

Exercise 16.3**Question 1:**

Which of the following can not be valid assignment of probabilities for outcomes of

sample space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

| Assignment | ω_1 | ω_2 | ω_3 | ω_4 | ω_5 | ω_6 | ω_7 |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|
| (a) | 0.1 | 0.01 | 0.05 | 0.03 | 0.01 | 0.2 | 0.6 |
| (b) | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ |
| (c) | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
| (d) | -0.1 | 0.2 | 0.3 | 0.4 | -0.2 | 0.1 | 0.3 |
| (e) | $\frac{1}{14}$ | $\frac{2}{14}$ | $\frac{3}{14}$ | $\frac{4}{14}$ | $\frac{5}{14}$ | $\frac{6}{14}$ | $\frac{15}{14}$ |

Answer

(a)

| ω_1 | ω_2 | ω_3 | ω_4 | ω_5 | ω_6 | ω_7 |
|------------|------------|------------|------------|------------|------------|------------|
| 0.1 | 0.01 | 0.05 | 0.03 | 0.01 | 0.2 | 0.6 |

Here, each of the numbers $p(\omega_i)$ is positive and less than 1.

Sum of probabilities

$$\begin{aligned}
 &= p(\omega_1) + p(\omega_2) + p(\omega_3) + p(\omega_4) + p(\omega_5) + p(\omega_6) + p(\omega_7) \\
 &= 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 \\
 &= 1
 \end{aligned}$$

Thus, the assignment is valid.

(b)

| | | | | | | |
|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| ω_1 | ω_2 | ω_3 | ω_4 | ω_5 | ω_6 | ω_7 |
| $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ |

Here, each of the numbers $p(\omega_i)$ is positive and less than 1.

Sum of probabilities

$$\begin{aligned}
 &= p(\omega_1) + p(\omega_2) + p(\omega_3) + p(\omega_4) + p(\omega_5) + p(\omega_6) + p(\omega_7) \\
 &= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = 7 \times \frac{1}{7} = 1
 \end{aligned}$$

Thus, the assignment is valid.

(c)

| | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|
| ω_1 | ω_2 | ω_3 | ω_4 | ω_5 | ω_6 | ω_7 |
| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |

Here, each of the numbers $p(\omega_i)$ is positive and less than 1.

Sum of probabilities

$$\begin{aligned}
 &= p(\omega_1) + p(\omega_2) + p(\omega_3) + p(\omega_4) + p(\omega_5) + p(\omega_6) + p(\omega_7) \\
 &= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7 \\
 &= 2.8 \neq 1
 \end{aligned}$$

Thus, the assignment is not valid.

(d)

| | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|
| ω_1 | ω_2 | ω_3 | ω_4 | ω_5 | ω_6 | ω_7 |
| -0.1 | 0.2 | 0.3 | 0.4 | -0.2 | 0.1 | 0.3 |

Here, $p(\omega_1)$ and $p(\omega_5)$ are negative.

Hence, the assignment is not valid.

(e)

| | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|
| ω_1 | ω_2 | ω_3 | ω_4 | ω_5 | ω_6 | ω_7 |
|------------|------------|------------|------------|------------|------------|------------|

| | | | | | | |
|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|
| $\frac{1}{14}$ | $\frac{2}{14}$ | $\frac{3}{14}$ | $\frac{4}{14}$ | $\frac{5}{14}$ | $\frac{6}{14}$ | $\frac{15}{14}$ |
|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|

Here, $p(\omega_7) = \frac{15}{14} > 1$

Hence, the assignment is not valid.

Question 2:

A coin is tossed twice, what is the probability that at least one tail occurs?

Answer

When a coin is tossed twice, the sample space is given by

$$S = \{HH, HT, TH, TT\}$$

Let A be the event of the occurrence of at least one tail.

Accordingly, $A = \{HT, TH, TT\}$

$$\begin{aligned} \therefore P(A) &= \frac{\text{Number of outcomes favourable to } A}{\text{Total number of possible outcomes}} \\ &= \frac{n(A)}{n(S)} \\ &= \frac{3}{4} \end{aligned}$$

Question 3:

A die is thrown, find the probability of following events:

- (i) A prime number will appear,
- (ii) A number greater than or equal to 3 will appear,
- (iii) A number less than or equal to one will appear,
- (iv) A number more than 6 will appear,
- (v) A number less than 6 will appear.

Answer

The sample space of the given experiment is given by

$$S = \{1, 2, 3, 4, 5, 6\}$$

(i) Let A be the event of the occurrence of a prime number.

Accordingly, $A = \{2, 3, 5\}$

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) Let B be the event of the occurrence of a number greater than or equal to 3.

Accordingly, $B = \{3, 4, 5, 6\}$

$$\therefore P(B) = \frac{\text{Number of outcomes favourable to B}}{\text{Total number of possible outcomes}} = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

(iii) Let C be the event of the occurrence of a number less than or equal to one.

Accordingly, $C = \{1\}$

$$\therefore P(C) = \frac{\text{Number of outcomes favourable to C}}{\text{Total number of possible outcomes}} = \frac{n(C)}{n(S)} = \frac{1}{6}$$

(iv) Let D be the event of the occurrence of a number greater than 6.

Accordingly, $D = \Phi$

$$\therefore P(D) = \frac{\text{Number of outcomes favourable to D}}{\text{Total number of possible outcomes}} = \frac{n(D)}{n(S)} = \frac{0}{6} = 0$$

(v) Let E be the event of the occurrence of a number less than 6.

Accordingly, $E = \{1, 2, 3, 4, 5\}$

$$\therefore P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}} = \frac{n(E)}{n(S)} = \frac{5}{6}$$

Question 4:

A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card.

Answer

(a) When a card is selected from a pack of 52 cards, the number of possible outcomes is 52 i.e., the sample space contains 52 elements.

Therefore, there are 52 points in the sample space.

(b) Let A be the event in which the card drawn is an ace of spades.

Accordingly, $n(A) = 1$

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{1}{52}$$

(c) (i) Let E be the event in which the card drawn is an ace.

Since there are 4 aces in a pack of 52 cards, $n(E) = 4$

$$\therefore P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}} = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(ii) Let F be the event in which the card drawn is black.

Since there are 26 black cards in a pack of 52 cards, $n(F) = 26$

$$\therefore P(F) = \frac{\text{Number of outcomes favourable to F}}{\text{Total number of possible outcomes}} = \frac{n(F)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

Question 5:

A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed.

Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12

Answer

Since the fair coin has 1 marked on one face and 6 on the other, and the die has six faces that are numbered 1, 2, 3, 4, 5, and 6, the sample space is given by

$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

Accordingly, $n(S) = 12$

(i) Let A be the event in which the sum of numbers that turn up is 3.

Accordingly, $A = \{(1, 2)\}$

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{1}{12}$$

(ii) Let B be the event in which the sum of numbers that turn up is 12.

Accordingly, $B = \{(6, 6)\}$

$$\therefore P(B) = \frac{\text{Number of outcomes favourable to B}}{\text{Total number of possible outcomes}} = \frac{n(B)}{n(S)} = \frac{1}{12}$$

Question 6:

There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Answer

There are four men and six women on the city council.

As one council member is to be selected for a committee at random, the sample space contains 10 (4 + 6) elements.

Let A be the event in which the selected council member is a woman.

Accordingly, $n(A) = 6$

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{6}{10} = \frac{3}{5}$$

Question 7:

A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Answer

Since the coin is tossed four times, there can be a maximum of 4 heads or tails.

When 4 heads turns up, $\text{Re } 1 + \text{Re } 1 + \text{Re } 1 + \text{Re } 1 = \text{Rs } 4$ is the gain.

When 3 heads and 1 tail turn up, $\text{Re } 1 + \text{Re } 1 + \text{Re } 1 - \text{Rs } 1.50 = \text{Rs } 3 - \text{Rs } 1.50 = \text{Rs } 1.50$ is the gain.

When 2 heads and 2 tails turns up, $\text{Re } 1 + \text{Re } 1 - \text{Rs } 1.50 - \text{Rs } 1.50 = -\text{Re } 1$, i.e., Re 1 is the loss.

When 1 head and 3 tails turn up, $\text{Re } 1 - \text{Rs } 1.50 - \text{Rs } 1.50 - \text{Rs } 1.50 = -\text{Rs } 3.50$, i.e., Rs 3.50 is the loss.

When 4 tails turn up, $-\text{Rs } 1.50 - \text{Rs } 1.50 - \text{Rs } 1.50 - \text{Rs } 1.50 = -\text{Rs } 6.00$, i.e., Rs 6.00 is the loss.

There are $2^4 = 16$ elements in the sample space S, which is given by:

$S = \{\text{HHHH}, \text{HHHT}, \text{HHTH}, \text{HTHH}, \text{TTHH}, \text{HHTT}, \text{HTTH}, \text{TTHH}, \text{HTHT}, \text{THHT}, \text{HTTT}, \text{THTT}, \text{TTHT}, \text{TTTH}, \text{TTTT}\}$

$\therefore n(S) = 16$

The person wins Rs 4.00 when 4 heads turn up, i.e., when the event {HHHH} occurs.

∴ Probability (of winning Rs 4.00) = $\frac{1}{16}$

The person wins Rs 1.50 when 3 heads and one tail turn up, i.e., when the event {HHHT, HHTH, HTHH, THHH} occurs.

∴ Probability (of winning Rs 1.50) = $\frac{4}{16} = \frac{1}{4}$

The person loses Re 1.00 when 2 heads and 2 tails turn up, i.e., when the event {HHTT, HTTH, TTHH, HTHT, THTH, THHT} occurs.

∴ Probability (of losing Re 1.00) = $\frac{6}{16} = \frac{3}{8}$

The person loses Rs 3.50 when 1 head and 3 tails turn up, i.e., when the event {HTTT, THTT, TTHT, TTTH} occurs.

Probability (of losing Rs 3.50) = $\frac{4}{16} = \frac{1}{4}$

The person loses Rs 6.00 when 4 tails turn up, i.e., when the event {TTTT} occurs.

Probability (of losing Rs 6.00) = $\frac{1}{16}$

Question 8:

Three coins are tossed once. Find the probability of getting

- (i) 3 heads (ii) 2 heads (iii) at least 2 heads
- (iv) at most 2 heads (v) no head (vi) 3 tails
- (vii) exactly two tails (viii) no tail (ix) at most two tails.

Answer

When three coins are tossed once, the sample space is given by

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

∴ Accordingly, $n(S) = 8$

It is known that the probability of an event A is given by

$$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(A)}{n(S)}$$

(i) Let B be the event of the occurrence of 3 heads. Accordingly, $B = \{HHH\}$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{1}{8}$$

(ii) Let C be the event of the occurrence of 2 heads. Accordingly, $C = \{HHT, HTH, THH\}$

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{3}{8}$$

(iii) Let D be the event of the occurrence of at least 2 heads.

Accordingly, $D = \{HHH, HHT, HTH, THH\}$

$$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iv) Let E be the event of the occurrence of at most 2 heads.

Accordingly, $E = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$$

(v) Let F be the event of the occurrence of no head.

Accordingly, $F = \{TTT\}$

$$\therefore P(F) = \frac{n(F)}{n(S)} = \frac{1}{8}$$

(vi) Let G be the event of the occurrence of 3 tails.

Accordingly, $G = \{TTT\}$

$$\therefore P(G) = \frac{n(G)}{n(S)} = \frac{1}{8}$$

(vii) Let H be the event of the occurrence of exactly 2 tails.

Accordingly, $H = \{HTT, THT, TTH\}$

$$\therefore P(H) = \frac{n(H)}{n(S)} = \frac{3}{8}$$

(viii) Let I be the event of the occurrence of no tail.

Accordingly, $I = \{HHH\}$

$$\therefore P(I) = \frac{n(I)}{n(S)} = \frac{1}{8}$$

(ix) Let J be the event of the occurrence of at most 2 tails.

Accordingly, $I = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$

$$\therefore P(J) = \frac{n(J)}{n(S)} = \frac{7}{8}$$

Question 9:

If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'.

Answer

It is given that $P(A) = \frac{2}{11}$.

$$\text{Accordingly, } P(\text{not A}) = 1 - P(A) = 1 - \frac{2}{11} = \frac{9}{11}$$

Question 10:

A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel (ii) an consonant

Answer

There are 13 letters in the word ASSASSINATION.

\therefore Hence, $n(S) = 13$

(i) There are 6 vowels in the given word.

$$\therefore \text{Probability (vowel)} = \frac{6}{13}$$

(ii) There are 7 consonants in the given word.

$$\therefore \text{Probability (consonant)} = \frac{7}{13}$$

Question 11:

In a lottery, person chooses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [**Hint:** order of the numbers is not important.]

Answer

Total number of ways in which one can choose six different numbers from 1 to 20

$$= {}^{20}C_6 = \frac{|20}{|6|20-6}} = \frac{|20}{|6|14}} = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = 38760$$

Hence, there are 38760 combinations of 6 numbers.

Out of these combinations, one combination is already fixed by the lottery committee.

$$\therefore \text{Required probability of winning the prize in the game} = \frac{1}{38760}$$

Question 12:

Check whether the following probabilities $P(A)$ and $P(B)$ are consistently defined

(i) $P(A) = 0.5$, $P(B) = 0.7$, $P(A \cap B) = 0.6$

(ii) $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$

Answer

(i) $P(A) = 0.5$, $P(B) = 0.7$, $P(A \cap B) = 0.6$

It is known that if E and F are two events such that $E \subset F$, then $P(E) \leq P(F)$.

However, here, $P(A \cap B) > P(A)$.

Hence, $P(A)$ and $P(B)$ are not consistently defined.

(ii) $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$

It is known that if E and F are two events such that $E \subset F$, then $P(E) \leq P(F)$.

Here, it is seen that $P(A \cup B) > P(A)$ and $P(A \cup B) > P(B)$.

Hence, $P(A)$ and $P(B)$ are consistently defined.

Question 13:

Fill in the blanks in following table:

| | | | | |
|--|-------------|-------------|-----------------|-----------------|
| | P(A) | P(B) | P(A ∩ B) | P(A ∪ B) |
|--|-------------|-------------|-----------------|-----------------|

| | | | | |
|--------------|---------------|---------------|----------------|-----|
| (i) | $\frac{1}{3}$ | $\frac{1}{5}$ | $\frac{1}{15}$ | ... |
| (ii) | 0.35 | ... | 0.25 | 0.6 |
| (iii) | 0.5 | 0.35 | ... | 0.7 |

Answer

$$(i) \text{ Here, } P(A) = \frac{1}{3}, P(B) = \frac{1}{5}, P(A \cap B) = \frac{1}{15}$$

$$\text{We know that } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore P(A \cup B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{15} = \frac{5+3-1}{15} = \frac{7}{15}$$

$$(ii) \text{ Here, } P(A) = 0.35, P(A \cap B) = 0.25, P(A \cup B) = 0.6$$

$$\text{We know that } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore 0.6 = 0.35 + P(B) - 0.25$$

$$\Rightarrow P(B) = 0.6 - 0.35 + 0.25$$

$$\Rightarrow P(B) = 0.5$$

$$(iii) \text{ Here, } P(A) = 0.5, P(B) = 0.35, P(A \cup B) = 0.7$$

$$\text{We know that } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore 0.7 = 0.5 + 0.35 - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = 0.5 + 0.35 - 0.7$$

$$\Rightarrow P(A \cap B) = 0.15$$

Question 14:

Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find $P(A \text{ or } B)$, if A and B are mutually exclusive events.

Answer

$$\text{Here, } P(A) = \frac{3}{5}, P(B) = \frac{1}{5}$$

For mutually exclusive events A and B,

$$P(A \text{ or } B) = P(A) + P(B)$$

$$\therefore P(A \text{ or } B) = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

Question 15:

If E and F are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$, find: (i) $P(E \text{ or } F)$, (ii) $P(\text{not } E \text{ and not } F)$.

Answer

Here, $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$, and $P(E \text{ and } F) = \frac{1}{8}$

(i) We know that $P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$

$$\therefore P(E \text{ or } F) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$$

(ii) From (i), $P(E \text{ or } F) = P(E \cup F) = \frac{5}{8}$

We have $(E \cup F)' = (E' \cap F')$ [By De Morgan's law]

$$\therefore P(E' \cap F') = P(E \cup F)'$$

$$\text{Now, } P(E \cup F)' = 1 - P(E \cup F) = 1 - \frac{5}{8} = \frac{3}{8}$$

$$\therefore P(E' \cap F') = \frac{3}{8}$$

$$\text{Thus, } P(\text{not } E \text{ and not } F) = \frac{3}{8}$$

Question 16:

Events E and F are such that $P(\text{not } E \text{ or not } F) = 0.25$, State whether E and F are mutually exclusive.

Answer

It is given that $P(\text{not } E \text{ or not } F) = 0.25$

$$\text{i.e., } P(E' \cup F') = 0.25$$

$$\Rightarrow P(E \cap F)' = 0.25 \quad \left[E' \cup F' = (E \cap F)' \right]$$

$$\text{Now, } P(E \cap F) = 1 - P(E \cap F)'$$

$$\Rightarrow P(E \cap F) = 1 - 0.25$$

$$\Rightarrow P(E \cap F) = 0.75 \neq 0$$

$$\Rightarrow E \cap F \neq \phi$$

Thus, E and F are not mutually exclusive.

Question 17:

A and B are events such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$.

Determine (i) $P(\text{not } A)$, (ii) $P(\text{not } B)$ and (iii) $P(A \text{ or } B)$.

Answer

It is given that $P(A) = 0.42$, $P(B) = 0.48$, $P(A \text{ and } B) = 0.16$

$$(i) P(\text{not } A) = 1 - P(A) = 1 - 0.42 = 0.58$$

$$(ii) P(\text{not } B) = 1 - P(B) = 1 - 0.48 = 0.52$$

$$(iii) \text{ We know that } P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\therefore P(A \text{ or } B) = 0.42 + 0.48 - 0.16 = 0.74$$

Question 18:

In Class XI of a school 40% of the students study Mathematics and 30% study Biology.

10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Answer

Let A be the event in which the selected student studies Mathematics and B be the event in which the selected student studies Biology.

$$\text{Accordingly, } P(A) = 40\% = \frac{40}{100} = \frac{2}{5}$$

$$P(B) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(A \text{ or } B) = 10\% = \frac{10}{100} = \frac{1}{10}$$

We know that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\therefore P(A \text{ or } B) = \frac{2}{5} + \frac{3}{10} - \frac{1}{10} = \frac{6}{10} = 0.6$$

Thus, the probability that the selected student will be studying Mathematics or Biology is 0.6.

Question 19:

In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Answer

Let A and B be the events of passing first and second examinations respectively.

Accordingly, $P(A) = 0.8$, $P(B) = 0.7$ and $P(A \text{ or } B) = 0.95$

We know that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\therefore 0.95 = 0.8 + 0.7 - P(A \text{ and } B)$$

$$\Rightarrow P(A \text{ and } B) = 0.8 + 0.7 - 0.95 = 0.55$$

Thus, the probability of passing both the examinations is 0.55.

Question 20:

The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Answer

Let A and B be the events of passing English and Hindi examinations respectively.

Accordingly, $P(A \text{ and } B) = 0.5$, $P(\text{not } A \text{ and not } B) = 0.1$, i.e., $P(A' \cap B') = 0.1$

$P(A) = 0.75$

$$\text{Now, } (A \cup B)' = (A' \cap B') \quad [\text{De Morgan's law}]$$

$$\therefore P(A \cup B)' = P(A' \cap B') = 0.1$$

$$P(A \cup B) = 1 - P(A \cup B)' = 1 - 0.1 = 0.9$$

We know that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\therefore 0.9 = 0.75 + P(B) - 0.5$$

$$\Rightarrow P(B) = 0.9 - 0.75 + 0.5$$

$$\Rightarrow P(B) = 0.65$$

Thus, the probability of passing the Hindi examination is 0.65.

Question 21:

In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

- (i) The student opted for NCC or NSS.
- (ii) The student has opted neither NCC nor NSS.
- (iii) The student has opted NSS but not NCC.

Answer

Let A be the event in which the selected student has opted for NCC and B be the event in which the selected student has opted for NSS.

Total number of students = 60

Number of students who have opted for NCC = 30

$$\therefore P(A) = \frac{30}{60} = \frac{1}{2}$$

Number of students who have opted for NSS = 32

$$\therefore P(B) = \frac{32}{60} = \frac{8}{15}$$

Number of students who have opted for both NCC and NSS = 24

$$\therefore P(A \text{ and } B) = \frac{24}{60} = \frac{2}{5}$$

(i) We know that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\therefore P(A \text{ or } B) = \frac{1}{2} + \frac{8}{15} - \frac{2}{5} = \frac{15+16-12}{30} = \frac{19}{30}$$

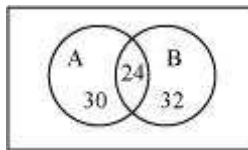
Thus, the probability that the selected student has opted for NCC or NSS is $\frac{19}{30}$.

(ii)

$$\begin{aligned}
 & P(\text{not A and not B}) \\
 &= P(A' \text{ and } B') \\
 &= P(A' \cap B') \\
 &= P(A \cup B)' \quad \left[(A' \cap B') = (A \cup B)' \text{ (by De Morgan's law)} \right] \\
 &= 1 - P(A \cup B) \\
 &= 1 - P(A \text{ or } B) \\
 &= 1 - \frac{19}{30} \\
 &= \frac{11}{30}
 \end{aligned}$$

Thus, the probability that the selected students has neither opted for NCC nor NSS is $\frac{11}{30}$.

(iii) The given information can be represented by a Venn diagram as



It is clear that

Number of students who have opted for NSS but not NCC

$$= n(B - A) = n(B) - n(A \cap B) = 32 - 24 = 8$$

Thus, the probability that the selected student has opted for NSS but not for NCC =

$$\frac{8}{60} = \frac{2}{15}$$