Exercise 7.4

Question 1:

If ${}^{n}C_{8} = {}^{n}C_{2}$, find ${}^{n}C_{2}$.

Answer

It is known that, ${}^{n}C_{a} = {}^{n}C_{b} \Rightarrow a = b \text{ or } n = a + b$

Therefore,

$${}^{n}C_{8} = {}^{n}C_{2} \Rightarrow n = 8 + 2 = 10$$

$$\therefore {}^{n}C_{2} = {}^{10}C_{2} = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10 \times 9 \times 8!}{2 \times 1 \times 8!} = 45$$

Question 2:

Determine *n* if

(i) ${}^{2n}C_{3} : {}^{n}C_{3} = 12 : 1$ (ii) ${}^{2n}C_{3} : {}^{n}C_{3} = 11 : 1$

Answer

(i)

$$\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1}$$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{12}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 12$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 12$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 12$$

$$\Rightarrow \frac{(2n-1)}{(n-2)} = 3$$

$$\Rightarrow 2n-1 = 3(n-2)$$

$$\Rightarrow 2n-1 = 3n-6$$

$$\Rightarrow 3n-2n = -1+6$$

$$\Rightarrow n = 5$$

(ii)

$$\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{11}{1}$$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = 11$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 11$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)}{n-2} = 11$$

$$\Rightarrow 4(2n-1) = 11(n-2)$$

$$\Rightarrow 8n-4 = 11n-22$$

$$\Rightarrow 11n-8n = -4+22$$

$\Rightarrow 3n = 18$

$\Rightarrow n = 6$

Question 3:

How many chords can be drawn through 21 points on a circle?

Answer

For drawing one chord on a circle, only 2 points are required.

To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted.

Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time.

Thus, required number of chords = $^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21!}{2!19!} = \frac{21 \times 20}{2} = 210$

Question 4:

In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Answer

A team of 3 boys and 3 girls is to be selected from 5 boys and 4 girls.

3 boys can be selected from 5 boys in $^5C_3$ ways.

3 girls can be selected from 4 girls in $^4C_3$ ways.

Therefore, by multiplication principle, number of ways in which a team of 3 boys and 3 girls can be selected $= {}^5C_3 \times {}^4C_3 = \frac{5!}{3!2!} \times \frac{4!}{3!1!}$

$= \frac{5 \times 4 \times 3!}{3! \times 2} \times \frac{4 \times 3!}{3!}$

$= 10 \times 4 = 40$

Question 5:

Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Answer

There are a total of 6 red balls, 5 white balls, and 5 blue balls.

9 balls have to be selected in such a way that each selection consists of 3 balls of each colour.

Here,

3 balls can be selected from 6 red balls in ${}^6C_3$ ways.

3 balls can be selected from 5 white balls in ${}^5C_3$ ways.

3 balls can be selected from 5 blue balls in ${}^5C_3$ ways.

Thus, by multiplication principle, required number of ways of selecting 9 balls

$$= {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{5!}{3!2!}$$
$$= \frac{6\times5\times4\times3!}{3!\times3\times2} \times \frac{5\times4\times3!}{3!\times2\times1} \times \frac{5\times4\times3!}{3!\times2\times1}$$
$$= 20\times10\times10 = 2000$$

Question 6:

Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Answer

In a deck of 52 cards, there are 4 aces. A combination of 5 cards have to be made in which there is exactly one ace.

Then, one ace can be selected in ${}^4C_1$ ways and the remaining 4 cards can be selected out of the 48 cards in ${}^{48}C_4$ ways.

Thus, by multiplication principle, required number of 5 card combinations

$$= {}^{48}C_4 \times {}^4C_1 = \frac{48!}{4!44!} \times \frac{4!}{1!3!}$$
$$= \frac{48\times47\times46\times45}{4\times3\times2\times1} \times 4$$
$$= 778320$$

Question 7:

In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Answer

Out of 17 players, 5 players are bowlers.

A cricket team of 11 players is to be selected in such a way that there are exactly 4 bowlers.

4 bowlers can be selected in $^5C_4$ ways and the remaining 7 players can be selected out of the 12 players in $^{12}C_7$ ways.

Thus, by multiplication principle, required number of ways of selecting cricket team

$$= {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!1!} \times \frac{12!}{7!5!} = 5 \times \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 3960$$

Question 8:

A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Answer

There are 5 black and 6 red balls in the bag.

2 black balls can be selected out of 5 black balls in $^5C_2$ ways and 3 red balls can be selected out of 6 red balls in $^6C_3$ ways.

Thus, by multiplication principle, required number of ways of selecting 2 black and 3 red balls $= {}^5C_2 \times {}^6C_3 = \frac{5!}{2!3!} \times \frac{6!}{3!3!} = \frac{5 \times 4}{2} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 10 \times 20 = 200$

Question 9:

In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Answer

There are 9 courses available out of which, 2 specific courses are compulsory for every student.

Therefore, every student has to choose 3 courses out of the remaining 7 courses. This can be chosen in $^7C_3$ ways.

Thus, required number of ways of choosing the programme

$$= {}^7C_3 = \frac{7!}{3!4!} = \frac{7\times6\times5\times4!}{3\times2\times1\times4!} = 35$$

NCERT Miscellaneous Solutions

Question 1:

How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

Answer

In the word DAUGHTER, there are 3 vowels namely, A, U, and E, and 5 consonants namely, D, G, H, T, and R.

Number of ways of selecting 2 vowels out of 3 vowels = ${}^3C_2 = 3$

Number of ways of selecting 3 consonants out of 5 consonants = ${}^5C_3 = 10$

Therefore, number of combinations of 2 vowels and 3 consonants = 3 × 10 = 30

Each of these 30 combinations of 2 vowels and 3 consonants can be arranged among themselves in 5! ways.

Hence, required number of different words = 30 × 5! = 3600

Question 2:

How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?

Answer

In the word EQUATION, there are 5 vowels, namely, A, E, I, O, and U, and 3 consonants, namely, Q, T, and N.

Since all the vowels and consonants have to occur together, both (AEIOU) and (QTN) can be assumed as single objects. Then, the permutations of these 2 objects taken all at a time are counted. This number would be ${}^2P_2 = 2!$

Corresponding to each of these permutations, there are 5! permutations of the five vowels taken all at a time and 3! permutations of the 3 consonants taken all at a time.

Hence, by multiplication principle, required number of words = 2! × 5! × 3!

= 1440

Question 3: