Exercise 11.1

Question 1:

Find the equation of the circle with centre (0, 2) and radius 2

Answer

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre (h, k) = (0, 2) and radius (r) = 2.

Therefore, the equation of the circle is

$(x - 0)^2 + (y - 2)^2 = 2^2$

$x^2 + y^2 + 4 - 4y = 4$

$x^2 + y^2 - 4y = 0$

Question 2:

Find the equation of the circle with centre (–2, 3) and radius 4

Answer

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre (h, k) = (–2, 3) and radius (r) = 4.

Therefore, the equation of the circle is

$(x + 2)^2 + (y - 3)^2 = (4)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$

$x^2 + y^2 + 4x - 6y - 3 = 0$

Question 3:

Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$

Answer

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre (h, k) = $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius (r) = $\frac{1}{12}$.

Therefore, the equation of the circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2$$
$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}=\frac{1}{144}$$
$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}-\frac{1}{144}=0$$
$$144x^2-144x+36+144y^2-72y+9-1=0$$
$$144x^2-144x+144y^2-72y+44=0$$
$$36x^2-36x+36y^2-18y+11=0$$
$$36x^2+36y^2-36x-18y+11=0$$

Question 4:

Find the equation of the circle with centre (1, 1) and radius $\sqrt{2}$

Answer

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre (h, k) = (1, 1) and radius (r) = $\sqrt{2}$.

Therefore, the equation of the circle is

$$(x-1)^2+(y-1)^2=\left(\sqrt{2}\right)^2$$
$$x^2-2x+1+y^2-2y+1=2$$
$$x^2+y^2-2x-2y=0$$

Question 5:

Find the equation of the circle with centre (–a, –b) and radius $\sqrt{a^2-b^2}$

Answer

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre (h, k) = (–a, –b) and radius (r) = $\sqrt{a^2-b^2}$.

Therefore, the equation of the circle is

$$(x+a)^2+(y+b)^2=\left(\sqrt{a^2-b^2}\right)^2$$
$$x^2+2ax+a^2+y^2+2by+b^2=a^2-b^2$$
$$x^2+y^2+2ax+2by+2b^2=0$$

Question 6:

Find the centre and radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$

Answer

The equation of the given circle is $(x + 5)^2 + (y - 3)^2 = 36$.

$(x + 5)^2 + (y - 3)^2 = 36$

$\Rightarrow \{x - (-5)\}^2 + (y - 3)^2 = 6^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where h = –5, k = 3, and r = 6.

Thus, the centre of the given circle is (–5, 3), while its radius is 6.

Question 7:

Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$

Answer

The equation of the given circle is $x^2 + y^2 - 4x - 8y - 45 = 0$.

$x^2 + y^2 - 4x - 8y - 45 = 0$

$\Rightarrow (x^2 - 4x) + (y^2 - 8y) = 45$

$\Rightarrow \{x^2 - 2(x)(2) + 2^2\} + \{y^2 - 2(y)(4)+ 4^2\} - 4 -16 = 45$

$\Rightarrow (x - 2)^2 + (y -4)^2 = 65$

$\Rightarrow (x - 2)^2 + (y -4)^2 = \left(\sqrt{65}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where h = 2, k = 4, and $r=\sqrt{65}$.

Thus, the centre of the given circle is (2, 4), while its radius is $\sqrt{65}$.

Question 8:

Find the centre and radius of the circle $x^2 + y^2 - 8x + 10y - 12 = 0$

Answer

The equation of the given circle is $x^2 + y^2 - 8x + 10y - 12 = 0$.

$x^2 + y^2 - 8x + 10y - 12 = 0$

$\Rightarrow (x^2 - 8x) + (y^2 + 10y) = 12$

$\Rightarrow \{x^2 - 2(x)(4) + 4^2\} + \{y^2 + 2(y)(5) + 5^2\} - 16 - 25 = 12$

$\Rightarrow (x - 4)^2 + (y + 5)^2 = 53$

$\Rightarrow (x-4)^2 + \{y-(-5)\}^2 = \left(\sqrt{53}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 4$, $k = -5$, and $r = \sqrt{53}$.

Thus, the centre of the given circle is (4, –5), while its radius is $\sqrt{53}$.

Question 9:

Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$

Answer

The equation of the given circle is $2x^2 + 2y^2 - x = 0$.

$$2x^2 + 2y^2 - x = 0$$

$$\Rightarrow \left(2x^2 - x\right) + 2y^2 = 0$$

$$\Rightarrow 2\left[\left(x^2 - \frac{x}{2}\right) + y^2\right] = 0$$

$$\Rightarrow \left\{x^2 - 2.x\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2\right\} + y^2 - \left(\frac{1}{4}\right)^2 = 0$$

$\Rightarrow \left(x - \frac{1}{4}\right)^2 + (y-0)^2 = \left(\frac{1}{4}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = \frac{1}{4}$, $k = 0$, and $r = \frac{1}{4}$.

Thus, the centre of the given circle is $\left(\frac{1}{4}, 0\right)$, while its radius is $\frac{1}{4}$.

Question 10:

Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x + y = 16$.

Answer

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (4, 1) and (6, 5),

$(4 - h)^2 + (1 - k)^2 = r^2$ … (1)

$(6 - h)^2 + (5 - k)^2 = r^2$ … (2)

Since the centre (h, k) of the circle lies on line $4x + y = 16$,

$4h + k = 16$ … (3)

From equations (1) and (2), we obtain

$(4 - h)^2 + (1 - k)^2 = (6 - h)^2 + (5 - k)^2$

$\Rightarrow 16 - 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 25 - 10k + k^2$

$\Rightarrow 16 - 8h + 1 - 2k = 36 - 12h + 25 - 10k$

$\Rightarrow 4h + 8k = 44$

$\Rightarrow h + 2k = 11$ … (4)

On solving equations (3) and (4), we obtain $h = 3$ and $k = 4$.

On substituting the values of h and k in equation (1), we obtain

$(4 - 3)^2 + (1 - 4)^2 = r^2$

$\Rightarrow (1)^2 + (-3)^2 = r^2$

$\Rightarrow 1 + 9 = r^2$

$\Rightarrow r^2 = 10$

$\Rightarrow r = \sqrt{10}$

Thus, the equation of the required circle is

$(x - 3)^2 + (y - 4)^2 = \left(\sqrt{10}\right)^2$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$

$x^2 + y^2 - 6x - 8y + 15 = 0$

Question 11:

Find the equation of the circle passing through the points (2, 3) and (−1, 1) and whose centre is on the line $x - 3y - 11 = 0$.

Answer

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (2, 3) and (−1, 1),

$(2 - h)^2 + (3 - k)^2 = r^2$ … (1)

$(-1 - h)^2 + (1 - k)^2 = r^2$ … (2)

Since the centre (h, k) of the circle lies on line $x - 3y - 11 = 0$,

$h - 3k = 11$ … (3)

From equations (1) and (2), we obtain

$(2 - h)^2 + (3 - k)^2 = (-1 - h)^2 + (1 - k)^2$

$\Rightarrow 4 - 4h + h^2 + 9 - 6k + k^2 = 1 + 2h + h^2 + 1 - 2k + k^2$

$\Rightarrow 4 - 4h + 9 - 6k = 1 + 2h + 1 - 2k$

$\Rightarrow 6h + 4k = 11$ … (4)

On solving equations (3) and (4), we obtain $h = \frac{7}{2}$ and $k = \frac{-5}{2}$.

On substituting the values of h and k in equation (1), we obtain

$$\left(2 - \frac{7}{2}\right)^2 + \left(3 + \frac{5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{4-7}{2}\right)^2 + \left(\frac{6+5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{-3}{2}\right)^2 + \left(\frac{11}{2}\right)^2 = r^2$$

$$\Rightarrow \frac{9}{4} + \frac{121}{4} = r^2$$

$$\Rightarrow \frac{130}{4} = r^2$$

Thus, the equation of the required circle is

$$\left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{130}{4}$$

$$\left(\frac{2x-7}{2}\right)^2 + \left(\frac{2y+5}{2}\right)^2 = \frac{130}{4}$$

$$4x^2 - 28x + 49 + 4y^2 + 20y + 25 = 130$$

$$4x^2 + 4y^2 - 28x + 20y - 56 = 0$$

$$4(x^2 + y^2 - 7x + 5y - 14) = 0$$

$$x^2 + y^2 - 7x + 5y - 14 = 0$$

Question 12:

Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).

Answer

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the radius of the circle is 5 and its centre lies on the x-axis, $k = 0$ and $r = 5$.

Now, the equation of the circle becomes $(x - h)^2 + y^2 = 25$.

It is given that the circle passes through point (2, 3).

$$\therefore (2-h)^2 + 3^2 = 25$$
$$\Rightarrow (2-h)^2 = 25-9$$
$$\Rightarrow (2-h)^2 = 16$$
$$\Rightarrow 2-h = \pm\sqrt{16} = \pm 4$$

If $2-h=4$, then $h=-2$.

If $2-h=-4$, then $h=6$.

When $h = -2$, the equation of the circle becomes

$(x + 2)^2 + y^2 = 25$

$x^2 + 4x + 4 + y^2 = 25$

$x^2 + y^2 + 4x - 21 = 0$

When $h = 6$, the equation of the circle becomes

$(x - 6)^2 + y^2 = 25$

$x^2 - 12x + 36 + y^2 = 25$

$x^2 + y^2 - 12x + 11 = 0$

Question 13:

Find the equation of the circle passing through (0, 0) and making intercepts a and b on the coordinate axes.

Answer

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the centre of the circle passes through (0, 0),

$(0 - h)^2 + (0 - k)^2 = r^2$

$\Rightarrow h^2 + k^2 = r^2$

The equation of the circle now becomes $(x - h)^2 + (y - k)^2 = h^2 + k^2$.

It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points $(a, 0)$ and $(0, b)$. Therefore,

$(a - h)^2 + (0 - k)^2 = h^2 + k^2$ … (1)

$(0 - h)^2 + (b - k)^2 = h^2 + k^2$ … (2)

From equation (1), we obtain

$a^2 - 2ah + h^2 + k^2 = h^2 + k^2$

$\Rightarrow a^2 - 2ah = 0$

$\Rightarrow a(a - 2h) = 0$

$\Rightarrow a = 0$ or $(a - 2h) = 0$

However, $a \neq 0$; hence, $(a - 2h) = 0 \Rightarrow h = \frac{a}{2}$.

From equation (2), we obtain

$h^2 + b^2 - 2bk + k^2 = h^2 + k^2$

$\Rightarrow b^2 - 2bk = 0$

$\Rightarrow b(b - 2k) = 0$

$\Rightarrow b = 0$ or$(b - 2k) = 0$

However, $b \neq 0$; hence, $(b - 2k) = 0 \Rightarrow k = \frac{b}{2}$.

Thus, the equation of the required circle is

$$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2$$

$$\Rightarrow\left(\frac{2x-a}{2}\right)^2+\left(\frac{2y-b}{2}\right)^2=\frac{a^2+b^2}{4}$$

$$\Rightarrow 4x^2-4ax+a^2+4y^2-4by+b^2=a^2+b^2$$

$$\Rightarrow 4x^2+4y^2-4ax-4by=0$$

$$\Rightarrow x^2+y^2-ax-by=0$$

Question 14:

Find the equation of a circle with centre (2, 2) and passes through the point (4, 5).

Answer

The centre of the circle is given as $(h, k) = (2, 2)$.

Since the circle passes through point (4, 5), the radius (r) of the circle is the distance between the points (2, 2) and (4, 5).

$$\therefore r=\sqrt{(2-4)^2+(2-5)^2}=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$$

Thus, the equation of the circle is

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y-2)^2=\left(\sqrt{13}\right)^2$$
$$x^2-4x+4+y^2-4y+4=13$$
$$x^2+y^2-4x-4y-5=0$$

Question 15:

Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

Answer

The equation of the given circle is $x^2 + y^2 = 25$.

$x^2 + y^2 = 25$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = 5^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 0$, $k = 0$, and $r = 5$.

$\therefore$Centre = (0, 0) and radius = 5

Distance between point (–2.5, 3.5) and centre (0, 0)

$$=\sqrt{(-2.5-0)^2+(3.5-0)^2}$$
$$=\sqrt{6.25+12.25}$$
$$=\sqrt{18.5}$$
$$=4.3\ (\text{approx.})<5$$

Since the distance between point (–2.5, 3.5) and centre (0, 0) of the circle is less than the radius of the circle, point (–2.5, 3.5) lies inside the circle.