

## Exercise 7.9

**Question 1:**

$$\int_{-1}^1 (x+1) dx$$

Answer

$$\text{Let } I = \int_{-1}^1 (x+1) dx$$

$$\int (x+1) dx = \frac{x^2}{2} + x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(1) - F(-1) \\ &= \left(\frac{1}{2} + 1\right) - \left(\frac{1}{2} - 1\right) \\ &= \frac{1}{2} + 1 - \frac{1}{2} + 1 \\ &= 2 \end{aligned}$$

**Question 2:**

$$\int_2^3 \frac{1}{x} dx$$

Answer

$$\text{Let } I = \int_2^3 \frac{1}{x} dx$$

$$\int \frac{1}{x} dx = \log|x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(3) - F(2) \\ &= \log|3| - \log|2| = \log \frac{3}{2} \end{aligned}$$

**Question 3:**

$$\int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$$

Answer

$$\text{Let } I = \int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$$

$$\begin{aligned} \int (4x^3 - 5x^2 + 6x + 9) dx &= 4 \left( \frac{x^4}{4} \right) - 5 \left( \frac{x^3}{3} \right) + 6 \left( \frac{x^2}{2} \right) + 9(x) \\ &= x^4 - \frac{5x^3}{3} + 3x^2 + 9x = F(x) \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$I = F(2) - F(1)$$

$$\begin{aligned} I &= \left\{ 2^4 - \frac{5 \cdot (2)^3}{3} + 3(2)^2 + 9(2) \right\} - \left\{ (1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1) \right\} \\ &= \left( 16 - \frac{40}{3} + 12 + 18 \right) - \left( 1 - \frac{5}{3} + 3 + 9 \right) \\ &= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9 \\ &= 33 - \frac{35}{3} \\ &= \frac{99 - 35}{3} \\ &= \frac{64}{3} \end{aligned}$$

**Question 4:**

$$\int_0^{\frac{\pi}{4}} \sin 2x dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \sin 2x dx$$

$$\int \sin 2x dx = \left( \frac{-\cos 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F\left(\frac{\pi}{4}\right) - F(0) \\ &= -\frac{1}{2} \left[ \cos 2\left(\frac{\pi}{4}\right) - \cos 0 \right] \\ &= -\frac{1}{2} \left[ \cos\left(\frac{\pi}{2}\right) - \cos 0 \right] \\ &= -\frac{1}{2} [0 - 1] \\ &= \frac{1}{2} \end{aligned}$$

**Question 5:**

$$\int_0^{\frac{\pi}{2}} \cos 2x \, dx$$

Answer

$$\begin{aligned} \text{Let } I &= \int_0^{\frac{\pi}{2}} \cos 2x \, dx \\ \int \cos 2x \, dx &= \left( \frac{\sin 2x}{2} \right) = F(x) \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F\left(\frac{\pi}{2}\right) - F(0) \\ &= \frac{1}{2} \left[ \sin 2\left(\frac{\pi}{2}\right) - \sin 0 \right] \\ &= \frac{1}{2} [\sin \pi - \sin 0] \\ &= \frac{1}{2} [0 - 0] = 0 \end{aligned}$$

**Question 6:**

$$\int_1^5 e^x \, dx$$

Answer

$$\text{Let } I = \int_4^5 e^x dx$$

$$\int e^x dx = e^x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(5) - F(4)$$

$$= e^5 - e^4$$

$$= e^4(e-1)$$

**Question 7:**

$$\int_0^{\frac{\pi}{4}} \tan x dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \tan x dx$$

$$\int \tan x dx = -\log |\cos x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\log \left| \cos \frac{\pi}{4} \right| + \log |\cos 0|$$

$$= -\log \left| \frac{1}{\sqrt{2}} \right| + \log |1|$$

$$= -\log(2)^{\frac{1}{2}}$$

$$= \frac{1}{2} \log 2$$

**Question 8:**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x dx$$

Answer

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

$$\int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right) \\ &= \log \left| \operatorname{cosec} \frac{\pi}{4} - \cot \frac{\pi}{4} \right| - \log \left| \operatorname{cosec} \frac{\pi}{6} - \cot \frac{\pi}{6} \right| \\ &= \log |\sqrt{2} - 1| - \log |2 - \sqrt{3}| \\ &= \log \left( \frac{\sqrt{2} - 1}{2 - \sqrt{3}} \right) \end{aligned}$$

**Question 9:**

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Answer

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(1) - F(0) \\ &= \sin^{-1}(1) - \sin^{-1}(0) \\ &= \frac{\pi}{2} - 0 \\ &= \frac{\pi}{2} \end{aligned}$$

**Question 10:**

$$\int_0^1 \frac{dx}{1+x^2}$$

Answer

$$\text{Let } I = \int_0^1 \frac{dx}{1+x^2}$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(1) - F(0) \\ &= \tan^{-1}(1) - \tan^{-1}(0) \\ &= \frac{\pi}{4} \end{aligned}$$

**Question 11:**

$$\int_2^3 \frac{dx}{x^2-1}$$

Answer

$$\text{Let } I = \int_2^3 \frac{dx}{x^2-1}$$

$$\int \frac{dx}{x^2-1} = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(3) - F(2) \\ &= \frac{1}{2} \left[ \log \left| \frac{3-1}{3+1} \right| - \log \left| \frac{2-1}{2+1} \right| \right] \\ &= \frac{1}{2} \left[ \log \left| \frac{2}{4} \right| - \log \left| \frac{1}{3} \right| \right] \\ &= \frac{1}{2} \left[ \log \frac{1}{2} - \log \frac{1}{3} \right] \\ &= \frac{1}{2} \left[ \log \frac{3}{2} \right] \end{aligned}$$

**Question 12:**

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$\int \cos^2 x \, dx = \int \left( \frac{1 + \cos 2x}{2} \right) dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2} \left( x + \frac{\sin 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \left[ F\left(\frac{\pi}{2}\right) - F(0) \right] \\ &= \frac{1}{2} \left[ \left( \frac{\pi}{2} + \frac{\sin \pi}{2} \right) - \left( 0 + \frac{\sin 0}{2} \right) \right] \\ &= \frac{1}{2} \left[ \frac{\pi}{2} + 0 - 0 - 0 \right] \\ &= \frac{\pi}{4} \end{aligned}$$

**Question 13:**

$$\int_2^3 \frac{x \, dx}{x^2 + 1}$$

Answer

$$\text{Let } I = \int_2^3 \frac{x}{x^2 + 1} dx$$

$$\int \frac{x}{x^2 + 1} dx = \frac{1}{2} \int \frac{2x}{x^2 + 1} dx = \frac{1}{2} \log(1 + x^2) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(3) - F(2) \\ &= \frac{1}{2} \left[ \log(1 + (3)^2) - \log(1 + (2)^2) \right] \\ &= \frac{1}{2} \left[ \log(10) - \log(5) \right] \\ &= \frac{1}{2} \log\left(\frac{10}{5}\right) = \frac{1}{2} \log 2 \end{aligned}$$

**Question 14:**

$$\int_0^1 \frac{2x+3}{5x^2+1} dx$$

Answer

$$\text{Let } I = \int_0^1 \frac{2x+3}{5x^2+1} dx$$

$$\begin{aligned} \int \frac{2x+3}{5x^2+1} dx &= \frac{1}{5} \int \frac{5(2x+3)}{5x^2+1} dx \\ &= \frac{1}{5} \int \frac{10x+15}{5x^2+1} dx \\ &= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5x^2+1} dx \\ &= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5\left(x^2+\frac{1}{5}\right)} dx \\ &= \frac{1}{5} \log(5x^2+1) + \frac{3}{5} \cdot \frac{1}{\frac{1}{\sqrt{5}}} \tan^{-1} \frac{x}{\frac{1}{\sqrt{5}}} \\ &= \frac{1}{5} \log(5x^2+1) + \frac{3}{\sqrt{5}} \tan^{-1}(\sqrt{5}x) \\ &= F(x) \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(1) - F(0) \\ &= \left\{ \frac{1}{5} \log(5+1) + \frac{3}{\sqrt{5}} \tan^{-1}(\sqrt{5}) \right\} - \left\{ \frac{1}{5} \log(1) + \frac{3}{\sqrt{5}} \tan^{-1}(0) \right\} \\ &= \frac{1}{5} \log 6 + \frac{3}{\sqrt{5}} \tan^{-1} \sqrt{5} \end{aligned}$$

**Question 15:**

$$\int_0^1 x e^{x^2} dx$$

Answer



$$\text{Let } I = \int_0^1 x e^{x^2} dx$$

$$\text{Put } x^2 = t \Rightarrow 2x dx = dt$$

As  $x \rightarrow 0, t \rightarrow 0$  and as  $x \rightarrow 1, t \rightarrow 1$ ,

$$\therefore I = \frac{1}{2} \int_0^1 e^t dt$$

$$\frac{1}{2} \int e^t dt = \frac{1}{2} e^t = F(t)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \frac{1}{2} e - \frac{1}{2} e^0$$

$$= \frac{1}{2} (e - 1)$$

**Question 16:**

$$\int_0^1 \frac{5x^2}{x^2 + 4x + 3}$$

Answer

$$I = \int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx$$

Let

Dividing  $5x^2$  by  $x^2 + 4x + 3$ , we obtain

$$I = \int_1^2 \left\{ 5 - \frac{20x + 15}{x^2 + 4x + 3} \right\} dx$$

$$= \int_1^2 5 dx - \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx$$

$$= [5x]_1^2 - \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx$$

$$I = 5 - I_1, \text{ where } I_1 = \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx \quad \dots(1)$$

$$\text{Consider } I_1 = \int_1^2 \frac{20x+15}{x^2+4x+8} dx$$

$$\begin{aligned} \text{Let } 20x+15 &= A \frac{d}{dx}(x^2+4x+3) + B \\ &= 2Ax + (4A+B) \end{aligned}$$

Equating the coefficients of  $x$  and constant term, we obtain

$$A = 10 \text{ and } B = -25$$

$$\Rightarrow I_1 = 10 \int_1^2 \frac{2x+4}{x^2+4x+3} dx - 25 \int_1^2 \frac{dx}{x^2+4x+3}$$

$$\text{Let } x^2+4x+3 = t$$

$$\Rightarrow (2x+4)dx = dt$$

$$\begin{aligned} \Rightarrow I_1 &= 10 \int \frac{dt}{t} - 25 \int \frac{dx}{(x+2)^2 - 1^2} \\ &= 10 \log t - 25 \left[ \frac{1}{2} \log \left( \frac{x+2-1}{x+2+1} \right) \right] \\ &= \left[ 10 \log(x^2+4x+3) \right]_1^2 - 25 \left[ \frac{1}{2} \log \left( \frac{x+1}{x+3} \right) \right]_1^2 \\ &= [10 \log 15 - 10 \log 8] - 25 \left[ \frac{1}{2} \log \frac{3}{5} - \frac{1}{2} \log \frac{2}{4} \right] \\ &= [10 \log(5 \times 3) - 10 \log(4 \times 2)] - \frac{25}{2} [\log 3 - \log 5 - \log 2 + \log 4] \\ &= [10 \log 5 + 10 \log 3 - 10 \log 4 - 10 \log 2] - \frac{25}{2} [\log 3 - \log 5 - \log 2 + \log 4] \\ &= \left[ 10 + \frac{25}{2} \right] \log 5 + \left[ -10 - \frac{25}{2} \right] \log 4 + \left[ 10 - \frac{25}{2} \right] \log 3 + \left[ -10 + \frac{25}{2} \right] \log 2 \\ &= \frac{45}{2} \log 5 - \frac{45}{2} \log 4 - \frac{5}{2} \log 3 + \frac{5}{2} \log 2 \\ &= \frac{45}{2} \log \frac{5}{4} - \frac{5}{2} \log \frac{3}{2} \end{aligned}$$

Substituting the value of  $I_1$  in (1), we obtain

$$\begin{aligned}
 I &= 5 - \left[ \frac{45}{2} \log \frac{5}{4} - \frac{5}{2} \log \frac{3}{2} \right] \\
 &= 5 - \frac{5}{2} \left[ 9 \log \frac{5}{4} - \log \frac{3}{2} \right]
 \end{aligned}$$

**Question 17:**

$$\int_0^{\frac{\pi}{4}} (2 \sec^2 x + x^3 + 2) dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{4}} (2 \sec^2 x + x^3 + 2) dx$$

$$\int (2 \sec^2 x + x^3 + 2) dx = 2 \tan x + \frac{x^4}{4} + 2x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
 I &= F\left(\frac{\pi}{4}\right) - F(0) \\
 &= \left\{ \left( 2 \tan \frac{\pi}{4} + \frac{1}{4} \left(\frac{\pi}{4}\right)^4 + 2 \left(\frac{\pi}{4}\right) \right) - (2 \tan 0 + 0 + 0) \right\} \\
 &= 2 \tan \frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2} \\
 &= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}
 \end{aligned}$$

**Question 18:**

$$\int_0^{\pi} \left( \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$$

Answer

$$\begin{aligned}\text{Let } I &= \int_0^{\pi} \left( \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx \\ &= - \int_0^{\pi} \left( \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right) dx \\ &= - \int_0^{\pi} \cos x \, dx\end{aligned}$$

$$\int \cos x \, dx = \sin x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}I &= F(\pi) - F(0) \\ &= \sin \pi - \sin 0 \\ &= 0\end{aligned}$$

**Question 19:**

$$\int_0^2 \frac{6x+3}{x^2+4} dx$$

Answer

$$\begin{aligned}\text{Let } I &= \int_0^2 \frac{6x+3}{x^2+4} dx \\ \int \frac{6x+3}{x^2+4} dx &= 3 \int \frac{2x+1}{x^2+4} dx \\ &= 3 \int \frac{2x}{x^2+4} dx + 3 \int \frac{1}{x^2+4} dx \\ &= 3 \log(x^2+4) + \frac{3}{2} \tan^{-1} \frac{x}{2} = F(x)\end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
 I &= F(2) - F(0) \\
 &= \left\{ 3 \log(2^2 + 4) + \frac{3}{2} \tan^{-1} \left( \frac{2}{2} \right) \right\} - \left\{ 3 \log(0 + 4) + \frac{3}{2} \tan^{-1} \left( \frac{0}{2} \right) \right\} \\
 &= 3 \log 8 + \frac{3}{2} \tan^{-1} 1 - 3 \log 4 - \frac{3}{2} \tan^{-1} 0 \\
 &= 3 \log 8 + \frac{3}{2} \left( \frac{\pi}{4} \right) - 3 \log 4 - 0 \\
 &= 3 \log \left( \frac{8}{4} \right) + \frac{3\pi}{8} \\
 &= 3 \log 2 + \frac{3\pi}{8}
 \end{aligned}$$

**Question 20:**

$$\int_0^1 \left( xe^x + \sin \frac{\pi x}{4} \right) dx$$

Answer

$$\text{Let } I = \int_0^1 \left( xe^x + \sin \frac{\pi x}{4} \right) dx$$

$$\begin{aligned}
 \int \left( xe^x + \sin \frac{\pi x}{4} \right) dx &= x \int e^x dx - \int \left\{ \left( \frac{d}{dx} x \right) \int e^x dx \right\} dx + \left\{ \frac{-\cos \frac{\pi x}{4}}{\frac{\pi}{4}} \right\} \\
 &= xe^x - \int e^x dx - \frac{4\pi}{\pi} \cos \frac{x}{4} \\
 &= xe^x - e^x - \frac{4\pi}{\pi} \cos \frac{x}{4} \\
 &= F(x)
 \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
 I &= F(1) - F(0) \\
 &= \left(1.e^1 - e^1 - \frac{4}{\pi} \cos \frac{\pi}{4}\right) - \left(0.e^0 - e^0 - \frac{4}{\pi} \cos 0\right) \\
 &= e - e - \frac{4}{\pi} \left(\frac{1}{\sqrt{2}}\right) + 1 + \frac{4}{\pi} \\
 &= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}
 \end{aligned}$$

**Question 21:**

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} \text{ equals}$$

**A.**  $\frac{\pi}{3}$

**B.**  $\frac{2\pi}{3}$

**C.**  $\frac{\pi}{6}$

**D.**  $\frac{\pi}{12}$

Answer

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
 \int_1^{\sqrt{3}} \frac{dx}{1+x^2} &= F(\sqrt{3}) - F(1) \\
 &= \tan^{-1} \sqrt{3} - \tan^{-1} 1 \\
 &= \frac{\pi}{3} - \frac{\pi}{4} \\
 &= \frac{\pi}{12}
 \end{aligned}$$

Hence, the correct Answer is D.

**Question 22:**

$$\int_0^2 \frac{dx}{4+9x^2} \text{ equals}$$

**A.**  $\frac{\pi}{6}$

**B.**  $\frac{\pi}{12}$

**C.**  $\frac{\pi}{24}$

**D.**  $\frac{\pi}{4}$

Answer

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2 + (3x)^2}$$

$$\text{Put } 3x = t \Rightarrow 3dx = dt$$

$$\begin{aligned} \therefore \int \frac{dx}{(2)^2 + (3x)^2} &= \frac{1}{3} \int \frac{dt}{(2)^2 + t^2} \\ &= \frac{1}{3} \left[ \frac{1}{2} \tan^{-1} \frac{t}{2} \right] \\ &= \frac{1}{6} \tan^{-1} \left( \frac{3x}{2} \right) \\ &= F(x) \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} &= F\left(\frac{2}{3}\right) - F(0) \\ &= \frac{1}{6} \tan^{-1}\left(\frac{3 \cdot 2}{2 \cdot 3}\right) - \frac{1}{6} \tan^{-1} 0 \\ &= \frac{1}{6} \tan^{-1} 1 - 0 \\ &= \frac{1}{6} \times \frac{\pi}{4} \\ &= \frac{\pi}{24}\end{aligned}$$

Hence, the correct Answer is C.