Question 1:

Decide, among the following sets, which sets are subsets of one and another:

A = {x: $x \in$ R and x satisfy $x^2 - 8x + 12 = 0$},

B = {2, 4, 6}, C = {2, 4, 6, 8…}, D = {6}.

Answer

A = {x: $x \in$ R and x satisfies $x^2 - 8x + 12 = 0$}

2 and 6 are the only solutions of $x^2 - 8x + 12 = 0$.

∴ A = {2, 6}

B = {2, 4, 6}, C = {2, 4, 6, 8 …}, D = {6}

∴ D ⊂ A ⊂ B ⊂ C

Hence, A ⊂ B, A ⊂ C, B ⊂ C, D ⊂ A, D ⊂ B, D ⊂ C

Question 2:

In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(i) If $x \in$ A and A ∈ B, then $x \in$ B

(ii) If A ⊂ B and B ∈ C, then A ∈ C

(iii) If A ⊂ B and B ⊂ C, then A ⊂ C

(iv) If A ⊄ B and B ⊄ C, then A ⊄ C

(v) If $x \in$ A and A ⊄ B, then $x \in$ B

(vi) If A ⊂ B and $x \notin$ B, then $x \notin$ A

Answer

(i) False

Let A = {1, 2} and B = {1, {1, 2}, {3}}

Now, $2 \in \{1,2\}$ and $\{1, 2\} \in \{\{3\},1,\{1,2\}\}$

∴ A ∈ B

However, $2 \notin \{\{3\},1,\{1,2\}\}$

(ii) False

Let $\mathrm{A} = \{2\}$, $\mathrm{B} = \{0,2\}$, and $\mathrm{C} = \{1,\{0,2\},3\}$

As $A \subset B$

$B \in C$

However, $A \notin C$

(iii) True

Let $A \subset B$ and $B \subset C$.

Let $x \in A$

$\Rightarrow x \in B \qquad [\because A \subset B]$

$\Rightarrow x \in C \qquad [\because B \subset C]$

$\therefore A \subset C$

(iv) False

Let $A = \{1, 2\}$, $B = \{0, 6, 8\}$, and $C = \{0, 1, 2, 6, 9\}$

Accordingly, $A \not\subset B$ and $B \not\subset C$.

However, $A \subset C$

(v) False

Let A = {3, 5, 7} and B = {3, 4, 6}

Now, $5 \in A$ and $A \not\subset B$

However, $5 \notin B$

(vi) True

Let $A \subset B$ and $x \notin B$.

To show: $x \notin A$

If possible, suppose $x \in A$.

Then, $x \in B$, which is a contradiction as $x \notin B$

$\therefore x \notin A$

Question 3:

Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. show that B = C.

Answer

Let, A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$.

To show: B = C

Let $x \in B$

$\Rightarrow x \in A \cup B \quad [B \subset A \cup B]$

$\Rightarrow x \in A \cup C \quad [A \cup B = A \cup C]$

$\Rightarrow x \in A$ or $x \in C$

Case I

$x \in A$

Also, $x \in B$

$\therefore\ x \in A \cap B$

$\Rightarrow x \in A \cap C \quad [\because A \cap B = A \cap C]$

$\therefore x \in A$ and $x \in C$

$\therefore x \in C$

$\therefore B \subset C$

Similarly, we can show that $C \subset B$.

$\therefore B = C$

Question 4:

Show that the following four conditions are equivalent:

(i) $A \subset B$ **(ii)** $A - B = \Phi$

(iii) $A \cup B = B$ **(iv)** $A \cap B = A$

Answer

First, we have to show that (i) $\Leftrightarrow$ (ii).

Let $A \subset B$

To show: $A - B \neq \Phi$

If possible, suppose $A - B \neq \Phi$

This means that there exists $x \in A$, $x \neq B$, which is not possible as $A \subset B$.

$\therefore A - B = \Phi$

$\therefore A \subset B \Rightarrow A - B = \Phi$

Let $A - B = \Phi$

To show: $A \subset B$

Let $x \in A$

Clearly, $x \in B$ because if $x \notin B$, then $A - B \neq \Phi$

$\therefore A - B = \Phi \Rightarrow A \subset B$

$\therefore$ (i) $\Leftrightarrow$ (ii)

Let A $\subset$ B

To show: $A \cup B = B$

Clearly, $B \subset A \cup B$

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

Case I: $x \in$ A

$\Rightarrow x \in B \qquad [\because A \subset B]$

$\therefore$ $A \cup B \subset B$

Case II: $x \in$ B

Then, $A \cup B = B$

Conversely, let $A \cup B = B$

Let $x \in$ A

$\Rightarrow x \in A \cup B \qquad [\because A \subset A \cup B]$

$\Rightarrow x \in B \qquad [\because A \cup B = B]$

$\therefore$ A $\subset$ B

Hence, (i) $\Leftrightarrow$ (iii)

Now, we have to show that (i) $\Leftrightarrow$ (iv).

Let A $\subset$ B

Clearly $A \cap B \subset A$

Let $x \in$ A

We have to show that $x \in A \cap B$

As A $\subset$ B, $x \in$ B

$\therefore$ $x \in A \cap B$

$\therefore$ $A \subset A \cap B$

Hence, A = A ∩ B

Conversely, suppose A ∩ B = A

Let $x \in$ A

$\Rightarrow$ $x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$

$\therefore A \subset B$

Hence, (i) $\Leftrightarrow$ (iv).

Question 5:

Show that if $A \subset B$, then $C - B \subset C - A$.

Answer

Let $A \subset B$

To show: $C - B \subset C - A$

Let $x \in C - B$

$\Rightarrow x \in C$ and $x \notin B$

$\Rightarrow x \in C$ and $x \notin A$ $[A \subset B]$

$\Rightarrow x \in C - A$

$\therefore C - B \subset C - A$

Question 6:

Assume that P (A) = P (B). Show that A = B.

Answer

Let $P(A) = P(B)$

To show: $A = B$

Let $x \in A$

$A \in P(A) = P(B)$

$\therefore x \in C$, for some $C \in P(B)$

Now, $C \subset B$

$\therefore x \in B$

$\therefore A \subset B$

Similarly, $B \subset A$

$\therefore A = B$

Question 7:

Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.

Answer

False

Let A = {0, 1} and B = {1, 2}

∴ A ∪ B = {0, 1, 2}

P(A) = {Φ, {0}, {1}, {0, 1}}

P(B) = {Φ, {1}, {2}, {1, 2}}

P(A ∪ B) = {Φ, {0}, {1}, {2}, {0, 1}, {1, 2}, {0, 2}, {0, 1, 2}}

P(A) ∪ P(B) = {Φ, {0}, {1}, {0, 1}, {2}, {1, 2}}

∴ P(A) ∪ P(B) ≠ P(A ∪ B)

Question 8:

Show that for any sets A and B,

A = (A ∩ B) ∪ (A – B) and A ∪ (B – A) = (A ∪ B)

Answer

To show: A = (A ∩ B) ∪ (A – B)

Let x ∈ A

We have to show that x ∈ (A ∩ B) ∪ (A – B)

Case I

x ∈ A ∩ B

Then, x ∈ (A ∩ B) ⊂ (A ∪ B) ∪ (A – B)

Case II

x ∉ A ∩ B

⇒ x ∉ A or x ∉ B

∴ x ∉ B [x ∉ A]

∴ x ∉ A – B ⊂ (A ∪ B) ∪ (A – B)

∴ A ⊂ (A ∩ B) ∪ (A – B) … (1)

It is clear that

A ∩ B ⊂ A and (A – B) ⊂ A

∴ (A ∩ B) ∪ (A – B) ⊂ A … (2)

From (1) and (2), we obtain

A = (A ∩ B) ∪ (A – B)

To prove: A ∪ (B – A) ⊂ A ∪ B

Let $x \in A \cup (B - A)$

$\Rightarrow x \in A$ or $x \in (B - A)$

$\Rightarrow x \in A$ or ($x \in B$ and $x \notin A$)

$\Rightarrow$ ($x \in A$ or $x \in B$) and ($x \in A$ or $x \notin A$)

$\Rightarrow x \in (A \cup B)$

$\therefore A \cup (B - A) \subset (A \cup B)$ … (3)

Next, we show that $(A \cup B) \subset A \cup (B - A)$.

Let $y \in A \cup B$

$\Rightarrow y \in A$ or $y \in B$

$\Rightarrow$ ($y \in A$ or $y \in B$) and ($y \in A$ or $y \notin A$)

$\Rightarrow y \in A$ or ($y \in B$ and $y \notin A$)

$\Rightarrow y \in A \cup (B - A)$

$\therefore A \cup B \subset A \cup (B - A)$ … (4)

Hence, from (3) and (4), we obtain $A \cup (B - A) = A \cup B$.

Question 9:

Using properties of sets show that

(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$.

Answer

(i) To show: $A \cup (A \cap B) = A$

We know that

$A \subset A$

$A \cap B \subset A$

$\therefore A \cup (A \cap B) \subset A$ … (1)

Also, $A \subset A \cup (A \cap B)$ … (2)

$\therefore$ From (1) and (2), $A \cup (A \cap B) = A$

(ii) To show: $A \cap (A \cup B) = A$

$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$

$= A \cup (A \cap B)$

$= A$ {from (1)}

Question 10:

Show that $A \cap B = A \cap C$ need not imply $B = C$.

Answer

Let $A = \{0, 1\}$, $B = \{0, 2, 3\}$, and $C = \{0, 4, 5\}$

Accordingly, $A \cap B = \{0\}$ and $A \cap C = \{0\}$

Here, $A \cap B = A \cap C = \{0\}$

However, $B \neq C$ [$2 \in B$ and $2 \notin C$]

Question 11:

Let A and B be sets. If $A \cap X = B \cap X = \Phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

(Hints $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and use distributive law)

Answer

Let A and B be two sets such that $A \cap X = B \cap X = f$ and $A \cup X = B \cup X$ for some set X.

To show: $A = B$

It can be seen that

$A = A \cap (A \cup X) = A \cap (B \cup X)$ [$A \cup X = B \cup X$]

$= (A \cap B) \cup (A \cap X)$ [Distributive law]

$= (A \cap B) \cup \Phi$ [$A \cap X = \Phi$]

$= A \cap B$ … (1)

Now, $B = B \cap (B \cup X)$

$= B \cap (A \cup X)$ [$A \cup X = B \cup X$]

$= (B \cap A) \cup (B \cap X)$ [Distributive law]

$= (B \cap A) \cup \Phi$ [$B \cap X = \Phi$]

$= B \cap A$

$= A \cap B$ … (2)

Hence, from (1) and (2), we obtain $A = B$.

Question 12:

Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \Phi$.

Answer

Let $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{2, 0\}$.

Accordingly, $A \cap B = \{1\}$, $B \cap C = \{2\}$, and $A \cap C = \{0\}$.

$\therefore$ $A \cap B$, $B \cap C$, and $A \cap C$ are non-empty.

However, $A \cap B \cap C = \Phi$

Question 13:

In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

Answer

Let U be the set of all students who took part in the survey.

Let T be the set of students taking tea.

Let C be the set of students taking coffee.

Accordingly, $n(U) = 600$, $n(T) = 150$, $n(C) = 225$, $n(T \cap C) = 100$

To find: Number of student taking neither tea nor coffee i.e., we have to find $n(T' \cap C')$.

$n(T' \cap C') = n(T \cup C)'$

$= n(U) - n(T \cup C)$

$= n(U) - [n(T) + n(C) - n(T \cap C)]$

$= 600 - [150 + 225 - 100]$

$= 600 - 275$

$= 325$

Hence, 325 students were taking neither tea nor coffee.

Question 14:

In a group of students 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Answer

Let U be the set of all students in the group.

Let E be the set of all students who know English.

Let H be the set of all students who know Hindi.

$\therefore H \cup E = U$

Accordingly, $n(H) = 100$ and $n(E) = 50$

$n(H \cap E)$ = 25

n(U) = n(H) + $n(E)$ – n(H ∩ E)

= 100 + 50 – 25

= 125

Hence, there are 125 students in the group.

Question 15:

In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I,11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

Answer

Let A be the set of people who read newspaper H.

Let B be the set of people who read newspaper T.

Let C be the set of people who read newspaper I.

Accordingly, n(A) = 25, n(B) = 26, and n(C) = 26

n(A ∩ C) = 9, n(A ∩ B) = 11, and n(B ∩ C) = 8

n(A ∩ B ∩ C) = 3

Let U be the set of people who took part in the survey.

(i) Accordingly,

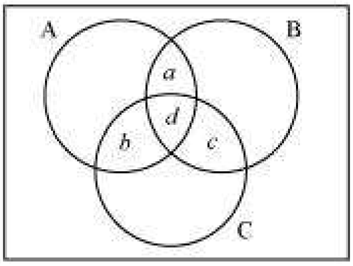
n(A ∪ B ∪ C) = n(A) + n(B) + n(C) – n(A ∩ B) – n(B ∩ C) – n(C ∩ A) + n(A ∩ B ∩ C)

= 25 + 26 + 26 – 11 – 8 – 9 + 3

= 52

Hence, 52 people read at least one of the newspapers.

(ii) Let a be the number of people who read newspapers H and T only.



Let b denote the number of people who read newspapers I and H only.

Let c denote the number of people who read newspapers T and I only.

Let d denote the number of people who read all three newspapers.

Accordingly, $d = n(A \cap B \cap C) = 3$

Now, $n(A \cap B) = a + d$

$n(B \cap C) = c + d$

$n(C \cap A) = b + d$

$\therefore a + d + c + d + b + d = 11 + 8 + 9 = 28$

$\Rightarrow a + b + c + d = 28 - 2d = 28 - 6 = 22$

Hence, (52 – 22) = 30 people read exactly one newspaper.

Question 16:

In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

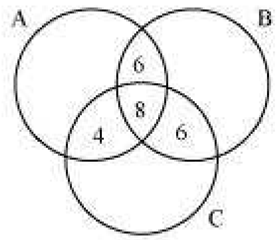
Answer

Let A, B, and C be the set of people who like product A, product B, and product C respectively.

Accordingly, $n(A) = 21$, $n(B) = 26$, $n(C) = 29$, $n(A \cap B) = 14$, $n(C \cap A) = 12$,

$n(B \cap C) = 14$, $n(A \cap B \cap C) = 8$

The Venn diagram for the given problem can be drawn as



It can be seen that number of people who like product C only is

{29 – (4 + 8 + 6)} = 11