

Exercise 2.2**Question 1:**

Let $A = \{1, 2, 3, \dots, 14\}$. Define a relation R from A to A by $R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in A\}$. Write down its domain, codomain and range.

Answer

The relation R from A to A is given as

$$R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in A\}$$

i.e., $R = \{(x, y) : 3x = y, \text{ where } x, y \in A\}$

$$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$$\therefore \text{Domain of } R = \{1, 2, 3, 4\}$$

The whole set A is the codomain of the relation R .

$$\therefore \text{Codomain of } R = A = \{1, 2, 3, \dots, 14\}$$

The range of R is the set of all second elements of the ordered pairs in the relation.

$$\therefore \text{Range of } R = \{3, 6, 9, 12\}$$

Question 2:

Define a relation R on the set \mathbf{N} of natural numbers by $R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4; x, y \in \mathbf{N}\}$. Depict this relationship using roster form. Write down the domain and the range.

Answer

$$R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4, x, y \in \mathbf{N}\}$$

The natural numbers less than 4 are 1, 2, and 3.

$$\therefore R = \{(1, 6), (2, 7), (3, 8)\}$$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$$\therefore \text{Domain of } R = \{1, 2, 3\}$$

The range of R is the set of all second elements of the ordered pairs in the relation.

$$\therefore \text{Range of } R = \{6, 7, 8\}$$

Question 3:

$A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. Define a relation R from A to B by $R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd; } x \in A, y \in B\}$. Write R in roster form.

Answer

$$A = \{1, 2, 3, 5\} \text{ and } B = \{4, 6, 9\}$$

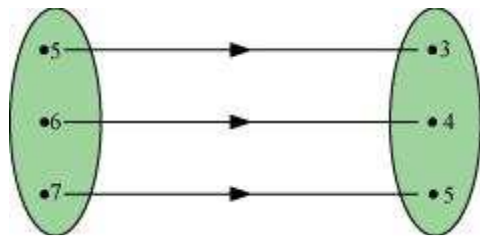
$$R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd; } x \in A, y \in B\}$$

$$\therefore R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$$

Question 4:

The given figure shows a relationship between the sets P and Q . write this relation (i) in set-builder form (ii) in roster form.

What is its domain and range?



Answer

According to the given figure, $P = \{5, 6, 7\}$, $Q = \{3, 4, 5\}$

(i) $R = \{(x, y) : y = x - 2; x \in P\}$ or $R = \{(x, y) : y = x - 2 \text{ for } x = 5, 6, 7\}$

(ii) $R = \{(5, 3), (6, 4), (7, 5)\}$

Domain of $R = \{5, 6, 7\}$

Range of $R = \{3, 4, 5\}$

Question 5:

Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by

$\{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R .

Answer

$A = \{1, 2, 3, 4, 6\}$, $R = \{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$

(i) $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$

(ii) Domain of $R = \{1, 2, 3, 4, 6\}$

(iii) Range of $R = \{1, 2, 3, 4, 6\}$

Question 6:

Determine the domain and range of the relation R defined by $R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$.

Answer

$R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

$\therefore R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

\therefore Domain of $R = \{0, 1, 2, 3, 4, 5\}$

Range of $R = \{5, 6, 7, 8, 9, 10\}$

Question 7:

Write the relation $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$ in roster form.

Answer

$R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$

The prime numbers less than 10 are 2, 3, 5, and 7.

$\therefore R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

Question 8:

Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A to B .

Answer

It is given that $A = \{x, y, z\}$ and $B = \{1, 2\}$.

$\therefore A \times B = \{(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)\}$

Since $n(A \times B) = 6$, the number of subsets of $A \times B$ is 2^6 .

Therefore, the number of relations from A to B is 2^6 .

Question 9:

Let R be the relation on \mathbf{Z} defined by $R = \{(a, b) : a, b \in \mathbf{Z}, a - b \text{ is an integer}\}$. Find the domain and range of R.

Answer

$$R = \{(a, b) : a, b \in \mathbf{Z}, a - b \text{ is an integer}\}$$

It is known that the difference between any two integers is always an integer.

$$\therefore \text{Domain of } R = \mathbf{Z}$$

$$\text{Range of } R = \mathbf{Z}$$