Exercise 3.1

Question 1:

Find the radian measures corresponding to the following degree measures:

(i) 25° (ii) – 47° 30' (iii) 240° (iv) 520°

Answer

(i) 25°

We know that 180° = π radian

$$\therefore 25° = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$$

(ii) –47° 30'

–47° 30' = $-47\frac{1}{2}$ degree [1° = 60']

$$= \frac{-95}{2} \text{ degree}$$

Since 180° = π radian

$$\frac{-95}{2} \text{ deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian} = \left(\frac{-19}{36 \times 2}\right)\pi \text{ radian} = \frac{-19}{72}\pi \text{ radian}$$

$$\therefore -47° 30' = \frac{-19}{72}\pi \text{ radian}$$

(iii) 240°

We know that 180° = π radian

$$\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

We know that 180° = π radian

$$\therefore 520° = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

Question 2:

Find the degree measures corresponding to the following radian measures

$\left(\text{Use } \pi = \frac{22}{7}\right)$.

(i) $\frac{11}{16}$ (ii) – 4 (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

Answer

(i) $\frac{11}{16}$

We know that π radian = 180°

$$\therefore \frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree} = \frac{45 \times 11}{\pi \times 4} \text{deg ree}$$

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree} = \frac{315}{8} \text{ deg ree}$$

$$= 39\frac{3}{8} \text{ deg ree}$$

$$= 39° + \frac{3 \times 60}{8} \text{ min utes} \qquad [1° = 60']$$

$$= 39° + 22' + \frac{1}{2} \text{ min utes}$$

$$= 39°22'30'' \qquad [1' = 60'']$$

(ii) – 4

We know that π radian = 180°

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ deg ree} = \frac{180 \times 7(-4)}{22} \text{ deg ree}$$

$$= \frac{-2520}{11} \text{ deg ree} = -229\frac{1}{11} \text{ deg ree}$$

$$= -229° + \frac{1 \times 60}{11} \text{ min utes} \qquad [1° = 60']$$

$$= -229° + 5' + \frac{5}{11} \text{ min utes}$$

$$= -229°5'27'' \qquad [1' = 60'']$$

(iii) $\frac{5\pi}{3}$

We know that π radian = 180°

$$\therefore \frac{5\pi}{3} \text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ deg ree} = 300°$$

(iv) $\frac{7\pi}{6}$

We know that π radian = 180°

$$\therefore \frac{7\pi}{6}\text{radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$$

Question 3:

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Answer

Number of revolutions made by the wheel in 1 minute = 360

∴Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of 6 × 2π radian, i.e.,

12 π radian

Thus, in one second, the wheel turns an angle of 12π radian.

Question 4:

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm $\left(\text{Use } \pi = \frac{22}{7}\right)$.

Answer

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then

$$\theta = \frac{l}{r}$$

Therefore, forr = 100 cm, l = 22 cm, we have

$$\theta = \frac{22}{100}\ \text{radian} = \frac{180}{\pi} \times \frac{22}{100}\ \text{degree} = \frac{180 \times 7 \times 22}{22 \times 100}\ \text{degree}$$

$$= \frac{126}{10}\ \text{degree} = 12\frac{3}{5}\ \text{degree} = 12°36' \qquad [1° = 60']$$

Thus, the required angle is 12°36’.

Question 5:

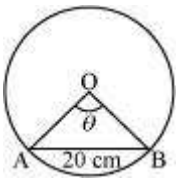
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Answer

Diameter of the circle = 40 cm

∴Radius (r) of the circle = $\frac{40}{2}\text{ cm} = 20\text{ cm}$

Let AB be a chord (length = 20 cm) of the circle.



In ΔOAB, OA = OB = Radius of circle = 20 cm

Also, AB = 20 cm

Thus, ΔOAB is an equilateral triangle.

∴θ = 60° = $\frac{\pi}{3}$ radian

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$.

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3}\text{ cm}$$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}\text{ cm}$.

Question 6:

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Answer

Let the radii of the two circles be r_1 and r_2. Let an arc of length *l* subtend an angle of 60° at the centre of the circle of radius r_1, while let an arc of length *l* subtend an angle of 75° at the centre of the circle of radius r_2.

Now, 60° = $\frac{\pi}{3}$ radian and 75° = $\frac{5\pi}{12}$ radian

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$ or $l = r\theta$.

$$\therefore l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

$$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

$$\Rightarrow r_1 = \frac{r_2 5}{4}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$$

Thus, the ratio of the radii is 5:4.

Question 7:

Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Answer

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$.

It is given that *r* = 75 cm

(i) Here, *l* = 10 cm

$$\theta = \frac{10}{75} \text{ radian} = \frac{2}{15} \text{ radian}$$

(ii) Here, *l* = 15 cm

$$\theta = \frac{15}{75} \text{ radian} = \frac{1}{5} \text{ radian}$$

(iii) Here, l = 21 cm

$$\theta = \frac{21}{75} \text{ radian} = \frac{7}{25} \text{ radian}$$