Exercise 11.1

Question 1:

If a line makes angles 90°, 135°, 45° with *x*, *y* and *z*-axes respectively, find its direction cosines.

Answer

Let direction cosines of the line be *l*, *m*, and *n*.

$l = \cos 90° = 0$

$m = \cos 135° = -\frac{1}{\sqrt{2}}$

$n = \cos 45° = \frac{1}{\sqrt{2}}$

Therefore, the direction cosines of the line are $0, -\frac{1}{\sqrt{2}}, \text{ and } \frac{1}{\sqrt{2}}.$

Question 2:

Find the direction cosines of a line which makes equal angles with the coordinate axes.

Answer

Let the direction cosines of the line make an angle *α* with each of the coordinate axes.

∴ $l = \cos\alpha$, $m = \cos\alpha$, $n = \cos\alpha$

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow 3\cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha = \frac{1}{3}$

$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$

Thus, the direction cosines of the line, which is equally inclined to the coordinate axes, are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \text{ and } \pm\frac{1}{\sqrt{3}}.$

Question 3:

If a line has the direction ratios −18, 12, −4, then what are its direction cosines?

Answer

If a line has direction ratios of −18, 12, and −4, then its direction cosines are

$$\frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$$

i.e., $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$

$\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

Thus, the direction cosines are $-\frac{9}{11}, \frac{6}{11}, \text{ and } \frac{-2}{11}$.

Question 4:

Show that the points (2, 3, 4), (−1, −2, 1), (5, 8, 7) are collinear.

Answer

The given points are A (2, 3, 4), B (− 1, − 2, 1), and C (5, 8, 7).

It is known that the direction ratios of line joining the points, (x_1, y_1, z_1) and (x_2, y_2, z_2), are given by, $x_2 - x_1$, $y_2 - y_1$, and $z_2 - z_1$.

The direction ratios of AB are (−1 − 2), (−2 − 3), and (1 − 4) i.e., −3, −5, and −3.

The direction ratios of BC are (5 − (− 1)), (8 − (− 2)), and (7 − 1) i.e., 6, 10, and 6.

It can be seen that the direction ratios of BC are −2 times that of AB i.e., they are proportional.

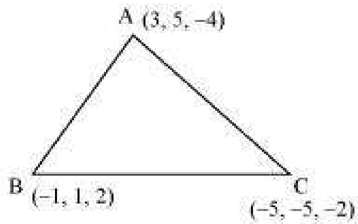
Therefore, AB is parallel to BC. Since point B is common to both AB and BC, points A, B, and C are collinear.

Question 5:

Find the direction cosines of the sides of the triangle whose vertices are (3, 5, − 4), (− 1, 1, 2) and (− 5, − 5, − 2)

Answer

The vertices of ΔABC are A (3, 5, −4), B (−1, 1, 2), and C (−5, −5, −2).



The direction ratios of side AB are (−1 − 3), (1 − 5), and (2 − (−4)) i.e., −4, −4, and 6.

$$\text{Then,} \sqrt{(-4)^2+(-4)^2+(6)^2} = \sqrt{16+16+36}$$
$$= \sqrt{68}$$
$$= 2\sqrt{17}$$

Therefore, the direction cosines of AB are

$$\frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}}, \frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}}, \frac{6}{\sqrt{(-4)^2+(-4)^2+(6)^2}}$$
$$\frac{-4}{2\sqrt{17}}, -\frac{4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}}$$
$$\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$$

The direction ratios of BC are (−5 − (−1)), (−5 − 1), and (−2 − 2) i.e., −4, −6, and −4.

Therefore, the direction cosines of BC are

$$\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$
$$\frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}$$

The direction ratios of CA are (−5 − 3), (−5 − 5), and (−2 − (−4)) i.e., −8, −10, and 2.

Therefore, the direction cosines of AC are

$$\frac{-8}{\sqrt{(-8)^2+(10)^2+(2)^2}},\frac{-5}{\sqrt{(-8)^2+(10)^2+(2)^2}},\frac{2}{\sqrt{(-8)^2+(10)^2+(2)^2}}$$

$$\frac{-8}{2\sqrt{42}},\frac{-10}{2\sqrt{42}},\frac{2}{2\sqrt{42}}$$

Exercise 11.2

Question 1:

Show that the three lines with direction cosines

$\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

Answer

Two lines with direction cosines, l_1, m_1, n_1 and l_2, m_2, n_2, are perpendicular to each other, if $l_1l_2 + m_1m_2 + n_1n_2 = 0$

(i) For the lines with direction cosines, $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$ and $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$, we obtain

$$\begin{aligned} l_1l_2 + m_1m_2 + n_1n_2 &= \frac{12}{13}\times\frac{4}{13} + \left(\frac{-3}{13}\right)\times\frac{12}{13} + \left(\frac{-4}{13}\right)\times\frac{3}{13} \\ &= \frac{48}{169} - \frac{36}{169} - \frac{12}{169} \\ &= 0 \end{aligned}$$

Therefore, the lines are perpendicular.

(ii) For the lines with direction cosines, $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$ and $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$, we obtain

$$\begin{aligned} l_1l_2 + m_1m_2 + n_1n_2 &= \frac{4}{13}\times\frac{3}{13} + \frac{12}{13}\times\left(\frac{-4}{13}\right) + \frac{3}{13}\times\frac{12}{13} \\ &= \frac{12}{169} - \frac{48}{169} + \frac{36}{169} \\ &= 0 \end{aligned}$$

Therefore, the lines are perpendicular.

(iii) For the lines with direction cosines, $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ and $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$, we obtain

$$l_1l_2 + m_1m_2 + n_1n_2 = \left(\frac{3}{13}\right)\times\left(\frac{12}{13}\right)+\left(\frac{-4}{13}\right)\times\left(\frac{-3}{13}\right)+\left(\frac{12}{13}\right)\times\left(\frac{-4}{13}\right)$$
$$= \frac{36}{169}+\frac{12}{169}-\frac{48}{169}$$
$$= 0$$

Therefore, the lines are perpendicular.

Thus, all the lines are mutually perpendicular.

Question 2:

Show that the line through the points (1, −1, 2) (3, 4, −2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

Answer

Let AB be the line joining the points, (1, −1, 2) and (3, 4, − 2), and CD be the line joining the points, (0, 3, 2) and (3, 5, 6).

The direction ratios, a_1, b_1, c_1, of AB are (3 − 1), (4 − (−1)), and (−2 − 2) i.e., 2, 5, and −4.

The direction ratios, a_2, b_2, c_2, of CD are (3 − 0), (5 − 3), and (6 −2) i.e., 3, 2, and 4.

AB and CD will be perpendicular to each other, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 3 + 5 \times 2 + (-4) \times 4$

$= 6 + 10 - 16$

$= 0$

Therefore, AB and CD are perpendicular to each other.

Question 3:

Show that the line through the points (4, 7, 8) (2, 3, 4) is parallel to the line through the points (−1, −2, 1), (1, 2, 5).

Answer

Let AB be the line through the points, (4, 7, 8) and (2, 3, 4), and CD be the line through the points, (−1, −2, 1) and (1, 2, 5).

The directions ratios, a_1, b_1, c_1, of AB are (2 − 4), (3 − 7), and (4 − 8) i.e., −2, −4, and −4.

The direction ratios, a_2, b_2, c_2, of CD are (1 − (−1)), (2 − (−2)), and (5 − 1) i.e., 2, 4, and 4.