## Miscellaneous Solutions

**Question 1:**

Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Answer

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1}x$.

Here, $\frac{13\pi}{6} \notin [0, \pi]$.

Now, $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ can be written as:

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right] = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right], \text{ where } \frac{\pi}{6} \in [0, \pi].$$

$$\therefore \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right] = \frac{\pi}{6}$$

**Question 2:**

Find the value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

Answer

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

Here, $\frac{7\pi}{6} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Now, $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ can be written as:

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)=\tan^{-1}\left[\tan\left(2\pi-\frac{5\pi}{6}\right)\right] \qquad \left[\tan(2\pi-x)=-\tan x\right]$$

$$=\tan^{-1}\left[-\tan\left(\frac{5\pi}{6}\right)\right]=\tan^{-1}\left[\tan\left(-\frac{5\pi}{6}\right)\right]=\tan^{-1}\left[\tan\left(\pi-\frac{5\pi}{6}\right)\right]$$

$$=\tan^{-1}\left[\tan\left(\frac{\pi}{6}\right)\right],\text{ where }\frac{\pi}{6}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

$$\therefore \tan^{-1}\left(\tan\frac{7\pi}{6}\right)=\tan^{-1}\left(\tan\frac{\pi}{6}\right)=\frac{\pi}{6}$$

**Question 3:**

Prove $2\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{24}{7}$

Answer

Let $\sin^{-1}\frac{3}{5}=x$. Then, $\sin x=\frac{3}{5}$.

$$\Rightarrow \cos x=\sqrt{1-\left(\frac{3}{5}\right)^2}=\frac{4}{5}$$

$$\therefore \tan x=\frac{3}{4}$$

$$\therefore x=\tan^{-1}\frac{3}{4}\Rightarrow \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}$$

Now, we have:

$$\text{L.H.S.} = 2\sin^{-1}\frac{3}{5} = 2\tan^{-1}\frac{3}{4}$$

$$= \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right) \qquad \left[2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{16-9}{16}}\right) = \tan^{-1}\left(\frac{3}{2}\times\frac{16}{7}\right)$$

$$= \tan^{-1}\frac{24}{7} = \text{R.H.S.}$$

**Question 4:**

Prove $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

Answer

Let $\sin^{-1}\frac{8}{17} = x$. Then, $\sin x = \frac{8}{17} \Rightarrow \cos x = \sqrt{1-\left(\frac{8}{17}\right)^2} = \sqrt{\frac{225}{289}} = \frac{15}{17}$.

$$\therefore \tan x = \frac{8}{15} \Rightarrow x = \tan^{-1}\frac{8}{15}$$

$$\therefore \sin^{-1}\frac{8}{17} = \tan^{-1}\frac{8}{15} \qquad ...(1)$$

Now, let $\sin^{-1}\frac{3}{5} = y$. Then, $\sin y = \frac{3}{5} \Rightarrow \cos y = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$.

$$\therefore \tan y = \frac{3}{4} \Rightarrow y = \tan^{-1}\frac{3}{4}$$

$$\therefore \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4} \qquad ...(2)$$

Now, we have:

$$\text{L.H.S.} = \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}$$

$$= \tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4} \qquad \left[\text{Using } (1) \text{ and } (2)\right]$$

$$= \tan^{-1}\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}$$

$$= \tan^{-1}\left(\frac{32+45}{60-24}\right) \qquad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\frac{77}{36} = \text{R.H.S.}$$

**Question 5:**

Prove $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

Answer

Let $\cos^{-1}\frac{4}{5}=x$. Then, $\cos x=\frac{4}{5}\Rightarrow \sin x=\sqrt{1-\left(\frac{4}{5}\right)^2}=\frac{3}{5}$.

$\therefore \tan x=\frac{3}{4}\Rightarrow x=\tan^{-1}\frac{3}{4}$

$\therefore \cos^{-1}\frac{4}{5}=\tan^{-1}\frac{3}{4} \qquad ...(1)$

Now, let $\cos^{-1}\frac{12}{13}=y$. Then, $\cos y=\frac{12}{13}\Rightarrow \sin y=\frac{5}{13}$.

$\therefore \tan y=\frac{5}{12}\Rightarrow y=\tan^{-1}\frac{5}{12}$

$\therefore \cos^{-1}\frac{12}{13}=\tan^{-1}\frac{5}{12} \qquad ...(2)$

Let $\cos^{-1}\frac{33}{65}=z$. Then, $\cos z=\frac{33}{65}\Rightarrow \sin z=\frac{56}{65}$.

$\therefore \tan z=\frac{56}{33}\Rightarrow z=\tan^{-1}\frac{56}{33}$

$\therefore \cos^{-1}\frac{33}{65}=\tan^{-1}\frac{56}{33} \qquad ...(3)$

Now, we will prove that:

$$\begin{aligned}
\text{L.H.S.} &= \cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13} \\
&= \tan^{-1}\frac{3}{4}+\tan^{-1}\frac{5}{12} && \left[\text{Using }(1)\text{ and }(2)\right] \\
&= \tan^{-1}\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}} && \left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right] \\
&= \tan^{-1}\frac{36+20}{48-15} \\
&= \tan^{-1}\frac{56}{33} \\
&= \tan^{-1}\frac{56}{33} && \left[\text{by }(3)\right] \\
&= \text{R.H.S.}
\end{aligned}$$

**Question 6:**

Prove $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

Answer

Let $\sin^{-1}\frac{3}{5}=x$. Then, $\sin x=\frac{3}{5}\Rightarrow \cos x=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{\frac{16}{25}}=\frac{4}{5}$.

$\therefore \tan x=\frac{3}{4}\Rightarrow x=\tan^{-1}\frac{3}{4}$

$\therefore \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}$ ...(1)

Now, let $\cos^{-1}\frac{12}{13}=y$. Then, $\cos y=\frac{12}{13}\Rightarrow \sin y=\frac{5}{13}$.

$\therefore \tan y=\frac{5}{12}\Rightarrow y=\tan^{-1}\frac{5}{12}$

$\therefore \cos^{-1}\frac{12}{13}=\tan^{-1}\frac{5}{12}$ ...(2)

Let $\sin^{-1}\frac{56}{65}=z$. Then, $\sin z=\frac{56}{65}\Rightarrow \cos z=\frac{33}{65}$.

$\therefore \tan z=\frac{56}{33}\Rightarrow z=\tan^{-1}\frac{56}{33}$

$\therefore \sin^{-1}\frac{56}{65}=\tan^{-1}\frac{56}{33}$ ...(3)

Now, we have:

$$\text{L.H.S.} = \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5}$$
$$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4} \qquad \left[\text{Using } (1) \text{ and } (2)\right]$$
$$= \tan^{-1}\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}.\frac{3}{4}} \qquad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$
$$= \tan^{-1}\frac{20+36}{48-15}$$
$$= \tan^{-1}\frac{56}{33}$$
$$= \sin^{-1}\frac{56}{65} = \text{R.H.S.} \qquad \left[\text{Using } (3)\right]$$

**Question 7:**

Prove $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

Answer

Let $\sin^{-1}\frac{5}{13} = x$. Then, $\sin x = \frac{5}{13} \Rightarrow \cos x = \frac{12}{13}$.

$\therefore \tan x = \frac{5}{12} \Rightarrow x = \tan^{-1}\frac{5}{12}$

$\therefore \sin^{-1}\frac{5}{13} = \tan^{-1}\frac{5}{12} \qquad ...(1)$

Let $\cos^{-1}\frac{3}{5} = y$. Then, $\cos y = \frac{3}{5} \Rightarrow \sin y = \frac{4}{5}$.

$\therefore \tan y = \frac{4}{3} \Rightarrow y = \tan^{-1}\frac{4}{3}$

$\therefore \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{4}{3} \qquad ...(2)$

Using (1) and (2), we have

$$\text{R.H.S.} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$$
$$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$$
$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right) \qquad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$
$$= \tan^{-1}\left(\frac{15+48}{36-20}\right)$$
$$= \tan^{-1}\frac{63}{16}$$
$$= \text{L.H.S.}$$

**Question 8:**

Prove $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

Answer

$$\text{L.H.S.} = \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right) \qquad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\left(\frac{7+5}{35-1}\right) + \tan^{-1}\left(\frac{8+3}{24-1}\right)$$

$$= \tan^{-1}\frac{12}{34} + \tan^{-1}\frac{11}{23}$$

$$= \tan^{-1}\frac{6}{17} + \tan^{-1}\frac{11}{23}$$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\times\frac{11}{23}}\right)$$

$$= \tan^{-1}\left(\frac{138+187}{391-66}\right)$$

$$= \tan^{-1}\left(\frac{325}{325}\right) = \tan^{-1}1$$

$$= \frac{\pi}{4} = \text{R.H.S.}$$

**Question 9:**

Prove $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x \in [0,\ 1]$

Answer

Let $x = \tan^2\theta$. Then, $\sqrt{x} = \tan\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$.

$$\therefore \frac{1-x}{1+x} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$$

Now, we have:

$$\text{R.H.S.} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{1}{2}\times 2\theta = \theta = \tan^{-1}\sqrt{x} = \text{L.H.S.}$$

**Question 10:**

Prove $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\ \frac{\pi}{4}\right)$

Answer

$$\text{Consider } \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$$

$$=\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1-\sin x}\right)^2} \qquad (\text{by rationalizing})$$

$$=\frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1+\sin x)(1-\sin x)}}{1+\sin x-1+\sin x}$$

$$=\frac{2\left(1+\sqrt{1-\sin^2 x}\right)}{2\sin x}=\frac{1+\cos x}{\sin x}=\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$=\cot\frac{x}{2}$$

$$\therefore \text{L.H.S.}=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\cot^{-1}\left(\cot\frac{x}{2}\right)=\frac{x}{2}=\text{R.H.S.}$$

**Question 11:**

Prove $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ -\frac{1}{\sqrt{2}}\le x\le 1$ [**Hint:** put $x$ = cos $2\theta$]

Answer

Put $x = \cos 2\theta$ so that $\theta = \frac{1}{2}\cos^{-1} x$. Then, we have:

$$\text{L.H.S.} = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right) = \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}1 - \tan^{-1}(\tan\theta) \qquad \left[\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y\right]$$

$$= \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x = \text{R.H.S.}$$

**Question 12:**

Prove $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Answer

$$\text{L.H.S.} = \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$
$$= \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right)$$
$$= \frac{9}{4}\left(\cos^{-1}\frac{1}{3}\right) \quad . \quad ....(1) \quad \left[\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

Now, let $\cos^{-1}\frac{1}{3} = x$. Then, $\cos x = \frac{1}{3} \Rightarrow \sin x = \sqrt{1-\left(\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3}$.

$$\therefore x = \sin^{-1}\frac{2\sqrt{2}}{3} \Rightarrow \cos^{-1}\frac{1}{3} = \sin^{-1}\frac{2\sqrt{2}}{3}$$
$$\therefore \text{L.H.S.} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3} = \text{R.H.S.}$$

**Question 13:**

Solve $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

Answer

$$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$$
$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x) \qquad \left[2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$
$$\Rightarrow \frac{2\cos x}{1-\cos^2 x} = 2\operatorname{cosec} x$$
$$\Rightarrow \frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x}$$
$$\Rightarrow \cos x = \sin x$$
$$\Rightarrow \tan x = 1$$
$$\therefore x = \frac{\pi}{4}$$

**Question 14:**

Solve $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x, (x > 0)$

Answer

$$\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$$

$$\Rightarrow \tan^{-1}1-\tan^{-1}x=\frac{1}{2}\tan^{-1}x \qquad \left[\tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\right]$$

$$\Rightarrow \frac{\pi}{4}=\frac{3}{2}\tan^{-1}x$$

$$\Rightarrow \tan^{-1}x=\frac{\pi}{6}$$

$$\Rightarrow x=\tan\frac{\pi}{6}$$

$$\therefore x=\frac{1}{\sqrt{3}}$$

**Question 15:**

Solve $\sin\left(\tan^{-1}x\right),\ |x|<1$ is equal to

**(A)** $\frac{x}{\sqrt{1-x^2}}$ **(B)** $\frac{1}{\sqrt{1-x^2}}$ **(C)** $\frac{1}{\sqrt{1+x^2}}$ **(D)** $\frac{x}{\sqrt{1+x^2}}$

Answer

Let $\tan^{-1}x = y$. Then, $\tan y = x \Rightarrow \sin y = \frac{x}{\sqrt{1+x^2}}$.

$$\therefore y=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\Rightarrow \tan^{-1}x=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$\therefore \sin\left(\tan^{-1}x\right)=\sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)=\frac{x}{\sqrt{1+x^2}}$$

The correct answer is D.

**Question 16:**

Solve $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$, then $x$ is equal to

**(A)** $0, \frac{1}{2}$ **(B)** $1, \frac{1}{2}$ **(C)** 0 **(D)** $\frac{1}{2}$

Answer

$$\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$$

$$\Rightarrow -2\sin^{-1}x=\frac{\pi}{2}-\sin^{-1}(1-x)$$

$$\Rightarrow -2\sin^{-1}x=\cos^{-1}(1-x) \qquad ...(1)$$

Let $\sin^{-1}x=\theta \Rightarrow \sin\theta = x \Rightarrow \cos\theta=\sqrt{1-x^2}$.

$$\therefore \theta=\cos^{-1}\left(\sqrt{1-x^2}\right)$$

$$\therefore \sin^{-1}x=\cos^{-1}\left(\sqrt{1-x^2}\right)$$

Therefore, from equation (1), we have

$$-2\cos^{-1}\left(\sqrt{1-x^2}\right)=\cos^{-1}(1-x)$$

Put $x$ = sin $y$. Then, we have:

$$-2\cos^{-1}\left(\sqrt{1-\sin^2 y}\right)=\cos^{-1}(1-\sin y)$$

$$\Rightarrow -2\cos^{-1}(\cos y)=\cos^{-1}(1-\sin y)$$

$$\Rightarrow -2y=\cos^{-1}(1-\sin y)$$

$$\Rightarrow 1-\sin y=\cos(-2y)=\cos 2y$$

$$\Rightarrow 1-\sin y=1-2\sin^2 y$$

$$\Rightarrow 2\sin^2 y-\sin y=0$$

$$\Rightarrow \sin y(2\sin y-1)=0$$

$$\Rightarrow \sin y=0 \text{ or } \frac{1}{2}$$

$$\therefore x=0 \text{ or } x=\frac{1}{2}$$

But, when $x=\frac{1}{2}$, it can be observed that:

$$\text{L.H.S.} = \sin^{-1}\left(1-\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$$
$$= \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$$
$$= -\sin^{-1}\frac{1}{2}$$
$$= -\frac{\pi}{6} \neq \frac{\pi}{2} \neq \text{R.H.S.}$$

$\therefore x = \frac{1}{2}$ is not the solution of the given equation.

Thus, $x$ = 0.

Hence, the correct answer is **C**.

**Question 17:**

Solve $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$ is equal to

**(A)** $\frac{\pi}{2}$ **(B).** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{4}$ **(D)** $\frac{-3\pi}{4}$

Answer

$$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\frac{x-y}{x+y}$$

$$=\tan^{-1}\left[\frac{\frac{x}{y}-\frac{x-y}{x+y}}{1+\left(\frac{x}{y}\right)\left(\frac{x-y}{x+y}\right)}\right] \qquad \left[\tan^{-1}y-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\right]$$

$$=\tan^{-1}\left[\frac{\frac{x(x+y)-y(x-y)}{y(x+y)}}{\frac{y(x+y)+x(x-y)}{y(x+y)}}\right]$$

$$=\tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right)$$

$$=\tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)=\tan^{-1}1=\frac{\pi}{4}$$

Hence, the correct answer is **C**.