Exercise 11.3

Question 1:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$

Answer

The given equation is $\frac{x^2}{36}+\frac{y^2}{16}=1$.

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain *a* = 6 and *b* = 4.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{36-16}=\sqrt{20}=2\sqrt{5}$

Therefore,

The coordinates of the foci are $\left(2\sqrt{5},0\right)$ and $\left(-2\sqrt{5},0\right)$.

The coordinates of the vertices are (6, 0) and (–6, 0).

Length of major axis = 2*a* = 12

Length of minor axis = 2*b* = 8

$\text{Eccentricity, } e=\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 16}{6}=\frac{16}{3}$

Question 2:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{4}+\frac{y^2}{25}=1$

Answer

The given equation is $\frac{x^2}{4}+\frac{y^2}{25}=1$ or $\frac{x^2}{2^2}+\frac{y^2}{5^2}=1$.

Here, the denominator of $\frac{y^2}{25}$ is greater than the denominator of $\frac{x^2}{4}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $b = 2$ and $a = 5$.

$$\therefore c=\sqrt{a^2-b^2}=\sqrt{25-4}=\sqrt{21}$$

Therefore,

The coordinates of the foci are $\left(0,\sqrt{21}\right)$ and $\left(0,-\sqrt{21}\right)$.

The coordinates of the vertices are (0, 5) and (0, –5)

Length of major axis = $2a = 10$

Length of minor axis = $2b = 4$

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{\sqrt{21}}{5}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{5}=\frac{8}{5}$

Question 3:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Answer

The given equation is $\frac{x^2}{16}+\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$.

Here, the denominator of $\frac{x^2}{16}$ is greater than the denominator of $\frac{y^2}{9}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain $a = 4$ and $b = 3$.

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{16-9} = \sqrt{7}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{7}, 0\right)$.

The coordinates of the vertices are $\left(\pm 4, 0\right)$.

Length of major axis = 2*a* = 8

Length of minor axis = 2*b* = 6

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{7}}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 9}{4} = \frac{9}{2}$

Question 4:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{25} + \frac{y^2}{100} = 1$

Answer

The given equation is $\frac{x^2}{25} + \frac{y^2}{100} = 1$ or $\frac{x^2}{5^2} + \frac{y^2}{10^2} = 1$.

Here, the denominator of $\frac{y^2}{100}$ is greater than the denominator of $\frac{x^2}{25}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain *b* = 5 and *a* = 10.

$\therefore\ c = \sqrt{a^2 - b^2} = \sqrt{100-25} = \sqrt{75} = 5\sqrt{3}$

Therefore,

The coordinates of the foci are $\left(0, \pm 5\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±10).

Length of major axis = 2*a* = 20

Length of minor axis = 2*b* = 10

Eccentricity, $e = \frac{c}{a} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 25}{10} = 5$

Question 5:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{49} + \frac{y^2}{36} = 1$

Answer

The given equation is $\frac{x^2}{49} + \frac{y^2}{36} = 1$ or $\frac{x^2}{7^2} + \frac{y^2}{6^2} = 1$.

Here, the denominator of $\frac{x^2}{49}$ is greater than the denominator of $\frac{y^2}{36}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain $a = 7$ and $b = 6$.

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{49 - 36} = \sqrt{13}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{13}, 0\right)$.

The coordinates of the vertices are (± 7, 0).

Length of major axis = 2*a* = 14

Length of minor axis = 2*b* = 12

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{13}}{7}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 36}{7} = \frac{72}{7}$

Question 6:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{100}+\frac{y^2}{400}=1$

Answer

The given equation is $\frac{x^2}{100}+\frac{y^2}{400}=1$ or $\frac{x^2}{10^2}+\frac{y^2}{20^2}=1$.

Here, the denominator of $\frac{y^2}{400}$ is greater than the denominator of $\frac{x^2}{100}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $b = 10$ and $a = 20$.

$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{400-100}=\sqrt{300}=10\sqrt{3}$

Therefore,

The coordinates of the foci are $\left(0,\pm 10\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±20)

Length of major axis = 2*a* = 40

Length of minor axis = 2*b* = 20

$\text{Eccentricity, } e=\frac{c}{a}=\frac{10\sqrt{3}}{20}=\frac{\sqrt{3}}{2}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 100}{20}=10$

Question 7:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $36x^2 + 4y^2 = 144$

Answer

The given equation is $36x^2 + 4y^2 = 144$.

It can be written as

$$36x^2 + 4y^2 = 144$$

$$\text{Or, } \frac{x^2}{4} + \frac{y^2}{36} = 1$$

$$\text{Or, } \frac{x^2}{2^2} + \frac{y^2}{6^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{6^2}$ is greater than the denominator of $\frac{x^2}{2^2}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 2$ and $a = 6$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$$

Therefore,

The coordinates of the foci are $\left(0, \pm 4\sqrt{2}\right)$.

The coordinates of the vertices are (0, ±6).

Length of major axis = $2a$ = 12

Length of minor axis = $2b$ = 4

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{6} = \frac{4}{3}$

Question 8:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $16x^2 + y^2 = 16$

Answer

The given equation is $16x^2 + y^2 = 16$.

It can be written as

$16x^2 + y^2 = 16$

Or, $\frac{x^2}{1} + \frac{y^2}{16} = 1$

Or, $\frac{x^2}{1^2} + \frac{y^2}{4^2} = 1$ ...(1)

Here, the denominator of $\frac{y^2}{4^2}$ is greater than the denominator of $\frac{x^2}{1^2}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 1$ and $a = 4$.

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{16-1} = \sqrt{15}$

Therefore,

The coordinates of the foci are $\left(0, \pm\sqrt{15}\right)$.

The coordinates of the vertices are (0, ±4).

Length of major axis = 2*a* = 8

Length of minor axis = 2*b* = 2

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{15}}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times1}{4} = \frac{1}{2}$

Question 9:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $4x^2 + 9y^2 = 36$

Answer

The given equation is $4x^2 + 9y^2 = 36$.

It can be written as

$$4x^2+9y^2=36$$

$$\text{Or, } \frac{x^2}{9}+\frac{y^2}{4}=1$$

$$\text{Or, } \frac{x^2}{3^2}+\frac{y^2}{2^2}=1 \qquad ...(1)$$

Here, the denominator of $\frac{x^2}{3^2}$ is greater than the denominator of $\frac{y^2}{2^2}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain a = 3 and b = 2.

$$\therefore c=\sqrt{a^2-b^2}=\sqrt{9-4}=\sqrt{5}$$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{5},0\right)$.

The coordinates of the vertices are (±3, 0).

Length of major axis = 2*a* = 6

Length of minor axis = 2*b* = 4

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{\sqrt{5}}{3}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{3}=\frac{8}{3}$

Question 10:

Find the equation for the ellipse that satisfies the given conditions: Vertices (±5, 0), foci (±4, 0)

Answer

Vertices (±5, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, *a* = 5 and *c* = 4.

It is known that $a^2 = b^2 + c^2$.

$\therefore 5^2 = b^2 + 4^2$

$\Rightarrow 25 = b^2 + 16$

$\Rightarrow b^2 = 25 - 16$

$\Rightarrow b = \sqrt{9} = 3$

Thus, the equation of the ellipse is $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ or $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

Question 11:

Find the equation for the ellipse that satisfies the given conditions: Vertices (0, ±13), foci (0, ±5)

Answer

Vertices (0, ±13), foci (0, ±5)

Here, the vertices are on the *y*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where *a* is the semi-major axis.

Accordingly, $a = 13$ and $c = 5$.

It is known that $a^2 = b^2 + c^2$.

$\therefore 13^2 = b^2 + 5^2$

$\Rightarrow 169 = b^2 + 25$

$\Rightarrow b^2 = 169 - 25$

$\Rightarrow b = \sqrt{144} = 12$

Thus, the equation of the ellipse is $\frac{x^2}{12^2} + \frac{y^2}{13^2} = 1$ or $\frac{x^2}{144} + \frac{y^2}{169} = 1$.

Question 12:

Find the equation for the ellipse that satisfies the given conditions: Vertices (±6, 0), foci (±4, 0)

Answer

Vertices (±6, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, *a* = 6, *c* = 4.

It is known that $a^2=b^2+c^2$.

$\therefore 6^2=b^2+4^2$

$\Rightarrow 36=b^2+16$

$\Rightarrow b^2=36-16$

$\Rightarrow b=\sqrt{20}$

Thus, the equation of the ellipse is $\frac{x^2}{6^2}+\frac{y^2}{\left(\sqrt{20}\right)^2}=1$ or $\frac{x^2}{36}+\frac{y^2}{20}=1$.

Question 13:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Answer

Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Here, the major axis is along the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, *a* = 3 and *b* = 2.

Thus, the equation of the ellipse is $\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$ i.e., $\frac{x^2}{9}+\frac{y^2}{4}=1$.

Question 14:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $\left(0,\ \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Answer

Ends of major axis $\left(0,\ \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Here, the major axis is along the *y*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where *a* is the semi-major axis.

Accordingly, *a* = $\sqrt{5}$ and *b* = 1.

Thus, the equation of the ellipse is $\frac{x^2}{1^2}+\frac{y^2}{\left(\sqrt{5}\right)^2}=1$ or $\frac{x^2}{1}+\frac{y^2}{5}=1$.

Question 15:

Find the equation for the ellipse that satisfies the given conditions: Length of major axis 26, foci (±5, 0)

Answer

Length of major axis = 26; foci = (±5, 0).

Since the foci are on the *x*-axis, the major axis is along the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, 2*a* = 26 ⇒ *a* = 13 and *c* = 5.

It is known that $a^2=b^2+c^2$.

$\therefore 13^2=b^2+5^2$

$\Rightarrow 169=b^2+25$

$\Rightarrow b^2=169-25$

$\Rightarrow b=\sqrt{144}=12$

Thus, the equation of the ellipse is $\frac{x^2}{13^2}+\frac{y^2}{12^2}=1$ or $\frac{x^2}{169}+\frac{y^2}{144}=1$.

Question 16:

Find the equation for the ellipse that satisfies the given conditions: Length of minor axis 16, foci (0, ±6)

Answer

Length of minor axis = 16; foci = (0, ±6).

Since the foci are on the *y*-axis, the major axis is along the *y*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where *a* is the semi-major axis.

Accordingly, 2*b* = 16 ⇒ *b* = 8 and *c* = 6.

It is known that $a^2=b^2+c^2$.

$\therefore a^2=8^2+6^2=64+36=100$

$\Rightarrow a=\sqrt{100}=10$

Thus, the equation of the ellipse is $\frac{x^2}{8^2}+\frac{y^2}{10^2}=1$ or $\frac{x^2}{64}+\frac{y^2}{100}=1$.

Question 17:

Find the equation for the ellipse that satisfies the given conditions: Foci (±3, 0), *a* = 4

Answer

Foci (±3, 0), *a* = 4

Since the foci are on the *x*-axis, the major axis is along the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, *c* = 3 and *a* = 4.

It is known that $a^2=b^2+c^2$.

$\therefore 4^2=b^2+3^2$

$\Rightarrow 16=b^2+9$

$\Rightarrow b^2=16-9=7$

Thus, the equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{7}=1$.

Question 18:

Find the equation for the ellipse that satisfies the given conditions: b = 3, c = 4, centre at the origin; foci on the x axis.

Answer

It is given that b = 3, c = 4, centre at the origin; foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, b = 3, c = 4.

It is known that $a^2=b^2+c^2$.

$\therefore a^2=3^2+4^2=9+16=25$

$\Rightarrow a=5$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{9}=1$.

Question 19:

Find the equation for the ellipse that satisfies the given conditions: Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Answer

Since the centre is at (0, 0) and the major axis is on the y-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \qquad ...(1)$$

Where, a is the semi-major axis

The ellipse passes through points (3, 2) and (1, 6). Hence,

$$\frac{9}{b^2}+\frac{4}{a^2}=1 \qquad ...(2)$$

$$\frac{1}{b^2}+\frac{36}{a^2}=1 \qquad ...(3)$$

On solving equations (2) and (3), we obtain b^2 = 10 and a^2 = 40.

Thus, the equation of the ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1 \text{ or } 4x^2+y^2=40$.

Question 20:

Find the equation for the ellipse that satisfies the given conditions: Major axis on the *x*-axis and passes through the points (4, 3) and (6, 2).

Answer

Since the major axis is on the *x*-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ...(1)$$

Where, a is the semi-major axis

The ellipse passes through points (4, 3) and (6, 2). Hence,

$$\frac{16}{a^2}+\frac{9}{b^2}=1 \qquad ...(2)$$

$$\frac{36}{a^2}+\frac{4}{b^2}=1 \qquad ...(3)$$

On solving equations (2) and (3), we obtain $a^2 = 52$ and $b^2 = 13$.

Thus, the equation of the ellipse is $\frac{x^2}{52}+\frac{y^2}{13}=1 \text{ or } x^2+4y^2=52$.