

**Exercise 5.1****Question 1:**

Express the given complex number in the form  $a + ib$ :  $(5i)\left(-\frac{3}{5}i\right)$

Answer

$$\begin{aligned} (5i)\left(-\frac{3}{5}i\right) &= -5 \times \frac{3}{5} \times i \times i \\ &= -3i^2 \\ &= -3(-1) \quad [i^2 = -1] \\ &= 3 \end{aligned}$$

**Question 2:**

Express the given complex number in the form  $a + ib$ :  $i^9 + i^{19}$

Answer

$$\begin{aligned} i^9 + i^{19} &= i^{4 \times 2 + 1} + i^{4 \times 4 + 3} \\ &= (i^4)^2 \cdot i + (i^4)^4 \cdot i^3 \\ &= 1 \times i + 1 \times (-i) \quad [i^4 = 1, i^3 = -i] \\ &= i + (-i) \\ &= 0 \end{aligned}$$

**Question 3:**

Express the given complex number in the form  $a + ib$ :  $i^{-39}$

Answer

$$\begin{aligned} i^{-39} &= i^{-4 \times 9 - 3} = (i^4)^{-9} \cdot i^{-3} \\ &= (1)^{-9} \cdot i^{-3} \quad [i^4 = 1] \\ &= \frac{1}{i^3} = \frac{1}{-i} \quad [i^3 = -i] \\ &= \frac{-1}{i} \times \frac{i}{i} \\ &= \frac{-i}{i^2} = \frac{-i}{-1} = i \quad [i^2 = -1] \end{aligned}$$

**Question 4:**

Express the given complex number in the form  $a + ib$ :  $3(7 + i7) + i(7 + i7)$

Answer

$$\begin{aligned} 3(7 + i7) + i(7 + i7) &= 21 + 21i + 7i + 7i^2 \\ &= 21 + 28i + 7 \times (-1) && [\because i^2 = -1] \\ &= 14 + 28i \end{aligned}$$

**Question 5:**

Express the given complex number in the form  $a + ib$ :  $(1 - i) - (-1 + i6)$

Answer

$$\begin{aligned} (1 - i) - (-1 + i6) &= 1 - i + 1 - 6i \\ &= 2 - 7i \end{aligned}$$

**Question 6:**

Express the given complex number in the form  $a + ib$ :  $\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$

Answer

$$\begin{aligned} &\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right) \\ &= \frac{1}{5} + \frac{2}{5}i - 4 - \frac{5}{2}i \\ &= \left(\frac{1}{5} - 4\right) + i\left(\frac{2}{5} - \frac{5}{2}\right) \\ &= \frac{-19}{5} + i\left(\frac{-21}{10}\right) \\ &= \frac{-19}{5} - \frac{21}{10}i \end{aligned}$$

**Question 7:**

Express the given complex number in the form  $a + ib$ :  $\left[\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right)\right] - \left(-\frac{4}{3} + i\right)$

Answer

$$\begin{aligned} & \left[ \left( \frac{1}{3} + i\frac{7}{3} \right) + \left( 4 + i\frac{1}{3} \right) \right] - \left( \frac{-4}{3} + i \right) \\ &= \frac{1}{3} + \frac{7}{3}i + 4 + \frac{1}{3}i + \frac{4}{3} - i \\ &= \left( \frac{1}{3} + 4 + \frac{4}{3} \right) + i \left( \frac{7}{3} + \frac{1}{3} - 1 \right) \\ &= \frac{17}{3} + i\frac{5}{3} \end{aligned}$$

**Question 8:**

Express the given complex number in the form  $a + ib$ :  $(1 - i)^4$

Answer

$$\begin{aligned} (1-i)^4 &= \left[ (1-i)^2 \right]^2 \\ &= \left[ 1^2 + i^2 - 2i \right]^2 \\ &= \left[ 1 - 1 - 2i \right]^2 \\ &= (-2i)^2 \\ &= (-2i) \times (-2i) \\ &= 4i^2 = -4 \quad [i^2 = -1] \end{aligned}$$

**Question 9:**

Express the given complex number in the form  $a + ib$ :  $\left( \frac{1}{3} + 3i \right)^3$

Answer

$$\begin{aligned}
 \left(\frac{1}{3} + 3i\right)^3 &= \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3} + 3i\right) \\
 &= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3} + 3i\right) \\
 &= \frac{1}{27} + 27(-i) + i + 9i^2 \quad [i^3 = -i] \\
 &= \frac{1}{27} - 27i + i - 9 \quad [i^2 = -1] \\
 &= \left(\frac{1}{27} - 9\right) + i(-27 + 1) \\
 &= \frac{-242}{27} - 26i
 \end{aligned}$$

**Question 10:**

Express the given complex number in the form  $a + ib$ :  $\left(-2 - \frac{1}{3}i\right)^3$

Answer

$$\begin{aligned}
 \left(-2 - \frac{1}{3}i\right)^3 &= (-1)^3 \left(2 + \frac{1}{3}i\right)^3 \\
 &= -\left[2^3 + \left(\frac{i}{3}\right)^3 + 3(2)\left(\frac{i}{3}\right)\left(2 + \frac{i}{3}\right)\right] \\
 &= -\left[8 + \frac{i^3}{27} + 2i\left(2 + \frac{i}{3}\right)\right] \\
 &= -\left[8 - \frac{i}{27} + 4i + \frac{2i^2}{3}\right] \quad [i^3 = -i] \\
 &= -\left[8 - \frac{i}{27} + 4i - \frac{2}{3}\right] \quad [i^2 = -1] \\
 &= -\left[\frac{22}{3} + \frac{107i}{27}\right] \\
 &= -\frac{22}{3} - \frac{107}{27}i
 \end{aligned}$$

**Question 11:**

Find the multiplicative inverse of the complex number  $4 - 3i$

Answer

$$\text{Let } z = 4 - 3i$$

$$\text{Then, } \bar{z} = 4 + 3i \text{ and } |z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$$

Therefore, the multiplicative inverse of  $4 - 3i$  is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

**Question 12:**

Find the multiplicative inverse of the complex number  $\sqrt{5} + 3i$

Answer

$$\text{Let } z = \sqrt{5} + 3i$$

$$\text{Then, } \bar{z} = \sqrt{5} - 3i \text{ and } |z|^2 = (\sqrt{5})^2 + 3^2 = 5 + 9 = 14$$

Therefore, the multiplicative inverse of  $\sqrt{5} + 3i$  is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5}-3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

**Question 13:**

Find the multiplicative inverse of the complex number  $-i$

Answer

$$\text{Let } z = -i$$

$$\text{Then, } \bar{z} = i \text{ and } |z|^2 = 1^2 = 1$$

Therefore, the multiplicative inverse of  $-i$  is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

**Question 14:**

Express the following expression in the form of  $a + ib$ .

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

Answer

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

$$= \frac{(3)^2 - (i\sqrt{5})^2}{\sqrt{3} + \sqrt{2}i - \sqrt{3} + \sqrt{2}i} \quad [(a+b)(a-b) = a^2 - b^2]$$

$$= \frac{9 - 5i^2}{2\sqrt{2}i}$$

$$= \frac{9 - 5(-1)}{2\sqrt{2}i} \quad [i^2 = -1]$$

$$= \frac{9 + 5}{2\sqrt{2}i} \times \frac{i}{i}$$

$$= \frac{14i}{2\sqrt{2}i^2}$$

$$= \frac{14i}{2\sqrt{2}(-1)}$$

$$= \frac{-7i}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-7\sqrt{2}i}{2}$$