Exercise 7.3

Question 1:

$\sin^2(2x+5)$

Answer

$$\sin^2(2x+5)=\frac{1-\cos 2(2x+5)}{2}=\frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx=\int \frac{1-\cos(4x+10)}{2}dx$$

$$=\frac{1}{2}\int 1\,dx-\frac{1}{2}\int \cos(4x+10)\,dx$$

$$=\frac{1}{2}x-\frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right)+\text{C}$$

$$=\frac{1}{2}x-\frac{1}{8}\sin(4x+10)+\text{C}$$

Question 2:

$\sin 3x\cos 4x$

Answer

It is known that, $\sin A\cos B=\frac{1}{2}\{\sin(A+B)+\sin(A-B)\}$

$$\therefore \int \sin 3x\cos 4x\,dx=\frac{1}{2}\int\{\sin(3x+4x)+\sin(3x-4x)\}\,dx$$

$$=\frac{1}{2}\int\{\sin 7x+\sin(-x)\}\,dx$$

$$=\frac{1}{2}\int\{\sin 7x-\sin x\}\,dx$$

$$=\frac{1}{2}\int \sin 7x\,dx-\frac{1}{2}\int \sin x\,dx$$

$$=\frac{1}{2}\left(\frac{-\cos 7x}{7}\right)-\frac{1}{2}(-\cos x)+\text{C}$$

$$=\frac{-\cos 7x}{14}+\frac{\cos x}{2}+\text{C}$$

Question 3:

cos 2x cos 4x cos 6x

Answer

It is known that, $\cos A \cos B = \frac{1}{2}\left\{\cos(A+B)+\cos(A-B)\right\}$

$$\therefore \int \cos 2x\left(\cos 4x \cos 6x\right)dx = \int \cos 2x\left[\frac{1}{2}\left\{\cos(4x+6x)+\cos(4x-6x)\right\}\right]dx$$

$$= \frac{1}{2}\int\left\{\cos 2x \cos 10x + \cos 2x \cos(-2x)\right\}dx$$

$$= \frac{1}{2}\int\left\{\cos 2x \cos 10x + \cos^2 2x\right\}dx$$

$$= \frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x)+\cos(2x-10x)\right\}+\left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$= \frac{1}{4}\int\left(\cos 12x + \cos 8x + 1 + \cos 4x\right)dx$$

$$= \frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}\right]+\text{C}$$

Question 4:

$\sin^3(2x+1)$

Answer

Let $I = \int \sin^3(2x+1)$

$$\Rightarrow \int \sin^3(2x+1)dx = \int \sin^2(2x+1)\cdot\sin(2x+1)dx$$

$$= \int\left(1-\cos^2(2x+1)\right)\sin(2x+1)dx$$

$$\text{Let} \cos(2x+1) = t$$

$$\Rightarrow -2\sin(2x+1)dx = dt$$

$$\Rightarrow \sin(2x+1)dx = \frac{-dt}{2}$$

$$\Rightarrow I = \frac{-1}{2}\int\left(1-t^2\right)dt$$

$$= \frac{-1}{2}\left\{t-\frac{t^3}{3}\right\}$$

$$= \frac{-1}{2}\left\{\cos\left(2x+1\right)-\frac{\cos^3\left(2x+1\right)}{3}\right\}$$

$$= \frac{-\cos\left(2x+1\right)}{2}+\frac{\cos^3\left(2x+1\right)}{6}+C$$

Question 5:

$\sin^3 x \cos^3 x$

Answer

$$\text{Let } I = \int \sin^3 x \cos^3 x \cdot dx$$

$$= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx$$

$$= \int \cos^3 x \left(1-\cos^2 x\right)\sin x \cdot dx$$

$$\text{Let} \cos x = t$$

$$\Rightarrow -\sin x \cdot dx = dt$$

$$\Rightarrow I = -\int t^3\left(1-t^2\right)dt$$

$$= -\int\left(t^3-t^5\right)dt$$

$$= -\left\{\frac{t^4}{4}-\frac{t^6}{6}\right\}+C$$

$$= -\left\{\frac{\cos^4 x}{4}-\frac{\cos^6 x}{6}\right\}+C$$

$$= \frac{\cos^6 x}{6}-\frac{\cos^4 x}{4}+C$$

Question 6:

$\sin x \sin 2x \sin 3x$

Answer

It is known that, $\sin A \sin B = \frac{1}{2}\left\{\cos(A-B)-\cos(A+B)\right\}$

$$\therefore \int \sin x \sin 2x \sin 3x \, dx = \int\left[\sin x \cdot \frac{1}{2}\left\{\cos(2x-3x)-\cos(2x+3x)\right\}\right] dx$$

$$= \frac{1}{2}\int\left(\sin x \cos(-x) - \sin x \cos 5x\right) dx$$

$$= \frac{1}{2}\int\left(\sin x \cos x - \sin x \cos 5x\right) dx$$

$$= \frac{1}{2}\int \frac{\sin 2x}{2} \, dx - \frac{1}{2}\int \sin x \cos 5x \, dx$$

$$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right] - \frac{1}{2}\int\left\{\frac{1}{2}\sin(x+5x)+\sin(x-5x)\right\} dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\int\left(\sin 6x + \sin(-4x)\right) dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{4}\right] + C$$

$$= \frac{-\cos 2x}{8} - \frac{1}{8}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{2}\right] + C$$

$$= \frac{1}{8}\left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x\right] + C$$

Question 7:

sin 4x sin 8x

Answer

It is known that, $\sin A \sin B = \frac{1}{2}\cos(A-B) - \cos(A+B)$

$$\therefore \int \sin 4x \sin 8x \, dx = \int \left\{ \frac{1}{2} \cos(4x - 8x) - \cos(4x + 8x) \right\} dx$$

$$= \frac{1}{2} \int \left(\cos(-4x) - \cos 12x \right) dx$$

$$= \frac{1}{2} \int \left(\cos 4x - \cos 12x \right) dx$$

$$= \frac{1}{2} \left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12} \right]$$

Question 8:

$$\frac{1 - \cos x}{1 + \cos x}$$

Answer

$$\frac{1 - \cos x}{1 + \cos x} = \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} \qquad \left[2\sin^2 \frac{x}{2} = 1 - \cos x \text{ and } 2\cos^2 \frac{x}{2} = 1 + \cos x \right]$$

$$= \tan^2 \frac{x}{2}$$

$$= \left(\sec^2 \frac{x}{2} - 1 \right)$$

$$\therefore \int \frac{1 - \cos x}{1 + \cos x} dx = \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= \left[\frac{\tan \frac{x}{2}}{\frac{1}{2}} - x \right] + C$$

$$= 2 \tan \frac{x}{2} - x + C$$

Question 9:

$$\frac{\cos x}{1 + \cos x}$$

Answer

$$\frac{\cos x}{1+\cos x}=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2} \text{ and } \cos x=2\cos^2\frac{x}{2}-1\right]$$

$$=\frac{1}{2}\left[1-\tan^2\frac{x}{2}\right]$$

$$\therefore \int\frac{\cos x}{1+\cos x}dx=\frac{1}{2}\int\left(1-\tan^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\int\left(1-\sec^2\frac{x}{2}+1\right)dx$$

$$=\frac{1}{2}\int\left(2-\sec^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\left[2x-\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]+C$$

$$=x-\tan\frac{x}{2}+C$$

Question 10:

$\sin^4 x$

Answer

$$\sin^4 x = \sin^2 x \sin^2 x$$
$$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$$
$$= \frac{1}{4}(1-\cos 2x)^2$$
$$= \frac{1}{4}\left[1+\cos^2 2x - 2\cos 2x\right]$$
$$= \frac{1}{4}\left[1+\left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right]$$
$$= \frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right]$$
$$= \frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right]$$

$$\therefore \int \sin^4 x \, dx = \frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] dx$$
$$= \frac{1}{4}\left[\frac{3}{2}x+\frac{1}{2}\left(\frac{\sin 4x}{4}\right) - \frac{2\sin 2x}{2}\right] + C$$
$$= \frac{1}{8}\left[3x+\frac{\sin 4x}{4} - 2\sin 2x\right] + C$$
$$= \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

Question 11:

$\cos^4 2x$

Answer

$$\cos^4 2x = \left(\cos^2 2x\right)^2$$
$$= \left(\frac{1+\cos 4x}{2}\right)^2$$
$$= \frac{1}{4}\left[1+\cos^2 4x + 2\cos 4x\right]$$
$$= \frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right)+2\cos 4x\right]$$
$$= \frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$
$$= \frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$
$$\therefore \int \cos^4 2x \, dx = \int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx$$
$$= \frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C$$

Question 12:

$$\frac{\sin^2 x}{1+\cos x}$$

Answer

$$\frac{\sin^2 x}{1+\cos x} = \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}} \quad \left[\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2};\ \cos x = 2\cos^2\frac{x}{2}-1\right]$$
$$= \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$
$$= 2\sin^2\frac{x}{2}$$
$$= 1-\cos x$$
$$\therefore \int \frac{\sin^2 x}{1+\cos x}dx = \int(1-\cos x)dx$$
$$= x-\sin x + C$$

Question 13:

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos \alpha}$$

Answer

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos \alpha}=\frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}} \qquad \left[\cos C-\cos D=-2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$=\frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=\frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$=2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$$=2\left[\cos(x)+\cos\alpha\right]$$

$$=2\cos x+2\cos\alpha$$

$$\therefore \int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos \alpha}dx=\int 2\cos x+2\cos\alpha$$

$$=2\left[\sin x+x\cos\alpha\right]+\text{C}$$

Question 14:

$$\frac{\cos x-\sin x}{1+\sin 2x}$$

Answer

$$\frac{\cos x - \sin x}{1 + \sin 2x} = \frac{\cos x - \sin x}{\left(\sin^2 x + \cos^2 x\right) + 2\sin x \cos x}$$

$$\left[\sin^2 x + \cos^2 x = 1;\ \ \sin 2x = 2\sin x \cos x\right]$$

$$= \frac{\cos x - \sin x}{\left(\sin x + \cos x\right)^2}$$

Let $\sin x + \cos x = t$

$\therefore \left(\cos x - \sin x\right) dx = dt$

$$\Rightarrow \int \frac{\cos x - \sin x}{1 + \sin 2x} dx = \int \frac{\cos x - \sin x}{\left(\sin x + \cos x\right)^2} dx$$

$$= \int \frac{dt}{t^2}$$

$$= \int t^{-2} dt$$

$$= -t^{-1} + \text{C}$$

$$= -\frac{1}{t} + \text{C}$$

$$= \frac{-1}{\sin x + \cos x} + \text{C}$$

Question 15:

$\tan^3 2x \sec 2x$

Answer

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right) \tan 2x \sec 2x$$
$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$

$$\therefore \int \tan^3 2x \sec 2x \ dx = \int \sec^2 2x \tan 2x \sec 2x \ dx - \int \tan 2x \sec 2x \ dx$$
$$= \int \sec^2 2x \tan 2x \sec 2x \ dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$\therefore 2 \sec 2x \tan 2x \ dx = dt$

$$\therefore \int \tan^3 2x \sec 2x \ dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

Question 16:

$\tan^4 x$

Answer

$\tan^4 x$

$= \tan^2 x \cdot \tan^2 x$

$= \left(\sec^2 x - 1\right) \tan^2 x$

$= \sec^2 x \tan^2 x - \tan^2 x$

$= \sec^2 x \tan^2 x - \left(\sec^2 x - 1\right)$

$= \sec^2 x \tan^2 x - \sec^2 x + 1$

$$\therefore \int \tan^4 x \ dx = \int \sec^2 x \tan^2 x \ dx - \int \sec^2 x \ dx + \int 1 \cdot dx$$
$$= \int \sec^2 x \tan^2 x \ dx - \tan x + x + C \qquad ...(1)$$

Consider $\int \sec^2 x \tan^2 x \ dx$

Let $\tan x = t \Rightarrow \sec^2 x \ dx = dt$

$$\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int \tan^4 x\, dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$

Question 17:

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$$

Answer

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

$$= \tan x \sec x + \cot x \operatorname{cosec} x$$

$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\, dx = \int (\tan x \sec x + \cot x \operatorname{cosec} x) dx$$

$$= \sec x - \operatorname{cosec} x + C$$

Question 18:

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$

Answer

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$

$$= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \qquad [\cos 2x = 1 - 2\sin^2 x]$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\therefore \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx = \int \sec^2 x\, dx = \tan x + C$$

Question 19:

$$\frac{1}{\sin x \cos^3 x}$$

Answer

$$\frac{1}{\sin x \cos^3 x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x}$$

$$= \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x \cos x}$$

$$= \tan x \sec^2 x + \frac{\frac{1 \cos^2 x}{\sin x \cos x}}{\cos^2 x}$$

$$= \tan x \sec^2 x + \frac{\sec^2 x}{\tan x}$$

$$\therefore \int \frac{1}{\sin x \cos^3 x} dx = \int \tan x \sec^2 x \, dx + \int \frac{\sec^2 x}{\tan x} \, dx$$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\Rightarrow \int \frac{1}{\sin x \cos^3 x} dx = \int t dt + \int \frac{1}{t} dt$$

$$= \frac{t^2}{2} + \log|t| + C$$

$$= \frac{1}{2} \tan^2 x + \log|\tan x| + C$$

Question 20:

$$\frac{\cos 2x}{(\cos x + \sin x)^2}$$

Answer

$$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{\cos 2x}{1+\sin 2x}$$

$$\therefore \int\frac{\cos 2x}{(\cos x+\sin x)^2}\, dx=\int\frac{\cos 2x}{(1+\sin 2x)}\, dx$$

Let $1+\sin 2x=t$

$\Rightarrow 2\cos 2x\, dx=dt$

$$\therefore \int\frac{\cos 2x}{(\cos x+\sin x)^2}dx=\frac{1}{2}\int\frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+\text{C}$$

$$=\frac{1}{2}\log|1+\sin 2x|+\text{C}$$

$$=\frac{1}{2}\log\left|(\sin x+\cos x)^2\right|+\text{C}$$

$$=\log|\sin x+\cos x|+\text{C}$$

Question 21:

$\sin^{-1}(\cos x)$

Answer

$\sin^{-1}(\cos x)$

Let $\cos x=t$

Then, $\sin x=\sqrt{1-t^2}$

$$\Rightarrow (-\sin x)dx = dt$$

$$dx = \frac{-dt}{\sin x}$$

$$dx = \frac{-dt}{\sqrt{1-t^2}}$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int \sin^{-1} t\left(\frac{-dt}{\sqrt{1-t^2}}\right)$$

$$= -\int \frac{\sin^{-1} t}{\sqrt{1-t^2}}dt$$

Let $\sin^{-1} t = u$

$$\Rightarrow \frac{1}{\sqrt{1-t^2}}dt = du$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int 4du$$

$$= -\frac{u^2}{2} + C$$

$$= \frac{-(\sin^{1} t)^2}{2} + C$$

$$= \frac{-\left[\sin^{-1}(\cos x)\right]^2}{2} + C \qquad \ldots(1)$$

It is known that,

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\therefore \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x) = \left(\frac{\pi}{2} - x\right)$$

Substituting in equation (1), we obtain

$$\int \sin^{-1}(\cos x)\,dx = \frac{-\left[\frac{\pi}{2}-x\right]^2}{2}+C$$

$$= -\frac{1}{2}\left(\frac{\pi^2}{2}+x^2-\pi x\right)+C$$

$$= -\frac{\pi^2}{8}-\frac{x^2}{2}+\frac{1}{2}\pi x+C$$

$$= \frac{\pi x}{2}-\frac{x^2}{2}+\left(C-\frac{\pi^2}{8}\right)$$

$$= \frac{\pi x}{2}-\frac{x^2}{2}+C_1$$

Question 22:

$$\frac{1}{\cos(x-a)\cos(x-b)}$$

Answer

$$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin\left[(x-b)-(x-a)\right]}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\frac{\left[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)\right]}{\cos(x-a)\cos(x-b)}$$

$$= \frac{1}{\sin(a-b)}\left[\tan(x-b)-\tan(x-a)\right]$$

$$\Rightarrow \int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)}\int\left[\tan(x-b)-\tan(x-a)\right]dx$$

$$= \frac{1}{\sin(a-b)}\left[-\log\left|\cos(x-b)\right|+\log\left|\cos(x-a)\right|\right]$$

$$= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

Question 23:

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\, dx$$ is equal to

A. tan x + cot x + C

B. tan x + cosec x + C

C. − tan x + cot x + C

D. tan x + sec x + C

Answer

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\, dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$= \int \left(\sec^2 x - \operatorname{cosec}^2 x \right) dx$$

$$= \tan x + \cot x + C$$

Hence, the correct Answer is A.

Question 24:

$$\int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx$$ equals

A. − cot (ex^x) + C

B. tan (xe^x) + C

C. tan (e^x) + C

D. cot (e^x) + C

Answer

$$\int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx$$

Let $ex^x = t$

$\Rightarrow \left(e^x \cdot x + e^x \cdot 1\right) dx = dt$

$e^x \left(x+1\right) dx = dt$

$$\therefore \int \frac{e^x \left(1+x\right)}{\cos^2\left(e^x x\right)}\, dx = \int \frac{dt}{\cos^2 t}$$

$$= \int \sec^2 t \; dt$$

$$= \tan t + C$$

$$= \tan\left(e^x \cdot x\right) + C$$

Hence, the correct Answer is B.