Exercise 2.1

Question 1:

If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of *x* and *y*.

Answer

It is given that $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$.

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$.

$\frac{x}{3}+1=\frac{5}{3}$

$\Rightarrow \frac{x}{3}=\frac{5}{3}-1 \quad y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \frac{x}{3}=\frac{2}{3} \quad \Rightarrow y=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow x=2 \quad \Rightarrow y=1$

$\therefore x = 2$ and $y = 1$

Question 2:

If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?

Answer

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

⇒ Number of elements in set B = 3

Number of elements in (A × B)

= (Number of elements in A) × (Number of elements in B)

= 3 × 3 = 9

Thus, the number of elements in (A × B) is 9.

Question 3:

If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

Answer

G = {7, 8} and H = {5, 4, 2}

We know that the Cartesian product P × Q of two non-empty sets P and Q is defined as

P × Q = {(*p*, *q*): *p*∈ P, *q* ∈ Q}

∴G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}

H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

Question 4:

State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If P = $\{m, n\}$ and Q = $\{n, m\}$, then P × Q = $\{(m, n), (n, m)\}$.
(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that $x \in$ A and $y \in$ B.
(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ Φ) = Φ.
Answer
(i) False
If P = $\{m, n\}$ and Q = $\{n, m\}$, then
P × Q = $\{(m, m), (m, n), (n, m), (n, n)\}$
(ii) True
(iii) True

Question 5:
If A = {−1, 1}, find A × A × A.
Answer
It is known that for any non-empty set A, A × A × A is defined as
A × A × A = $\{(a, b, c): a, b, c \in A\}$
It is given that A = {−1, 1}
∴ A × A × A = {(−1, −1, −1), (−1, −1, 1), (−1, 1, −1), (−1, 1, 1), (1, −1, −1), (1, −1, 1), (1, 1, −1), (1, 1, 1)}

Question 6:
If A × B = $\{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B.
Answer
It is given that A × B = $\{(a, x), (a, y), (b, x), (b, y)\}$
We know that the Cartesian product of two non-empty sets P and Q is defined as P × Q = $\{(p, q): p \in P, q \in Q\}$
∴ A is the set of all first elements and B is the set of all second elements.
Thus, A = $\{a, b\}$ and B = $\{x, y\}$

Question 7:
Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that
(i) A × (B ∩ C) = (A × B) ∩ (A × C)
(ii) A × C is a subset of B × D
Answer
(i) To verify: A × (B ∩ C) = (A × B) ∩ (A × C)
We have B ∩ C = {1, 2, 3, 4} ∩ {5, 6} = Φ
∴L.H.S. = A × (B ∩ C) = A × Φ = Φ
A × B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}
A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
∴ R.H.S. = (A × B) ∩ (A × C) = Φ
∴L.H.S. = R.H.S
Hence, A × (B ∩ C) = (A × B) ∩ (A × C)
(ii) To verify: A × C is a subset of B × D
A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}
We can observe that all the elements of set A × C are the elements of set B × D.
Therefore, A × C is a subset of B × D.

Question 8:

Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

Answer

A = {1, 2} and B = {3, 4}

∴A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}

⇒ n(A × B) = 4

We know that if C is a set with n(C) = m, then n[P(C)] = 2^m.

Therefore, the set A × B has 2^4 = 16 subsets. These are

Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)},
{(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)},
{(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)},
{(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

Question 9:

Let A and B be two sets such that n(A) = 3 and n (B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y and z are distinct elements.

Answer

It is given that n(A) = 3 and n(B) = 2; and (x, 1), (y, 2), (z, 1) are in A × B.

We know that A = Set of first elements of the ordered pair elements of A × B

B = Set of second elements of the ordered pair elements of A × B.

∴ x, y, and z are the elements of A; and 1 and 2 are the elements of B.

Since n(A) = 3 and n(B) = 2, it is clear that A = {x, y, z} and B = {1, 2}.

Question 10:

The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.

Answer

We know that if n(A) = p and n(B) = q, then n(A × B) = pq.

∴ n(A × A) = n(A) × n(A)

It is given that n(A × A) = 9

∴ n(A) × n(A) = 9

⇒ n(A) = 3

The ordered pairs (–1, 0) and (0, 1) are two of the nine elements of A × A.

We know that A × A = {(a, a): a ∈ A}. Therefore, –1, 0, and 1 are elements of A.

Since n(A) = 3, it is clear that A = {–1, 0, 1}.

The remaining elements of set A × A are (–1, –1), (–1, 1), (0, –1), (0, 0),
(1, –1), (1, 0), and (1, 1)

Exercise 2.2

Question 1:

Let A = {1, 2, 3, … , 14}. Define a relation R from A to A by R = {(x, y): $3x - y = 0$, where $x, y \in$ A}. Write down its domain, codomain and range.

Answer

The relation R from A to A is given as

R = {(x, y): $3x - y = 0$, where $x, y \in$ A}

i.e., R = {(x, y): $3x = y$, where $x, y \in$ A}

∴R = {(1, 3), (2, 6), (3, 9), (4, 12)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

∴Domain of R = {1, 2, 3, 4}

The whole set A is the codomainof the relation R.

∴Codomain of R = A = {1, 2, 3, …, 14}

The range of R is the set of all second elements of the ordered pairs in the relation.

∴Range of R = {3, 6, 9, 12}

Question 2:

Define a relation R on the set **N** of natural numbers by R = {(x, y): $y = x + 5$, x is a natural number less than 4; $x, y \in \mathbf{N}$}. Depict this relationship using roster form. Write down the domain and the range.

Answer

R = {(x, y): $y = x + 5$, x is a natural number less than 4, $x, y \in \mathbf{N}$}

The natural numbers less than 4 are 1, 2, and 3.

∴R = {(1, 6), (2, 7), (3, 8)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

∴ Domain of R = {1, 2, 3}

The range of R is the set of all second elements of the ordered pairs in the relation.

∴ Range of R = {6, 7, 8}

Question 3:

A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference between x and y is odd; $x \in$ A, $y \in$ B}. Write R in roster form.

Answer

A = {1, 2, 3, 5} and B = {4, 6, 9}

R = {(x, y): the difference between x and y is odd; $x \in$ A, $y \in$ B}

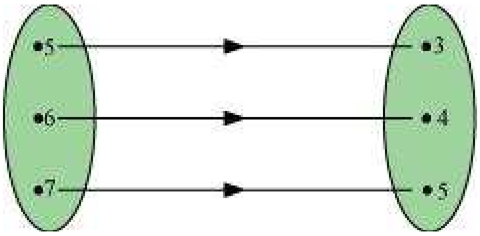
∴R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

Question 4:

The given figure shows a relationship between the sets P and Q. write this relation

(i) in set-builder form (ii) in roster form.

What is its domain and range?



Answer

According to the given figure, P = {5, 6, 7}, Q = {3, 4, 5}

(i) R = {(x, y): $y = x - 2$; $x \in$ P} or R = {(x, y): $y = x - 2$ for x = 5, 6, 7}

(ii) R = {(5, 3), (6, 4), (7, 5)}

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

Question 5:

Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by

{(a, b): a, $b \in$ A, b is exactly divisible by a}.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

Answer

A = {1, 2, 3, 4, 6}, R = {(a, b): a, $b \in$ A, b is exactly divisible by a}

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

Question 6:

Determine the domain and range of the relation R defined by R = {(x, x + 5): $x \in$ {0, 1, 2, 3, 4, 5}}.

Answer

R = {(x, x + 5): $x \in$ {0, 1, 2, 3, 4, 5}}

∴ R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

∴Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {5, 6, 7, 8, 9, 10}

Question 7:

Write the relation R = {(x, x^3): x is a prime number less than 10} in roster form.

Answer

R = {(x, x^3): x is a prime number less than 10}

The prime numbers less than 10 are 2, 3, 5, and 7.

∴R = {(2, 8), (3, 27), (5, 125), (7, 343)}

Question 8:

Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

Answer

It is given that A = {x, y, z} and B = {1, 2}.

∴ A × B = {(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)}

Since n(A × B) = 6, the number of subsets of A × B is 2^6.

Therefore, the number of relations from A to B is 2^6.

Question 9:

Let R be the relation on **Z** defined by R = {(a, b): a, $b \in$ **Z**, $a - b$ is an integer}. Find the domain and range of R.

Answer

R = {(a, b): a, $b \in$ **Z**, $a - b$ is an integer}

It is known that the difference between any two integers is always an integer.

∴Domain of R = **Z**

Range of R = **Z**

Exercise 2.3

Question 1:
Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.
(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
(iii) {(1, 3), (1, 5), (2, 5)}
Answer
(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.
Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.
Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}
(iii) {(1, 3), (1, 5), (2, 5)}
Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Question 2:
Find the domain and range of the following real function:
(i) $f(x) = -|x|$ (ii) $f(x)=\sqrt{9-x^2}$
Answer
(i) $f(x) = -|x|$, $x \in$ R

We know that $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$$\therefore f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$$

Since $f(x)$ is defined for $x \in$ **R**, the domain of f is **R.**
It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.
∴The range of f is $(-\infty, 0]$.
(ii) $f(x)=\sqrt{9-x^2}$
Since $\sqrt{9-x^2}$ is defined for all real numbers that are greater than or equal to −3 and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \leq x \leq 3\}$ or $[-3, 3]$.
For any value of x such that $-3 \leq x \leq 3$, the value of $f(x)$ will lie between 0 and 3.
∴The range of $f(x)$ is $\{x: 0 \leq x \leq 3\}$ or $[0, 3]$.

Question 3:
A function f is defined by $f(x) = 2x - 5$. Write down the values of
(i) $f(0)$, (ii) $f(7)$, (iii) $f(-3)$

Answer
The given function is $f(x) = 2x - 5$.
Therefore,
(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$
(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$
(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

Question 4:
The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$.
Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of C, when $t(C) = 212$
Answer
The given function is $t(C) = \frac{9C}{5} + 32$.
Therefore,
(i) $t(0) = \frac{9 \times 0}{5} + 32 = 0 + 32 = 32$
(ii) $t(28) = \frac{9 \times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5}$
(iii) $t(-10) = \frac{9 \times (-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$
(iv) It is given that $t(C) = 212$

$$\therefore 212 = \frac{9C}{5} + 32$$
$$\Rightarrow \frac{9C}{5} = 212 - 32$$
$$\Rightarrow \frac{9C}{5} = 180$$
$$\Rightarrow 9C = 180 \times 5$$
$$\Rightarrow C = \frac{180 \times 5}{9} = 100$$

Thus, the value of t, when $t(C) = 212$, is 100.

Question 5:
Find the range of each of the following functions.
(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$.
(ii) $f(x) = x^2 + 2$, x, is a real number.
(iii) $f(x) = x$, x is a real number
Answer
(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	…
$f(x)$	1.97	1.7	−0.7	−1	−4	−5.5	−10	−13	…

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2.
i.e., range of $f = (-\infty, 2)$

Alter:

Let $x > 0$

$\Rightarrow 3x > 0$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow f(x) < 2$

$\therefore$ Range of $f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2$, x, is a real number

The values of $f(x)$ for various values of real numbers x can be written in the tabular form as

x	0	±0.3	±0.8	±1	±2	±3		…
$f(x)$	2	2.09	2.64	3	6	11		…..

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.
i.e., range of $f = [2, \infty)$

Alter:

Let x be any real number.

Accordingly,

$x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow x^2 + 2 \geq 2$

$\Rightarrow f(x) \geq 2$

$\therefore$ Range of $f = [2, \infty)$

(iii) $f(x) = x$, x is a real number

It is clear that the range of f is the set of all real numbers.

$\therefore$ Range of f = **R**

Question 1:

The relation f is defined by $f(x)=\begin{cases} x^2, & 0\le x\le 3 \\ 3x, & 3\le x\le 10 \end{cases}$

The relation g is defined by $g(x)=\begin{cases} x^2, & 0\le x\le 2 \\ 3x, & 2\le x\le 10 \end{cases}$

Show that f is a function and g is not a function.

Answer

The relation f is defined as $f(x)=\begin{cases} x^2, & 0\le x\le 3 \\ 3x, & 3\le x\le 10 \end{cases}$

It is observed that for

$0 \le x < 3$, $f(x) = x^2$

$3 < x \le 10$, $f(x) = 3x$

Also, at $x = 3$, $f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$

i.e., at $x = 3$, $f(x) = 9$

Therefore, for $0 \le x \le 10$, the images of $f(x)$ are unique.

Thus, the given relation is a function.

The relation g is defined as $g(x)=\begin{cases} x^2, & 0\le x\le 2 \\ 3x, & 2\le x\le 10 \end{cases}$

It can be observed that for $x = 2$, $g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6. Hence, this relation is not a function.

Question 2:

If $f(x) = x^2$, find $\dfrac{f(1.1)-f(1)}{(1.1-1)}$.

Answer

$f(x)=x^2$

$$\therefore \frac{f(1.1)-f(1)}{(1.1-1)}=\frac{(1.1)^2-(1)^2}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1$$

Question 3:

Find the domain of the function $f(x)=\dfrac{x^2+2x+1}{x^2-8x+12}$

Answer

The given function is $f(x)=\dfrac{x^2+2x+1}{x^2-8x+12}$.

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}=\frac{x^2+2x+1}{(x-6)(x-2)}$$

It can be seen that function *f* is defined for all real numbers except at $x = 6$ and $x = 2$.
Hence, the domain of *f* is **R** – {2, 6}.

Question 4:

Find the domain and the range of the real function *f* defined by $f(x)=\sqrt{(x-1)}$.
Answer

The given real function is $f(x)=\sqrt{x-1}$.

It can be seen that $\sqrt{x-1}$ is defined for $(x - 1) \geq 0$.

i.e., $f(x)=\sqrt{(x-1)}$ is defined for $x \geq 1$.
Therefore, the domain of *f* is the set of all real numbers greater than or equal to 1 i.e., the domain of *f* = $[1, \infty)$.

As $x \geq 1 \Rightarrow (x - 1) \geq 0 \Rightarrow \sqrt{x-1} \geq 0$
Therefore, the range of *f* is the set of all real numbers greater than or equal to 0 i.e., the range of *f* = $[0, \infty)$.

Question 5:
Find the domain and the range of the real function *f* defined by $f(x) = |x - 1|$.
Answer
The given real function is $f(x) = |x - 1|$.
It is clear that $|x - 1|$ is defined for all real numbers.
∴Domain of *f* = **R**
Also, for $x \in$ **R**, $|x - 1|$ assumes all real numbers.
Hence, the range of *f* is the set of all non-negative real numbers.

Question 6:

Let $f=\left\{\left(x, \frac{x^2}{1+x^2}\right): x \in \mathbf{R}\right\}$ be a function from **R** into **R**. Determine the range of *f*.
Answer

$$f=\left\{\left(x, \frac{x^2}{1+x^2}\right): x \in \mathbf{R}\right\}$$

$$=\left\{(0, 0), \left(\pm 0.5, \frac{1}{5}\right), \left(\pm 1, \frac{1}{2}\right), \left(\pm 1.5, \frac{9}{13}\right), \left(\pm 2, \frac{4}{5}\right), \left(3, \frac{9}{10}\right), \left(4, \frac{16}{17}\right), \ldots\right\}$$

The range of *f* is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.
[Denominator is greater numerator]
Thus, range of *f* = [0, 1)

Question 7:

Let f, g: $\mathbf{R} \to \mathbf{R}$ be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\frac{f}{g}$.

Answer

f, g: $\mathbf{R} \to \mathbf{R}$ is defined as $f(x) = x + 1$, $g(x) = 2x - 3$

$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$

$\therefore (f + g)(x) = 3x - 2$

$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$

$\therefore (f - g)(x) = -x + 4$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0,\ x \in \mathbf{R}$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3},\ 2x-3 \neq 0 \text{ or } 2x \neq 3$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3},\ x \neq \frac{3}{2}$$

Question 8:

Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function from **Z** to **Z** defined by $f(x) = ax + b$, for some integers a, b. Determine a, b.

Answer

$f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$

$f(x) = ax + b$

$(1, 1) \in f$

$\Rightarrow f(1) = 1$

$\Rightarrow a \times 1 + b = 1$

$\Rightarrow a + b = 1$

$(0, -1) \in f$

$\Rightarrow f(0) = -1$

$\Rightarrow a \times 0 + b = -1$

$\Rightarrow b = -1$

On substituting $b = -1$ in $a + b = 1$, we obtain $a + (-1) = 1 \Rightarrow a = 1 + 1 = 2$.

Thus, the respective values of a and b are 2 and -1.

Question 9:

Let R be a relation from **N** to **N** defined by $R = \{(a, b): a, b \in \mathbf{N} \text{ and } a = b^2\}$. Are the following true?

(i) $(a, a) \in R$, for all $a \in \mathbf{N}$ (ii) $(a, b) \in R$, implies $(b, a) \in R$

(iii) $(a, b) \in R$, $(b, c) \in R$ implies $(a, c) \in R$.

Justify your answer in each case.

Answer

$R = \{(a, b): a, b \in \mathbf{N} \text{ and } a = b^2\}$

(i) It can be seen that $2 \in \mathbf{N}$;however, $2 \neq 2^2 = 4$.

Therefore, the statement "$(a, a) \in R$, for all $a \in \mathbf{N}$" is not true.

(ii) It can be seen that $(9, 3) \in \mathbf{N}$ because $9, 3 \in \mathbf{N}$ and $9 = 3^2$.

Now, $3 \neq 9^2 = 81$; therefore, $(3, 9) \notin \mathbf{N}$

Therefore, the statement "(a, b) ∈ R, implies (b, a) ∈ R" is not true.
(iii) It can be seen that (9, 3) ∈ R, (16, 4) ∈ R because 9, 3, 16, 4 ∈ **N** and $9 = 3^2$ and $16 = 4^2$.
Now, $9 \neq 4^2 = 16$; therefore, (9, 4) ∉ **N**
Therefore, the statement "(a, b) ∈ R, (b, c) ∈ R implies (a, c) ∈ R" is not true.

Question 10:
Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Are the following true?
(i) f is a relation from A to B (ii) f is a function from A to B.
Justify your answer in each case.
Answer
A = {1, 2, 3, 4} and B = {1, 5, 9, 11, 15, 16}
∴A × B = {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)}
It is given that f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}
(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A × B.
It is observed that f is a subset of A × B.
Thus, f is a relation from A to B.
(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation f is not a function.

Question 11:
Let f be the subset of **Z** × **Z** defined by f = {(ab, a + b): a, b ∈ **Z**}. Is f a function from **Z** to **Z**: justify your answer.
Answer
The relation f is defined as f = {(ab, a + b): a, b ∈ **Z**}
We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B.
Since 2, 6, −2, −6 ∈ **Z**, (2 × 6, 2 + 6), (−2 × −6, −2 + (−6)) ∈ f
i.e., (12, 8), (12, −8) ∈ f
It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and −8. Thus, relation f is not a function.

Question 12:
Let A = {9, 10, 11, 12, 13} and let f: A → **N** be defined by $f(n)$ = the highest prime factor of n. Find the range of f.
Answer
A = {9, 10, 11, 12, 13}
f: A → **N** is defined as
$f(n)$ = The highest prime factor of n
Prime factor of 9 = 3
Prime factors of 10 = 2, 5
Prime factor of 11 = 11
Prime factors of 12 = 2, 3
Prime factor of 13 = 13
∴$f(9)$ = The highest prime factor of 9 = 3

$f(10)$ = The highest prime factor of 10 = 5
$f(11)$ = The highest prime factor of 11 = 11
$f(12)$ = The highest prime factor of 12 = 3
$f(13)$ = The highest prime factor of 13 = 13
The range of f is the set of all $f(n)$, where $n \in A$.
$\therefore$Range of $f = \{3, 5, 11, 13\}$