Here, l = 35.5, C = 37, f = 26, h = 5, and N = 100

$$\therefore \text{Median} = 35.5 + \frac{50-37}{26} \times 5 = 35.5 + \frac{13 \times 5}{26} = 35.5 + 2.5 = 38$$

Thus, mean deviation about the median is given by,

$$\text{M.D.}(\text{M}) = \frac{1}{\text{N}} \sum_{i=1}^{8} f_i \left| x_i - \text{M} \right| = \frac{1}{100} \times 735 = 7.35$$

Exercise 15.2

Question 1:

Find the mean and variance for the data 6, 7, 10, 12, 13, 4, 8, 12

Answer

6, 7, 10, 12, 13, 4, 8, 12

Mean, $\bar{x} = \frac{\sum_{i=1}^{8} x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

The following table is obtained.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	−3	9
7	−2	4
10	−1	1
12	3	9
13	4	16
4	−5	25
8	−1	1
12	3	9
		74

$$\text{Variance}\left(\sigma^2\right) = \frac{1}{n}\sum_{i=1}^{8}(x_i - \bar{x})^2 = \frac{1}{8} \times 74 = 9.25$$

Question 2:

Find the mean and variance for the first *n* natural numbers

Answer

The mean of first *n* natural numbers is calculated as follows.

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$\therefore \text{Mean} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$\text{Variance}\left(\sigma^2\right) = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[x_i - \left(\frac{n+1}{2}\right)\right]^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - \frac{1}{n}\sum_{i=1}^{n}2\left(\frac{n+1}{2}\right)x_i + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2$$

$$= \frac{1}{n}\frac{n(n+1)(2n+1)}{6} - \left(\frac{n+1}{n}\right)\left[\frac{n(n+1)}{2}\right] + \frac{(n+1)^2}{4n} \times n$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4}$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= (n+1)\left[\frac{4n+2-3n-3}{12}\right]$$

$$= \frac{(n+1)(n-1)}{12}$$

$$= \frac{n^2-1}{12}$$

Question 3:

Find the mean and variance for the first 10 multiples of 3

Answer

The first 10 multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Here, number of observations, n = 10

$$\text{Mean, } \bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{165}{10} = 16.5$$

The following table is obtained.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
3	−13.5	182.25
6	−10.5	110.25
9	−7.5	56.25
12	−4.5	20.25
15	−1.5	2.25
18	1.5	2.25
21	4.5	20.25
24	7.5	56.25
27	10.5	110.25
30	13.5	182.25
		742.5

$$\text{Variance}\left(\sigma^2\right) = \frac{1}{n}\sum_{i=1}^{10}(x_i - \bar{x})^2 = \frac{1}{10} \times 742.5 = 74.25$$

Question 4:

Find the mean and variance for the data

xi	6	10	14	18	24	28	30
f i	2	4	7	12	8	4	3

Answer

The data is obtained in tabular form as follows.

x_i	f_i	f_ix_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
6	2	12	–13	169	338
10	4	40	–9	81	324
14	7	98	–5	25	175
18	12	216	–1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
	40	760			1736

Here, N = 40, $\sum_{i=1}^{7} f_i x_i = 760$

$$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_i x_i}{N} = \frac{760}{40} = 19$$

$$\text{Variance } = (\sigma^2) = \frac{1}{N}\sum_{i=1}^{7} f_i (x_i - \bar{x})^2 = \frac{1}{40} \times 1736 = 43.4$$

Question 5:

Find the mean and variance for the data

xi	92	93	97	98	102	104	109
f i	3	2	3	2	6	3	3

Answer

The data is obtained in tabular form as follows.

x_i	f_i	f_ix_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
92	3	276	–8	64	192
93	2	186	–7	49	98
97	3	291	–3	9	27
98	2	196	–2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243
	22	2200			640

Here, N = 22, $\sum_{i=1}^{7} f_ix_i = 2200$

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{7} f_ix_i = \frac{1}{22}\times 2200 = 100$$

$$\text{Variance}\left(\sigma^2\right) = \frac{1}{N}\sum_{i=1}^{7} f_i(x_i - \bar{x})^2 = \frac{1}{22}\times 640 = 29.09$$

Question 6:

Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Answer

The data is obtained in tabular form as follows.

x_i	f_i	$f_i = \frac{x_i - 64}{1}$	y_i^2	$f_i y_i$	$f_i y_i^2$
60	2	−4	16	−8	32
61	1	−3	9	−3	9
62	12	−2	4	−24	48
63	29	−1	1	−29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
	100	220		0	286

Mean, $\bar{x} = A \frac{\sum_{i=1}^{9} f_i y_i}{N} \times h = 64 + \frac{0}{100} \times 1 = 64 + 0 = 64$

$$\text{Variance}, \sigma^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{9} f_i y_i^2 - \left(\sum_{i=1}^{9} f_i y_i\right)^2\right]$$

$$= \frac{1}{100^2}[100 \times 286 - 0]$$

$$= 2.86$$

$$\therefore \text{Standard deviation}(\sigma) = \sqrt{2.86} = 1.69$$

Question 7:

Find the mean and variance for the following frequency distribution.

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210

Frequencies	2	3	5	10	3	5	2

Answer

Class	**Frequency f_i**	**Mid-point x_i**	$y_i = \frac{x_i - 105}{30}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-30	2	15	−3	9	−6	18
30-60	3	45	−2	4	−6	12
60-90	5	75	−1	1	−5	5
90-120	10	105	0	0	0	0
120-150	3	135	1	1	3	3
150-180	5	165	2	4	10	20
180-210	2	195	3	9	6	18
	30				2	76

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{7} f_i y_i}{N} \times h = 105 + \frac{2}{30} \times 30 = 105 + 2 = 107$

$$\text{Variance}\left(\sigma^2\right) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right]$$

$$= \frac{(30)^2}{(30)^2}\left[30 \times 76 - (2)^2\right]$$

$$= 2280 - 4$$

$$= 2276$$

Question 8:

Find the mean and variance for the following frequency distribution.

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Answer

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 25}{10}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-10	5	5	−2	4	−10	20
10-20	8	15	−1	1	−8	8
20-30	15	25	0	0	0	0
30-40	16	35	1	1	16	16
40-50	6	45	2	4	12	24
	50				10	68

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{5} f_i y_i}{N} \times h = 25 + \frac{10}{50} \times 10 = 25 + 2 = 27$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{5} f_i y_i^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$

$$= \frac{(10)^2}{(50)^2}\left[50 \times 68 - (10)^2\right]$$

$$= \frac{1}{25}\left[3400 - 100\right] = \frac{3300}{25}$$

$$= 132$$

Question 9:

Find the mean, variance and standard deviation using short-cut method

Height in cms	No. of children
70-75	3
75-80	4
80-85	7
85-90	7
90-95	15
95-100	9
100-105	6
105-110	6
110-115	3

Answer

Class Interval	**Frequency f_i**	**Mid-point x_i**	$y_i = \frac{x_i - 92.5}{5}$	y_i^2	$f_i y_i$	$f_i y_i^2$
70-75	3	72.5	−4	16	−12	48
75-80	4	77.5	−3	9	−12	36
80-85	7	82.5	−2	4	−14	28
85-90	7	87.5	−1	1	−7	7
90-95	15	92.5	0	0	0	0

95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	60				6	254

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{9} f_i y_i}{N} \times h = 92.5 + \frac{6}{60} \times 5 = 92.5 + 0.5 = 93$

$$\text{Variance}\left(\sigma^2\right) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{9} f_i y_i^2 - \left(\sum_{i=1}^{9} f_i y_i\right)^2\right]$$

$$= \frac{(5)^2}{(60)^2}\left[60 \times 254 - (6)^2\right]$$

$$= \frac{25}{3600}(15204) = 105.58$$

$$\therefore \text{Standard deviation}(\sigma) = \sqrt{105.58} = 10.27$$

Question 10:

The diameters of circles (in mm) drawn in a design are given below:

Diameters	No. of children
33-36	15
37-40	17
41-44	21

45-48	22
49-52	25

Answer

Class Interval	**Frequency f_i**	**Mid-point x_i**	$y_i = \frac{x_i - 42.5}{4}$	f_i^2	$f_i y_i$	$f_i y_i^2$
32.5-36.5	15	34.5	–2	4	–30	60
36.5-40.5	17	38.5	–1	1	–17	17
40.5-44.5	21	42.5	0	0	0	0
44.5-48.5	22	46.5	1	1	22	22
48.5-52.5	25	50.5	2	4	50	100
	100				25	199

Here, N = 100, h = 4

Let the assumed mean, A, be 42.5.

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{5} f_i y_i}{N} \times h = 42.5 + \frac{25}{100} \times 4 = 43.5$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{5} f_i y_i^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$
$$= \frac{16}{10000}\left[100\times 199 - (25)^2\right]$$
$$= \frac{16}{10000}[19900 - 625]$$
$$= \frac{16}{10000}\times 19275$$
$$= 30.84$$

$\therefore$ Standard deviation $(\sigma) = 5.55$

Exercise 15.3

Question 1:

From the data given below state which group is more variable, A or B?

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group A	9	17	32	33	40	10	9