

Exercise 3.4**Question 1:**

Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

Answer

$$\tan x = \sqrt{3}$$

It is known that $\tan \frac{\pi}{3} = \sqrt{3}$ and $\tan \left(\frac{4\pi}{3} \right) = \tan \left(\pi + \frac{\pi}{3} \right) = \tan \frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$.

$$\text{Now, } \tan x = \tan \frac{\pi}{3}$$

$$\Rightarrow x = n\pi + \frac{\pi}{3}, \text{ where } n \in Z$$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}, \text{ where } n \in Z$

Question 2:

Find the principal and general solutions of the equation $\sec x = 2$

Answer

$$\sec x = 2$$

It is known that $\sec \frac{\pi}{3} = 2$ and $\sec \frac{5\pi}{3} = \sec \left(2\pi - \frac{\pi}{3} \right) = \sec \frac{\pi}{3} = 2$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$.

$$\text{Now, } \sec x = \sec \frac{\pi}{3}$$

$$\Rightarrow \cos x = \cos \frac{\pi}{3} \quad \left[\sec x = \frac{1}{\cos x} \right]$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in \mathbf{Z}$

Question 3:

Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

Answer

$$\cot x = -\sqrt{3}$$

It is known that $\cot \frac{\pi}{6} = \sqrt{3}$

$$\therefore \cot\left(\pi - \frac{\pi}{6}\right) = -\cot \frac{\pi}{6} = -\sqrt{3} \text{ and } \cot\left(2\pi - \frac{\pi}{6}\right) = -\cot \frac{\pi}{6} = -\sqrt{3}$$

i.e., $\cot \frac{5\pi}{6} = -\sqrt{3}$ and $\cot \frac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are $x = \frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

Now, $\cot x = \cot \frac{5\pi}{6}$

$$\Rightarrow \tan x = \tan \frac{5\pi}{6} \quad \left[\cot x = \frac{1}{\tan x} \right]$$

$$\Rightarrow x = n\pi + \frac{5\pi}{6}, \text{ where } n \in \mathbf{Z}$$

Therefore, the general solution is $x = n\pi + \frac{5\pi}{6}$, where $n \in \mathbf{Z}$

Question 4:

Find the general solution of $\operatorname{cosec} x = -2$

Answer

$$\operatorname{cosec} x = -2$$

It is known that

$$\operatorname{cosec} \frac{\pi}{6} = 2$$

$$\therefore \operatorname{cosec} \left(\pi + \frac{\pi}{6} \right) = -\operatorname{cosec} \frac{\pi}{6} = -2 \text{ and } \operatorname{cosec} \left(2\pi - \frac{\pi}{6} \right) = -\operatorname{cosec} \frac{\pi}{6} = -2$$

$$\text{i.e., } \operatorname{cosec} \frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec} \frac{11\pi}{6} = -2$$

Therefore, the principal solutions are $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

$$\text{Now, } \operatorname{cosec} x = \operatorname{cosec} \frac{7\pi}{6}$$

$$\Rightarrow \sin x = \sin \frac{7\pi}{6} \quad \left[\operatorname{cosec} x = \frac{1}{\sin x} \right]$$

$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Therefore, the general solution is $x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbb{Z}$

Question 5:

Find the general solution of the equation $\cos 4x = \cos 2x$

Answer

$$\cos 4x = \cos 2x$$

$$\Rightarrow \cos 4x - \cos 2x = 0$$

$$\Rightarrow -2 \sin \left(\frac{4x+2x}{2} \right) \sin \left(\frac{4x-2x}{2} \right) = 0$$

$$\left[\because \cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \right]$$

$$\Rightarrow \sin 3x \sin x = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad \sin x = 0$$

$$\therefore 3x = n\pi \quad \text{or} \quad x = n\pi, \text{ where } n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{n\pi}{3} \quad \text{or} \quad x = n\pi, \text{ where } n \in \mathbb{Z}$$

Question 6:

Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$

Answer

$$\cos 3x + \cos x - \cos 2x = 0$$

$$\Rightarrow 2 \cos\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \right]$$

$$\Rightarrow 2 \cos 2x \cos x - \cos 2x = 0$$

$$\Rightarrow \cos 2x (2 \cos x - 1) = 0$$

$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad 2 \cos x - 1 = 0$$

$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$\therefore 2x = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x = \cos \frac{\pi}{3}, \text{ where } n \in Z$$

$$\Rightarrow x = (2n+1)\frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

Question 7:

Find the general solution of the equation $\sin 2x + \cos x = 0$

Answer

$$\sin 2x + \cos x = 0$$

$$\Rightarrow 2 \sin x \cos x + \cos x = 0$$

$$\Rightarrow \cos x (2 \sin x + 1) = 0$$

$$\Rightarrow \cos x = 0 \quad \text{or} \quad 2 \sin x + 1 = 0$$

$$\text{Now, } \cos x = 0 \Rightarrow \cos x = (2n+1)\frac{\pi}{2}, \text{ where } n \in Z$$

$$2 \sin x + 1 = 0$$

$$\Rightarrow \sin x = \frac{-1}{2} = -\sin \frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = \sin \frac{7\pi}{6}$$

$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n \frac{7\pi}{6}$, $n \in Z$.

Question 8:

Find the general solution of the equation $\sec^2 2x = 1 - \tan 2x$

Answer

$$\sec^2 2x = 1 - \tan 2x$$

$$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$$

$$\Rightarrow \tan^2 2x + \tan 2x = 0$$

$$\Rightarrow \tan 2x(\tan 2x + 1) = 0$$

$$\Rightarrow \tan 2x = 0 \quad \text{or} \quad \tan 2x + 1 = 0$$

Now, $\tan 2x = 0$

$$\Rightarrow \tan 2x = \tan 0$$

$$\Rightarrow 2x = n\pi + 0, \text{ where } n \in Z$$

$$\Rightarrow x = \frac{n\pi}{2}, \text{ where } n \in Z$$

$$\tan 2x + 1 = 0$$

$$\Rightarrow \tan 2x = -1 = -\tan \frac{\pi}{4} = \tan \left(\pi - \frac{\pi}{4} \right) = \tan \frac{3\pi}{4}$$

$$\Rightarrow 2x = n\pi + \frac{3\pi}{4}, \text{ where } n \in Z$$

$$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}, \text{ where } n \in Z$$

Therefore, the general solution is $\frac{n\pi}{2}$ or $\frac{n\pi}{2} + \frac{3\pi}{8}, n \in Z$.

Question 9:

Find the general solution of the equation $\sin x + \sin 3x + \sin 5x = 0$

Answer

$$\sin x + \sin 3x + \sin 5x = 0$$

$$(\sin x + \sin 5x) + \sin 3x = 0$$

$$\Rightarrow \left[2 \sin \left(\frac{x+5x}{2} \right) \cos \left(\frac{x-5x}{2} \right) \right] + \sin 3x = 0 \quad \left[\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right]$$

$$\Rightarrow 2 \sin 3x \cos(-2x) + \sin 3x = 0$$

$$\Rightarrow 2 \sin 3x \cos 2x + \sin 3x = 0$$

$$\Rightarrow \sin 3x (2 \cos 2x + 1) = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2 \cos 2x + 1 = 0$$

Now, $\sin 3x = 0 \Rightarrow 3x = n\pi$, where $n \in Z$

i.e., $x = \frac{n\pi}{3}$, where $n \in Z$

$$2 \cos 2x + 1 = 0$$

$$\Rightarrow \cos 2x = \frac{-1}{2} = -\cos \frac{\pi}{3} = \cos \left(\pi - \frac{\pi}{3} \right)$$

$$\Rightarrow \cos 2x = \cos \frac{2\pi}{3}$$

$$\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}, \text{ where } n \in Z$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

Therefore, the general solution is $\frac{n\pi}{3}$ or $n\pi \pm \frac{\pi}{3}$, $n \in Z$