec{a}+\vec{b}+\vec{c}$ be inclined to $\vec{a},\vec{b},$ and $\vec{c}$ at angles $\theta_1,\ \theta_2,$ and θ_3 respectively.

Then, we have:

$$\cos\theta_1=\frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$=\frac{|\vec{a}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}\qquad[\vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}=0]$$

$$=\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\theta_2=\frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}=\frac{\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{b}|}$$

$$=\frac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{b}|}\qquad[\vec{a}\cdot\vec{b}=\vec{c}\cdot\vec{b}=0]$$

$$=\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\theta_3=\frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}=\frac{\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}$$

$$=\frac{|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}\qquad[\vec{a}\cdot\vec{c}=\vec{b}\cdot\vec{c}=0]$$

$$=\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

Now, as $|\vec{a}| = |\vec{b}| = |\vec{c}|$, $\cos\theta_1 = \cos\theta_2 = \cos\theta_3$.

$\therefore \theta_1 = \theta_2 = \theta_3$

Hence, the vector $\left(\vec{a}+\vec{b}+\vec{c}\right)$ is equally inclined to $\vec{a}, \vec{b},$ and $\vec{c}$.

Question 15:

Prove that $\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right) = |\vec{a}|^2 + |\vec{b}|^2$, if and only if $\vec{a}, \vec{b}$ are perpendicular, given $\vec{a} \neq \vec{0},\ \vec{b} \neq \vec{0}$.

Answer

$\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right) = |\vec{a}|^2 + |\vec{b}|^2$

$\Leftrightarrow \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b} = |\vec{a}|^2 + |\vec{b}|^2$ $\quad$ [Distributivity of scalar products over addition]

$\Leftrightarrow |\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ $\quad$ $\left[\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}\ \text{(Scalar product is commutative)}\right]$

$\Leftrightarrow 2\vec{a}\cdot\vec{b} = 0$

$\Leftrightarrow \vec{a}\cdot\vec{b} = 0$

$\therefore \vec{a}$ and $\vec{b}$ are perpendicular. $\quad$ $\left[\vec{a} \neq \vec{0},\ \vec{b} \neq \vec{0}\ \text{(Given)}\right]$

Question 16:

If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} \geq 0$ only when

(A) $0 < \theta < \frac{\pi}{2}$ (B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$ (D) $0 \leq \theta \leq \pi$

Answer

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

It is known that $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$.

$\therefore \vec{a}\cdot\vec{b}\geq 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta \geq 0$

$\Rightarrow \cos\theta \geq 0 \qquad \left[|\vec{a}| \text{ and } |\vec{b}| \text{ are positive}\right]$

$\Rightarrow 0\leq\theta\leq\frac{\pi}{2}$

Hence, $\vec{a}.\vec{b}\geq 0$ when $0\leq\theta\leq\frac{\pi}{2}$.

The correct answer is B.

Question 17:

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta=\frac{\pi}{4}$ (B) $\theta=\frac{\pi}{3}$ (C) $\theta=\frac{\pi}{2}$ (D) $\theta=\frac{2\pi}{3}$

Answer

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ be the angle between them.

Then, $|\vec{a}|=|\vec{b}|=1$.

Now, $\vec{a}+\vec{b}$ is a unit vector if $|\vec{a}+\vec{b}|=1$.

$$\left|\vec{a}+\vec{b}\right|=1$$

$$\Rightarrow\left(\vec{a}+\vec{b}\right)^2=1$$

$$\Rightarrow\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)=1$$

$$\Rightarrow\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}=1$$

$$\Rightarrow\left|\vec{a}\right|^2+2\vec{a}.\vec{b}+\left|\vec{b}\right|^2=1$$

$$\Rightarrow 1^2+2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta+1^2=1$$

$$\Rightarrow 1+2.1.1\cos\theta+1=1$$

$$\Rightarrow\cos\theta=-\frac{1}{2}$$

$$\Rightarrow\theta=\frac{2\pi}{3}$$

Hence, $\vec{a}+\vec{b}$ is a unit vector if $\theta=\frac{2\pi}{3}$.

The correct answer is D.

Question 18:

The value of $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$ is

(A) 0 (B) –1 (C) 1 (D) 3

Answer

$$\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$$

$$=\hat{i}\cdot\hat{i}+\hat{j}\cdot\left(-\hat{j}\right)+\hat{k}\cdot\hat{k}$$

$$=1-\hat{j}\cdot\hat{j}+1$$

$$=1-1+1$$

$$=1$$

The correct answer is C.

Question 19:

If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$ when θ is equal to

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

Answer

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$$\left|\vec{a}\cdot\vec{b}\right| = \left|\vec{a}\times\vec{b}\right|$$

$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$

$$\Rightarrow \cos\theta = \sin\theta \qquad \left[|\vec{a}| \text{ and } |\vec{b}| \text{ are positive}\right]$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, $\left|\vec{a}.\vec{b}\right| = \left|\vec{a}\times\vec{b}\right|$ when θ is equal to $\frac{\pi}{4}$.

The correct answer is B.