Exercise 13.4

Question 1:

State which of the following are **not** the probability distributions of a random variable. Give reasons for your answer.

(i)

X	0	1	2
P (X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P (X)	0.1	0.5	0.2	− 0.1	0.3

(iii)

Y	−1	0	1
P (Y)	0.6	0.1	0.2

(iv)

Z	3	2	1	0	−1
P (Z)	0.3	0.2	0.4	0.1	0.05

Answer

It is known that the sum of all the probabilities in a probability distribution is one.

(i) Sum of the probabilities = $0.4 + 0.4 + 0.2 = 1$

Therefore, the given table is a probability distribution of random variables.

(ii) It can be seen that for $X = 3$, $P(X) = -0.1$

It is known that probability of any observation is not negative. Therefore, the given table is not a probability distribution of random variables.

(iii) Sum of the probabilities = $0.6 + 0.1 + 0.2 = 0.9 \neq 1$

Therefore, the given table is not a probability distribution of random variables.

(iv) Sum of the probabilities = 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 ≠ 1

Therefore, the given table is not a probability distribution of random variables.

Question 2:

An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represents the number of black balls. What are the possible values of X? Is X a random variable?

Answer

The two balls selected can be represented as BB, BR, RB, RR, where B represents a black ball and R represents a red ball.

X represents the number of black balls.

∴X (BB) = 2

X (BR) = 1

X (RB) = 1

X (RR) = 0

Therefore, the possible values of X are 0, 1, and 2.

Yes, X is a random variable.

Question 3:

Let X represents the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Answer

A coin is tossed six times and X represents the difference between the number of heads and the number of tails.

∴ X (6 H, 0T) $= |6-0| = 6$

X (5 H, 1 T) $= |5-1| = 4$

X (4 H, 2 T) $=|4-2|=2$

X (3 H, 3 T) $=|3-3|=0$

X (2 H, 4 T) $=|2-4|=2$

X (1 H, 5 T) $=|1-5|=4$

X (0H, 6 T) $=|0-6|=6$

Thus, the possible values of X are 6, 4, 2, and 0.

Question 4:

Find the probability distribution of

(i) number of heads in two tosses of a coin

(ii) number of tails in the simultaneous tosses of three coins

(iii) number of heads in four tosses of a coin

Answer

(i) When one coin is tossed twice, the sample space is

{HH, HT, TH, TT}

Let X represent the number of heads.

∴ X (HH) = 2, X (HT) = 1, X (TH) = 1, X (TT) = 0

Therefore, X can take the value of 0, 1, or 2.

It is known that,

$$P(HH)=P(HT)=P(TH)=P(TT)=\frac{1}{4}$$

P (X = 0) = P (TT) $=\frac{1}{4}$

P (X = 1) = P (HT) + P (TH) $=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

P (X = 2) = P (HH) $=\frac{1}{4}$

Thus, the required probability distribution is as follows.

X	0	1	2
P (X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii) When three coins are tossed simultaneously, the sample space is

$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Let X represent the number of tails.

It can be seen that X can take the value of 0, 1, 2, or 3.

P (X = 0) = P (HHH) = $\frac{1}{8}$

P (X = 1) = P (HHT) + P (HTH) + P (THH) = $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

P (X = 2) = P (HTT) + P (THT) + P (TTH) = $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

P (X = 3) = P (TTT) = $\frac{1}{8}$

Thus, the probability distribution is as follows.

X	0	1	2	3
P (X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) When a coin is tossed four times, the sample space is

$$S=\left\{\begin{matrix} HHHH, HHHT, HHTH, HHTT, HTHT, HTHH, HTTH, HTTT, \\ THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT \end{matrix}\right\}$$

Let X be the random variable, which represents the number of heads.

It can be seen that X can take the value of 0, 1, 2, 3, or 4.

P (X = 0) = P (TTTT) = $\frac{1}{16}$

P (X = 1) = P (TTTH) + P (TTHT) + P (THTT) + P (HTTT)

$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{4}{16}=\frac{1}{4}$

P (X = 2) = P (HHTT) + P (THHT) + P (TTHH) + P (HTTH) + P (HTHT) + P (THTH)

$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{6}{16}=\frac{3}{8}$

P (X = 3) = P (HHHT) + P (HHTH) + P (HTHH) P (THHH)

$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{4}{16}=\frac{1}{4}$

P (X = 4) = P (HHHH) = $\frac{1}{16}$

Thus, the probability distribution is as follows.

X	0	1	2	3	4
P (X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Question 5:

Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(i) number greater than 4

(ii) six appears on at least one die

Answer

When a die is tossed two times, we obtain (6 × 6) = 36 number of observations.

Let X be the random variable, which represents the number of successes.

i. Here, success refers to the number greater than 4.

P (X = 0) = P (number less than or equal to 4 on both the tosses) = $\frac{4}{6}\times\frac{4}{6}=\frac{4}{9}$

P (X = 1) = P (number less than or equal to 4 on first toss and greater than 4 on second toss) + P (number greater than 4 on first toss and less than or equal to 4 on second toss)

$$=\frac{4}{6}\times\frac{2}{6}+\frac{4}{6}\times\frac{2}{6}=\frac{4}{9}$$

P (X = 2) = P (number greater than 4 on both the tosses)

$$=\frac{2}{6}\times\frac{2}{6}=\frac{1}{9}$$

Thus, the probability distribution is as follows.

X	1	1	2
P (X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) Here, success means six appears on at least one die.

P (Y = 0) = P (six does not appear on any of the dice) $=\frac{5}{6}\times\frac{5}{6}=\frac{25}{36}$

P (Y = 1) = P (six appears on at least one of the dice) = $\frac{11}{36}$

Thus, the required probability distribution is as follows.

Y	0	1
P (Y)	$\frac{25}{36}$	$\frac{11}{36}$

Question 6:

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Answer

It is given that out of 30 bulbs, 6 are defective.

⇒ Number of non-defective bulbs = 30 − 6 = 24

4 bulbs are drawn from the lot with replacement.

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

∴ P (X = 0) = P (4 non-defective and 0 defective) $= {}^4C_0 \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} \frac{4}{5} = \frac{256}{625}$

P (X = 1) = P (3 non-defective and 1 defective) $= {}^4C_1 \cdot \left(\frac{1}{5}\right) \cdot \left(\frac{4}{5}\right)^3 = \frac{256}{625}$

P (X = 2) = P (2 non-defective and 2 defective) $= {}^4C_2 \cdot \left(\frac{1}{5}\right)^2 \cdot \left(\frac{4}{5}\right)^2 = \frac{96}{625}$

P (X = 3) = P (1 non-defective and 3 defective) $= {}^4C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right) = \frac{16}{625}$

P (X = 4) = P (0 non-defective and 4 defective) $= {}^4C_4 \cdot \left(\frac{1}{5}\right)^4 \cdot \left(\frac{4}{5}\right)^0 = \frac{1}{625}$

Therefore, the required probability distribution is as follows.

X	0	1	2	3	4
P (X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Question 7:

A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

Answer

Let the probability of getting a tail in the biased coin be *x*.

∴ P (T) = x

⇒ P (H) = 3x

For a biased coin, P (T) + P (H) = 1

$\Rightarrow x+3x=1$

$\Rightarrow 4x=1$

$\Rightarrow x=\frac{1}{4}$

$\therefore \text{P}(\text{T})=\frac{1}{4} \text{ and } \text{P}(\text{H})=\frac{3}{4}$

When the coin is tossed twice, the sample space is {HH, TT, HT, TH}.

Let X be the random variable representing the number of tails.

∴ P (X = 0) = P (no tail) = P (H) × P (H) $=\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$

P (X = 1) = P (one tail) = P (HT) + P (TH)

$=\frac{3}{4}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{3}{4}$

$=\frac{3}{16}+\frac{3}{16}$

$=\frac{3}{8}$

P (X = 2) = P (two tails) = P (TT) $=\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$

Therefore, the required probability distribution is as follows.

X	0	1	2
P (X)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

Question 8:

A random variable X has the following probability distribution.

X	0	1	2	3	4	5	6	7
P (X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine

(i) k

(ii) P (X < 3)

(iii) P (X > 6)

(iv) P (0 < X < 3)

Answer

(i) It is known that the sum of probabilities of a probability distribution of random variables is one.

$$\therefore 0+k+2k+2k+3k+k^2+2k^2+\left(7k^2+k\right)=1$$

$$\Rightarrow 10k^2+9k-1=0$$

$$\Rightarrow \left(10k-1\right)\left(k+1\right)=0$$

$$\Rightarrow k=-1,\frac{1}{10}$$

$k = -1$ is not possible as the probability of an event is never negative.

$$\therefore k=\frac{1}{10}$$

(ii) P (X < 3) = P (X = 0) + P (X = 1) + P (X = 2)

$= 0 + k + 2k$

$= 3k$

$= 3 \times \frac{1}{10}$

$= \frac{3}{10}$

(iii) P (X > 6) = P (X = 7)

$= 7k^2 + k$

$= 7 \times \left(\frac{1}{10}\right)^2 + \frac{1}{10}$

$= \frac{7}{100} + \frac{1}{10}$

$= \frac{17}{100}$

(iv) P (0 < X < 3) = P (X = 1) + P (X = 2)

$= k + 2k$

$= 3k$

$= 3 \times \frac{1}{10}$

$= \frac{3}{10}$

Question 9:

The random variable X has probability distribution P(X) of the following form, where *k* is some number:

$$P(X) = \begin{cases} k, \text{ if } x = 0 \\ 2k, \text{ if } x = 1 \\ 3k, \text{ if } x = 2 \\ 0, \text{ otherwise} \end{cases}$$

(a) Determine the value of *k*.

(b) Find P(X < 2), P(X ≥ 2), P(X ≥ 2).

Answer

(a) It is known that the sum of probabilities of a probability distribution of random variables is one.

$\therefore k + 2k + 3k + 0 = 1$

$\Rightarrow 6k = 1$

$\Rightarrow k = \frac{1}{6}$

(b) $P(X < 2) = P(X = 0) + P(X = 1)$

$= k + 2k$

$= 3k$

$= \frac{3}{6}$

$= \frac{1}{2}$

$$
\begin{aligned}
P(X \le 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\
&= k + 2k + 3k \\
&= 6k \\
&= \frac{6}{6} \\
&= 1
\end{aligned}
$$

$$\begin{aligned}P(X \geq 2) &= P(X = 2) + P(X > 2)\\ &= 3k + 0\\ &= 3k\\ &= \frac{3}{6}\\ &= \frac{1}{2}\end{aligned}$$

Question 10:

Find the mean number of heads in three tosses of a fair coin.

Answer

Let X denote the success of getting heads.

Therefore, the sample space is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

It can be seen that X can take the value of 0, 1, 2, or 3.

$$\begin{aligned}\therefore P(X = 0) &= P(TTT)\\ &= P(T)\cdot P(T)\cdot P(T)\\ &= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\ &= \frac{1}{8}\end{aligned}$$

$\therefore$ P (X = 1) = P (HHT) + P (HTH) + P (THH)

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$= \frac{3}{8}$$

$\therefore$P(X = 2) = P (HHT) + P (HTH) + P (THH)

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$= \frac{3}{8}$$

$$\therefore P(X=3) = P(HHH)$$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$= \frac{1}{8}$$

Therefore, the required probability distribution is as follows.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean of X E(X), μ = $\sum X_i P(X_i)$

$$= 0\times\frac{1}{8}+1\times\frac{3}{8}+2\times\frac{3}{8}+3\times\frac{1}{8}$$

$$= \frac{3}{8}+\frac{3}{4}+\frac{3}{8}$$

$$= \frac{3}{2}$$

$$= 1.5$$

Question 11:

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

Answer

Here, X represents the number of sixes obtained when two dice are thrown simultaneously. Therefore, X can take the value of 0, 1, or 2.

∴ P (X = 0) = P (not getting six on any of the dice) = $\frac{25}{36}$

P (X = 1) = P (six on first die and no six on second die) + P (no six on first die and six on second die)

$$=2\left(\frac{1}{6}\times\frac{5}{6}\right)=\frac{10}{36}$$

P (X = 2) = P (six on both the dice) = $\frac{1}{36}$

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Then, expectation of X = E(X) = $\sum X_i P(X_i)$

$$=0\times\frac{25}{36}+1\times\frac{10}{36}+2\times\frac{1}{36}$$

$$=\frac{1}{3}$$

Question 12:

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find E(X).

Answer

The two positive integers can be selected from the first six positive integers without replacement in 6 × 5 = 30 ways

X represents the larger of the two numbers obtained. Therefore, X can take the value of 2, 3, 4, 5, or 6.

For X = 2, the possible observations are (1, 2) and (2, 1).

$$\therefore P(X=2)=\frac{2}{30}=\frac{1}{15}$$

For X = 3, the possible observations are (1, 3), (2, 3), (3, 1), and (3, 2).

$$\therefore P(X=3)=\frac{4}{30}=\frac{2}{15}$$

For X = 4, the possible observations are (1, 4), (2, 4), (3, 4), (4, 3), (4, 2), and (4, 1).

$$\therefore P(X=4)=\frac{6}{30}=\frac{1}{5}$$

For X = 5, the possible observations are (1, 5), (2, 5), (3, 5), (4, 5), (5, 4), (5, 3), (5, 2), and (5, 1).

$$\therefore P(X=5)=\frac{8}{30}=\frac{4}{15}$$

For X = 6, the possible observations are (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 4), (6, 3), (6, 2), and (6, 1).

$$\therefore P(X=6)=\frac{10}{30}=\frac{1}{3}$$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

$$\begin{aligned}\text{Then, } E(X) &= \sum X_i P(X_i)\\ &= 2\cdot\frac{1}{15}+3\cdot\frac{2}{15}+4\cdot\frac{1}{5}+5\cdot\frac{4}{15}+6\cdot\frac{1}{3}\\ &= \frac{2}{15}+\frac{2}{5}+\frac{4}{5}+\frac{4}{3}+2\\ &= \frac{70}{15}\\ &= \frac{14}{3}\end{aligned}$$

Question 13:

Let X denotes the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

Answer

When two fair dice are rolled, 6 × 6 = 36 observations are obtained.

P(X = 2) = P(1, 1) = $\frac{1}{36}$

P(X = 3) = P (1, 2) + P(2, 1) = $\frac{2}{36}=\frac{1}{18}$

P(X = 4) = P(1, 3) + P(2, 2) + P(3, 1) = $\frac{3}{36}=\frac{1}{12}$

P(X = 5) = P(1, 4) + P(2, 3) + P(3, 2) + P(4, 1) = $\frac{4}{36}=\frac{1}{9}$

P(X = 6) = P(1, 5) + P (2, 4) + P(3, 3) + P(4, 2) + P(5, 1) = $\frac{5}{36}$

P(X = 7) = P(1, 6) + P(2, 5) + P(3, 4) + P(4, 3) + P(5, 2) + P(6, 1) $=\frac{6}{36}=\frac{1}{6}$

P(X = 8) = P(2, 6) + P(3, 5) + P(4, 4) + P(5, 3) + P(6, 2) = $\frac{5}{36}$

P(X = 9) = P(3, 6) + P(4, 5) + P(5, 4) + P(6, 3) = $\frac{4}{36}=\frac{1}{9}$

P(X = 10) = P(4, 6) + P(5, 5) + P(6, 4) = $\frac{3}{36}=\frac{1}{12}$

P(X = 11) = P(5, 6) + P(6, 5) = $\frac{2}{36}=\frac{1}{18}$

P(X = 12) = P(6, 6) = $\frac{1}{36}$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

$$\text{Then, } E(X) = \sum X_i \cdot P(X_i)$$

$$= 2\times\frac{1}{36}+3\times\frac{1}{18}+4\times\frac{1}{12}+5\times\frac{1}{9}+6\times\frac{5}{36}+7\times\frac{1}{6}$$

$$+8\times\frac{5}{36}+9\times\frac{1}{9}+10\times\frac{1}{12}+11\times\frac{1}{18}+12\times\frac{1}{36}$$

$$=\frac{1}{18}+\frac{1}{6}+\frac{1}{3}+\frac{5}{9}+\frac{5}{6}+\frac{7}{6}+\frac{10}{9}+1+\frac{5}{6}+\frac{11}{18}+\frac{1}{3}$$

$$=7$$

$$E(X^2) = \sum X_i^2 \cdot P(X_i)$$

$$= 4\times\frac{1}{36}+9\times\frac{1}{18}+16\times\frac{1}{12}+25\times\frac{1}{9}+36\times\frac{5}{36}+49\times\frac{1}{6}$$

$$+64\times\frac{5}{36}+81\times\frac{1}{9}+100\times\frac{1}{12}+121\times\frac{1}{18}+144\times\frac{1}{36}$$

$$=\frac{1}{9}+\frac{1}{2}+\frac{4}{3}+\frac{25}{9}+5+\frac{49}{6}+\frac{80}{9}+9+\frac{25}{3}+\frac{121}{18}+4$$

$$=\frac{987}{18}=\frac{329}{6}=54.833$$

$$\text{Then, } \mathrm{Var}(X) = E(X^2) - [E(X)]^2$$

$$= 54.833 - (7)^2$$

$$= 54.833 - 49$$

$$= 5.833$$

$$\therefore \text{Standard deviation} = \sqrt{\mathrm{Var}(X)}$$

$$= \sqrt{5.833}$$

$$= 2.415$$

Question 14:

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

Answer

There are 15 students in the class. Each student has the same chance to be chosen.

Therefore, the probability of each student to be selected is $\frac{1}{15}$.

The given information can be compiled in the frequency table as follows.

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

P(X = 14) = $\frac{2}{15}$, P(X = 15) = $\frac{1}{15}$, P(X = 16) = $\frac{2}{15}$, P(X = 16) = $\frac{3}{15}$,

P(X = 18) = $\frac{1}{15}$, P(X = 19) = $\frac{2}{15}$, P(X = 20) = $\frac{3}{15}$, P(X = 21) = $\frac{1}{15}$

Therefore, the probability distribution of random variable X is as follows.

X	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Then, mean of X = E(X)

$$=\sum X_i P(X_i)$$

$$=14\times\frac{2}{15}+15\times\frac{1}{15}+16\times\frac{2}{15}+17\times\frac{3}{15}+18\times\frac{1}{15}+19\times\frac{2}{15}+20\times\frac{3}{15}+21\times\frac{1}{15}$$

$$=\frac{1}{15}(28+15+32+51+18+38+60+21)$$

$$=\frac{263}{15}$$

$$=17.53$$

E(X^2) = $\sum X_i^2 P(X_i)$

$$= (14)^2 \cdot \frac{2}{15} + (15)^2 \cdot \frac{1}{15} + (16)^2 \cdot \frac{2}{15} + (17)^2 \cdot \frac{3}{15} +$$
$$(18)^2 \cdot \frac{1}{15} + (19)^2 \cdot \frac{2}{15} + (20)^2 \cdot \frac{3}{15} + (21)^2 \cdot \frac{1}{15}$$
$$= \frac{1}{15} \cdot (392 + 225 + 512 + 867 + 324 + 722 + 1200 + 441)$$
$$= \frac{4683}{15}$$
$$= 312.2$$

$$\therefore \text{Variance}(X) = E(X^2) - [E(X)]^2$$
$$= 312.2 - \left(\frac{263}{15}\right)^2$$
$$= 312.2 - 307.4177$$
$$= 4.7823$$
$$\approx 4.78$$

$$\text{Standard derivation} = \sqrt{\text{Variance}(X)}$$
$$= \sqrt{4.78}$$
$$= 2.186 \approx 2.19$$

Question 15:

In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var(X).

Answer

It is given that P(X = 0) = 30% = $\frac{30}{100} = 0.3$

$$P(X = 1) = 70\% = \frac{70}{100} = 0.7$$

Therefore, the probability distribution is as follows.

X	0	1
P(X)	0.3	0.7

$$\text{Then, } E(X) = \sum X_i P(X_i)$$
$$= 0 \times 0.3 + 1 \times 0.7$$
$$= 0.7$$

$$E(X^2) = \sum X_i^2 P(X_i)$$
$$= 0^2 \times 0.3 + (1)^2 \times 0.7$$
$$= 0.7$$

It is known that, Var (X) = $E(X^2) - [E(X)]^2$

= 0.7 − $(0.7)^2$

= 0.7 − 0.49

= 0.21

Question 16:

The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(A) 1 (B) 2 (C) 5 (D) $\frac{8}{3}$

Answer

Let X be the random variable representing a number on the die.

The total number of observations is six.

$$\therefore P(X=1) = \frac{3}{6} = \frac{1}{2}$$

$$P(X=2) = \frac{2}{6} = \frac{1}{3}$$

$$P(X=5) = \frac{1}{6}$$

Therefore, the probability distribution is as follows.

X	1	2	5
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Mean = E(X) = $\sum p_i x_i$

$= \frac{1}{2} \times 1 + \frac{1}{3} \times 2 + \frac{1}{6} \cdot 5$

$= \frac{1}{2} + \frac{2}{3} + \frac{5}{6}$

$= \frac{3+4+5}{6}$

$= \frac{12}{6}$

$= 2$

The correct answer is B.

Question 17:

Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

(A) $\frac{37}{221}$ (B) $\frac{5}{13}$ (C) $\frac{1}{13}$ (D) $\frac{2}{13}$

Answer

Let X denote the number of aces obtained. Therefore, X can take any of the values of 0, 1, or 2.

In a deck of 52 cards, 4 cards are aces. Therefore, there are 48 non-ace cards.

∴ P (X = 0) = P (0 ace and 2 non-ace cards) = $\frac{{}^4C_0 \times {}^{48}C_2}{{}^{52}C_2} = \frac{1128}{1326}$

P (X = 1) = P (1 ace and 1 non-ace cards) = $\frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{192}{1326}$

P (X = 2) = P (2 ace and 0 non- ace cards) = $\frac{{}^4C_2 \times {}^{48}C_0}{{}^{52}C_2} = \frac{6}{1326}$

Thus, the probability distribution is as follows.

X	0	1	2

P(X)	$\frac{1128}{1326}$	$\frac{192}{1326}$	$\frac{6}{1326}$

Then, E(X) = $\sum p_i x_i$

$$= 0 \times \frac{1128}{1326} + 1 \times \frac{192}{1326} + 2 \times \frac{6}{1326}$$

$$= \frac{204}{1326}$$

$$= \frac{2}{13}$$

Therefore, the correct answer is D.