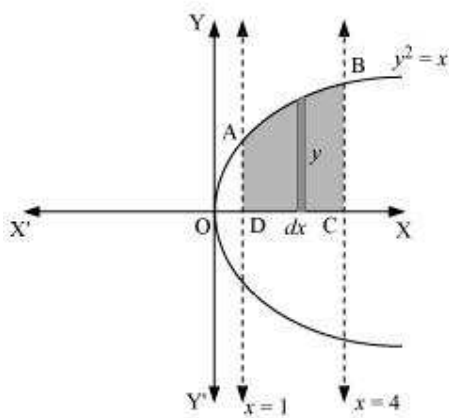


Exercise 8.1

Question 1:

Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis.

Answer



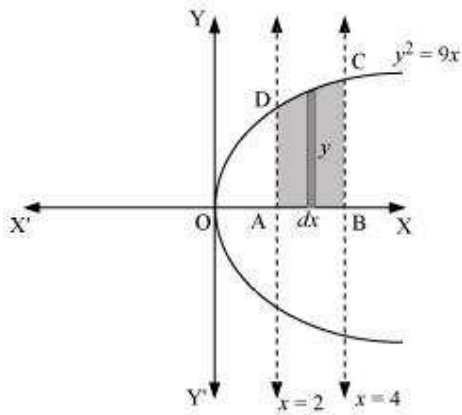
The area of the region bounded by the curve, $y^2 = x$, the lines, $x = 1$ and $x = 4$, and the x-axis is the area ABCD.

$$\begin{aligned}\text{Area of ABCD} &= \int_1^4 y \, dx \\ &= \int_1^4 \sqrt{x} \, dx \\ &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\ &= \frac{2}{3} \left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right] \\ &= \frac{2}{3} [8 - 1] \\ &= \frac{14}{3} \text{ units}\end{aligned}$$

Question 2:

Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x -axis in the first quadrant.

Answer



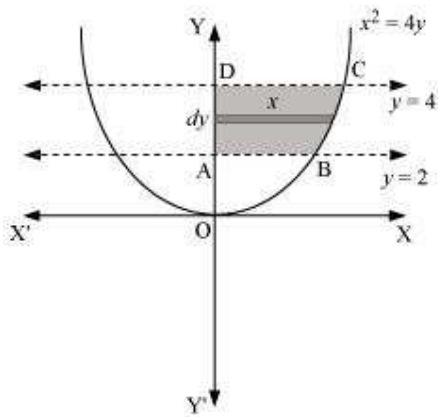
The area of the region bounded by the curve, $y^2 = 9x$, $x = 2$, and $x = 4$, and the x -axis is the area ABCD.

$$\begin{aligned}
 \text{Area of ABCD} &= \int_2^4 y \, dx \\
 &= \int_2^4 3\sqrt{x} \, dx \\
 &= 3 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4 \\
 &= 2 \left[x^{\frac{3}{2}} \right]_2^4 \\
 &= 2 \left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] \\
 &= 2 \left[8 - 2\sqrt{2} \right] \\
 &= (16 - 4\sqrt{2}) \text{ units}
 \end{aligned}$$

Question 3:

Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y -axis in the first quadrant.

Answer



The area of the region bounded by the curve, $x^2 = 4y$, $y = 2$, and $y = 4$, and the y -axis is the area ABCD.

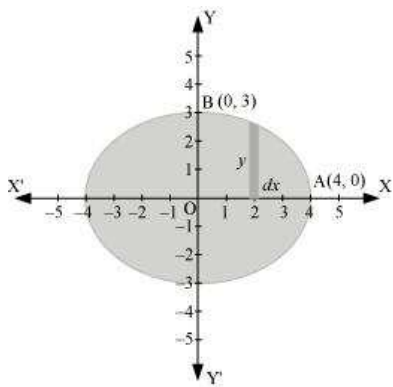
$$\begin{aligned}
 \text{Area of ABCD} &= \int_2^4 x \, dy \\
 &= \int_2^4 2\sqrt{y} \, dy \\
 &= 2 \int_2^4 \sqrt{y} \, dy \\
 &= 2 \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4 \\
 &= \frac{4}{3} \left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] \\
 &= \frac{4}{3} [8 - 2\sqrt{2}] \\
 &= \left(\frac{32 - 8\sqrt{2}}{3} \right) \text{ units}
 \end{aligned}$$

Question 4:

Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Answer

The given equation of the ellipse, $\frac{x^2}{16} + \frac{y^2}{9} = 1$, can be represented as



It can be observed that the ellipse is symmetrical about x-axis and y-axis.

\therefore Area bounded by ellipse = 4 \times Area of OAB

$$\begin{aligned}
 \text{Area of OAB} &= \int_0^4 y \, dx \\
 &= \int_0^4 3\sqrt{1 - \frac{x^2}{16}} \, dx \\
 &= \frac{3}{4} \int_0^4 \sqrt{16 - x^2} \, dx \\
 &= \frac{3}{4} \left[\frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_0^4 \\
 &= \frac{3}{4} [2\sqrt{16 - 16} + 8\sin^{-1}(1) - 0 - 8\sin^{-1}(0)] \\
 &= \frac{3}{4} \left[\frac{8\pi}{2} \right] \\
 &= \frac{3}{4} [4\pi] \\
 &= 3\pi
 \end{aligned}$$

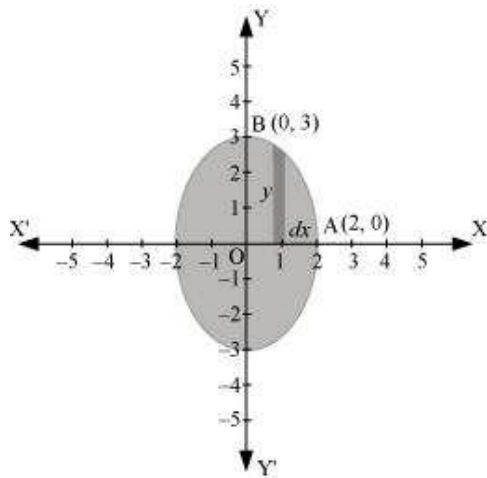
Therefore, area bounded by the ellipse = $4 \times 3\pi = 12\pi$ units

Question 5:

Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Answer

The given equation of the ellipse can be represented as



$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\Rightarrow y = 3\sqrt{1 - \frac{x^2}{4}} \quad \dots(1)$$

It can be observed that the ellipse is symmetrical about x-axis and y-axis.

\therefore Area bounded by ellipse = $4 \times$ Area OAB

$$\begin{aligned}
 \therefore \text{Area of OAB} &= \int_0^2 y \, dx \\
 &= \int_0^2 3\sqrt{1-\frac{x^2}{4}} \, dx \quad [\text{Using (1)}] \\
 &= \frac{3}{2} \int_0^2 \sqrt{4-x^2} \, dx \\
 &= \frac{3}{2} \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\
 &= \frac{3}{2} \left[\frac{2\pi}{2} \right] \\
 &= \frac{3\pi}{2}
 \end{aligned}$$

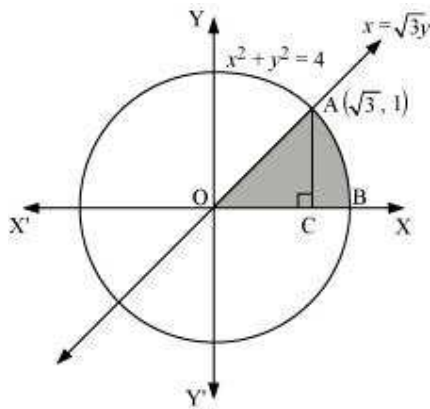
Therefore, area bounded by the ellipse = $4 \times \frac{3\pi}{2} = 6\pi$ units

Question 6:

Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$

Answer

The area of the region bounded by the circle, $x^2 + y^2 = 4, x = \sqrt{3}y$, and the x-axis is the area OAB.



The point of intersection of the line and the circle in the first quadrant is $(\sqrt{3}, 1)$.

Area OAB = Area Δ OCA + Area ACB

$$\text{Area of OAC} = \frac{1}{2} \times \text{OC} \times \text{AC} = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2} \quad \dots(1)$$

$$\begin{aligned} \text{Area of ABC} &= \int_{\sqrt{3}}^2 y \, dx \\ &= \int_{\sqrt{3}}^2 \sqrt{4-x^2} \, dx \\ &= \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{\sqrt{3}}^2 \\ &= \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} \sqrt{4-3} - 2 \sin^{-1} \left(\frac{\sqrt{3}}{2} \right) \right] \\ &= \left[\pi - \frac{\sqrt{3}\pi}{2} - 2 \left(\frac{\pi}{3} \right) \right] \\ &= \left[\pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} \right] \\ &= \left[\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right] \quad \dots(2) \end{aligned}$$

Therefore, area enclosed by x -axis, the line $x = \sqrt{3}y$, and the circle $x^2 + y^2 = 4$ in the first

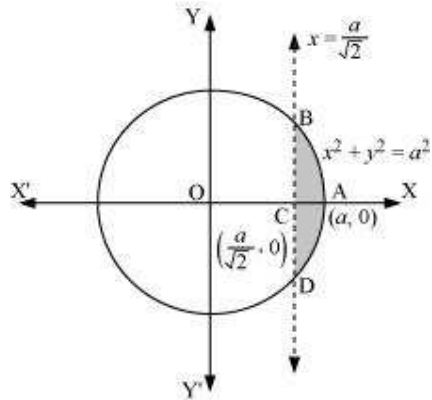
$$\text{quadrant} = \frac{\sqrt{3}\pi}{2} + \frac{3\sqrt{3}}{2} - \frac{2\pi}{3} = \frac{3\sqrt{3}\pi}{2} - \frac{2\pi}{3} \text{ units}$$

Question 7:

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$

Answer

The area of the smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$, is the area ABCDA.



It can be observed that the area ABCD is symmetrical about x -axis.

$$\therefore \text{Area ABCD} = 2 \times \text{Area ABC}$$

$$\begin{aligned}
 \text{Area of } ABC &= \int_{\frac{a}{\sqrt{2}}}^a y \, dx \\
 &= \int_{\frac{a}{\sqrt{2}}}^a \sqrt{a^2 - x^2} \, dx \\
 &= \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_{\frac{a}{\sqrt{2}}}^a \\
 &= \left[\frac{a^2}{2} \left(\frac{\pi}{2} \right) - \frac{a}{2\sqrt{2}} \sqrt{a^2 - \frac{a^2}{2}} - \frac{a^2}{2} \sin^{-1} \left(\frac{1}{\sqrt{2}} \right) \right] \\
 &= \frac{a^2 \pi}{4} - \frac{a}{2\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - \frac{a^2}{2} \left(\frac{\pi}{4} \right) \\
 &= \frac{a^2 \pi}{4} - \frac{a^2}{4} - \frac{a^2 \pi}{8} \\
 &= \frac{a^2}{4} \left[\pi - 1 - \frac{\pi}{2} \right] \\
 &= \frac{a^2}{4} \left[\frac{\pi}{2} - 1 \right] \\
 \Rightarrow \text{Area } ABCD &= 2 \left[\frac{a^2}{4} \left(\frac{\pi}{2} - 1 \right) \right] = \frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right)
 \end{aligned}$$

Therefore, the area of smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$, is $\frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right)$ units.

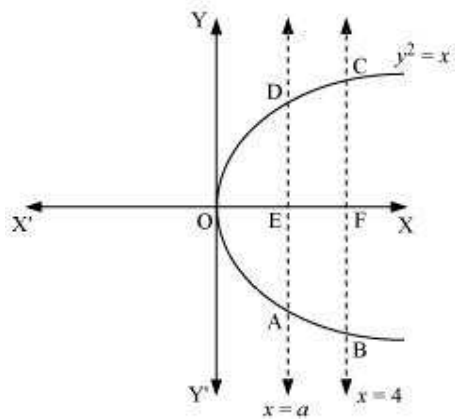
Question 8:

The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a .

Answer

The line, $x = a$, divides the area bounded by the parabola and $x = 4$ into two equal parts.

\therefore Area OAD = Area ABCD



It can be observed that the given area is symmetrical about x-axis.

\Rightarrow Area OED = Area EFCD

$$\begin{aligned}
 \text{Area } OED &= \int_0^a y \, dx \\
 &= \int_0^a \sqrt{x} \, dx \\
 &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^a \\
 &= \frac{2}{3}(a)^{\frac{3}{2}} \quad \dots(1)
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of } EFCD &= \int_0^4 \sqrt{x} \, dx \\
 &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 \\
 &= \frac{2}{3} \left[8 - a^{\frac{3}{2}} \right] \quad \dots(2)
 \end{aligned}$$

From (1) and (2), we obtain

$$\frac{2}{3}(a)^{\frac{3}{2}} = \frac{2}{3} \left[8 - (a)^{\frac{3}{2}} \right]$$

$$\Rightarrow 2 \cdot (a)^{\frac{3}{2}} = 8$$

$$\Rightarrow (a)^{\frac{3}{2}} = 4$$

$$\Rightarrow a = (4)^{\frac{2}{3}}$$

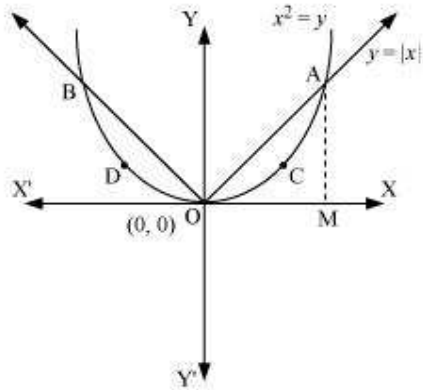
Therefore, the value of a is $(4)^{\frac{2}{3}}$.

Question 9:

Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$

Answer

The area bounded by the parabola, $x^2 = y$, and the line, $y = |x|$, can be represented as



The given area is symmetrical about y-axis.

$$\therefore \text{Area OACO} = \text{Area ODBO}$$

The point of intersection of parabola, $x^2 = y$, and line, $y = x$, is A (1, 1).

$$\text{Area of OACO} = \text{Area } \triangle OAB - \text{Area OBACO}$$

$$\therefore \text{Area of } \triangle OAB = \frac{1}{2} \times OB \times AB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$$

$$\text{Area of OBACO} = \int_0^1 y \, dx = \int_0^1 x^2 \, dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3}$$

$$\Rightarrow \text{Area of OACO} = \text{Area of } \triangle OAB - \text{Area of OBACO}$$

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6}$$

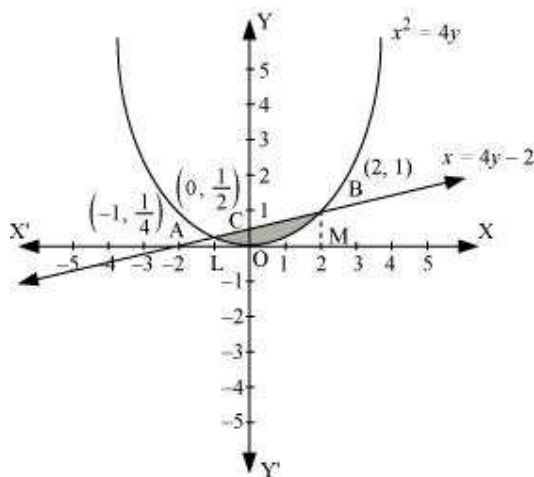
$$\text{Therefore, required area} = 2 \left[\frac{1}{6} \right] = \frac{1}{3} \text{ units}$$

Question 10:

Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

Answer

The area bounded by the curve, $x^2 = 4y$, and line, $x = 4y - 2$, is represented by the shaded area OBAO.



Let A and B be the points of intersection of the line and parabola.

Coordinates of point A are $\left(-1, \frac{1}{4}\right)$.

Coordinates of point B are $(2, 1)$.

We draw AL and BM perpendicular to x-axis.

It can be observed that,

Area OBAO = Area OBCO + Area OACO ... (1)

Then, Area OBCO = Area OMBC – Area OMBO

$$\begin{aligned}
 &= \int_0^2 \frac{x+2}{4} dx - \int_0^2 \frac{x^2}{4} dx \\
 &= \frac{1}{4} \left[\frac{x^2}{2} + 2x \right]_0^2 - \frac{1}{4} \left[\frac{x^3}{3} \right]_0^2 \\
 &= \frac{1}{4} [2+4] - \frac{1}{4} \left[\frac{8}{3} \right] \\
 &= \frac{3}{2} - \frac{2}{3} \\
 &= \frac{5}{6}
 \end{aligned}$$

Similarly, Area OACO = Area OLAC – Area OLAO

$$\begin{aligned}
 &= \int_{-1}^0 \frac{x+2}{4} dx - \int_{-1}^0 \frac{x^2}{4} dx \\
 &= \frac{1}{4} \left[\frac{x^2}{2} + 2x \right]_{-1}^0 - \frac{1}{4} \left[\frac{x^3}{3} \right]_{-1}^0 \\
 &= -\frac{1}{4} \left[\frac{(-1)^2}{2} + 2(-1) \right] - \left[-\frac{1}{4} \left(\frac{(-1)^3}{3} \right) \right] \\
 &= -\frac{1}{4} \left[\frac{1}{2} - 2 \right] - \frac{1}{12} \\
 &= \frac{1}{2} - \frac{1}{8} - \frac{1}{12} \\
 &= \frac{7}{24}
 \end{aligned}$$

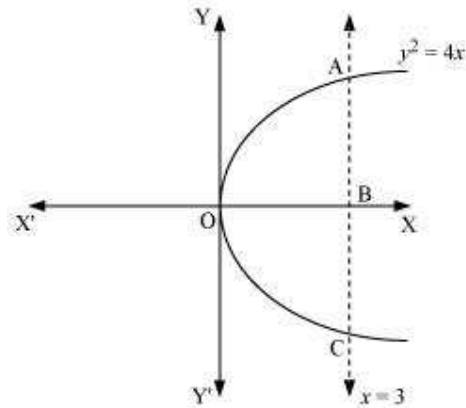
Therefore, required area = $\left(\frac{5}{6} + \frac{7}{24} \right) = \frac{9}{8}$ units

Question 11:

Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$

Answer

The region bounded by the parabola, $y^2 = 4x$, and the line, $x = 3$, is the area OACO.



The area OACO is symmetrical about x -axis.

\therefore Area of OACO = 2 (Area of OAB)

$$\begin{aligned}
 \text{Area OACO} &= 2 \left[\int_0^3 y \, dx \right] \\
 &= 2 \int_0^3 2\sqrt{x} \, dx \\
 &= 4 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^3 \\
 &= \frac{8}{3} \left[(3)^{\frac{3}{2}} \right] \\
 &= 8\sqrt{3}
 \end{aligned}$$

Therefore, the required area is $8\sqrt{3}$ units.

Question 12:

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

A. π

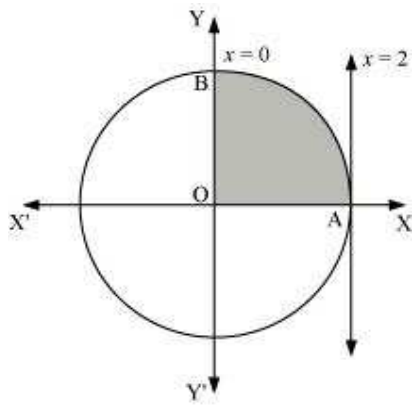
B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer

The area bounded by the circle and the lines, $x = 0$ and $x = 2$, in the first quadrant is represented as



$$\begin{aligned}
 \therefore \text{Area OAB} &= \int_0^2 y \, dx \\
 &= \int_0^2 \sqrt{4-x^2} \, dx \\
 &= \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\
 &= 2 \left(\frac{\pi}{2} \right) \\
 &= \pi \text{ units}
 \end{aligned}$$

Thus, the correct answer is A.

Question 13:

Area of the region bounded by the curve $y^2 = 4x$, y -axis and the line $y = 3$ is

A. 2

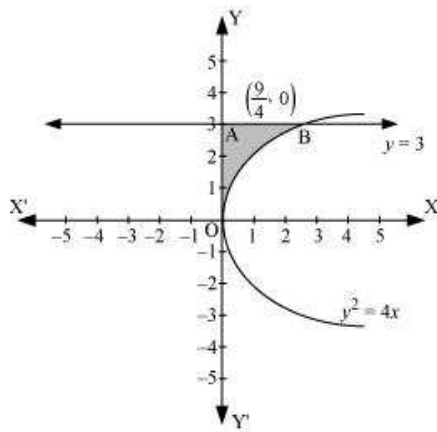
B. $\frac{9}{4}$

C. $\frac{9}{3}$

D. $\frac{9}{2}$

Answer

The area bounded by the curve, $y^2 = 4x$, y -axis, and $y = 3$ is represented as



$$\begin{aligned}
 \therefore \text{Area OAB} &= \int_0^3 x \, dy \\
 &= \int_0^3 \frac{y^2}{4} \, dy \\
 &= \frac{1}{4} \left[\frac{y^3}{3} \right]_0^3 \\
 &= \frac{1}{12} (27) \\
 &= \frac{9}{4} \text{ units}
 \end{aligned}$$

Thus, the correct answer is B.