

Exercise 3.2**Question 1:**

Find the values of other five trigonometric functions if $\cos x = -\frac{1}{2}$, x lies in third quadrant.

Answer

$$\cos x = -\frac{1}{2}$$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$$

Since x lies in the 3rd quadrant, the value of $\sin x$ will be negative.

$$\therefore \sin x = -\frac{\sqrt{3}}{2}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

Question 2:

Find the values of other five trigonometric functions if $\sin x = \frac{3}{5}$, x lies in second quadrant.

Answer

$$\sin x = \frac{3}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$$

$$\Rightarrow \cos^2 x = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm \frac{4}{5}$$

Since x lies in the 2nd quadrant, the value of $\cos x$ will be negative

$$\therefore \cos x = -\frac{4}{5}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

Question 3:

Find the values of other five trigonometric functions if $\cot x = \frac{3}{4}$, x lies in third quadrant.

Answer

$$\cot x = \frac{3}{4}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm \frac{5}{3}$$

Since x lies in the 3rd quadrant, the value of $\sec x$ will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(-\frac{3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(-\frac{3}{5}\right) = -\frac{4}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

Question 4:

Find the values of other five trigonometric functions if $\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Answer

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm \frac{12}{13}$$

Since x lies in the 4th quadrant, the value of $\sin x$ will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

Question 5:

Find the values of other five trigonometric functions if $\tan x = -\frac{5}{12}$, x lies in second quadrant.

Answer

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm \frac{13}{12}$$

Since x lies in the 2nd quadrant, the value of $\sec x$ will be negative.

$$\therefore \sec x = -\frac{13}{12}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

$$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Question 6:

Find the value of the trigonometric function $\sin 765^\circ$

Answer

It is known that the values of $\sin x$ repeat after an interval of 2π or 360° .

$$\therefore \sin 765^\circ = \sin(2 \times 360^\circ + 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

Question 7:

Find the value of the trigonometric function $\operatorname{cosec}(-1410^\circ)$

Answer

It is known that the values of $\operatorname{cosec} x$ repeat after an interval of 2π or 360° .

$$\begin{aligned} \therefore \operatorname{cosec}(-1410^\circ) &= \operatorname{cosec}(-1410^\circ + 4 \times 360^\circ) \\ &= \operatorname{cosec}(-1410^\circ + 1440^\circ) \\ &= \operatorname{cosec}30^\circ = 2 \end{aligned}$$

Question 8:

Find the value of the trigonometric function $\tan \frac{19\pi}{3}$

Answer

It is known that the values of $\tan x$ repeat after an interval of π or 180° .

$$\therefore \tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan \left(6\pi + \frac{\pi}{3} \right) = \tan \frac{\pi}{3} = \tan 60^\circ = \sqrt{3}$$

Question 9:

Find the value of the trigonometric function $\sin \left(-\frac{11\pi}{3} \right)$

Answer

It is known that the values of $\sin x$ repeat after an interval of 2π or 360° .

$$\therefore \sin \left(-\frac{11\pi}{3} \right) = \sin \left(-\frac{11\pi}{3} + 2 \times 2\pi \right) = \sin \left(\frac{\pi}{3} \right) = \frac{\sqrt{3}}{2}$$

Question 10:

Find the value of the trigonometric function $\cot \left(-\frac{15\pi}{4} \right)$

Answer

It is known that the values of $\cot x$ repeat after an interval of π or 180° .

$$\therefore \cot \left(-\frac{15\pi}{4} \right) = \cot \left(-\frac{15\pi}{4} + 4\pi \right) = \cot \frac{\pi}{4} = 1$$