Exercise 9.3

Question 1:

$$\frac{x}{a}+\frac{y}{b}=1$$

Answer

$$\frac{x}{a}+\frac{y}{b}=1$$

Differentiating both sides of the given equation with respect to *x*, we get:

$$\frac{1}{a}+\frac{1}{b}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{1}{a}+\frac{1}{b}y'=0$$

Again, differentiating both sides with respect to *x*, we get:

$$0+\frac{1}{b}y''=0$$

$$\Rightarrow \frac{1}{b}y''=0$$

$$\Rightarrow y''=0$$

Hence, the required differential equation of the given curve is $y''=0.$

Question 2:

$$y^2=a\left(b^2-x^2\right)$$

Answer

$$y^2=a\left(b^2-x^2\right)$$

Differentiating both sides with respect to *x*, we get:

$$2y\frac{dy}{dx}=a\left(-2x\right)$$

$$\Rightarrow 2yy'=-2ax$$

$$\Rightarrow yy'=-ax \qquad ...(1)$$

Again, differentiating both sides with respect to *x*, we get:

$y' \cdot y' + yy'' = -a$

$\Rightarrow (y')^2 + yy'' = -a$ ...(2)

Dividing equation (2) by equation (1), we get:

$$\frac{(y')^2 + yy''}{yy'} = \frac{-a}{-ax}$$

$\Rightarrow xyy'' + x(y')^2 - yy'' = 0$

This is the required differential equation of the given curve.

Question 3:

$y = a\,e^{3x} + b\,e^{-2x}$

Answer

$y = ae^{3x} + be^{-2x}$ ...(1)

Differentiating both sides with respect to *x*, we get:

$y' = 3ae^{3x} - 2be^{-2x}$ ...(2)

Again, differentiating both sides with respect to *x*, we get:

$y'' = 9ae^{3x} + 4be^{-2x}$ ...(3)

Multiplying equation (1) with (2) and then adding it to equation (2), we get:

$(2ae^{3x} + 2be^{-2x}) + (3ae^{3x} - 2bc^{-2x}) = 2y + y'$

$\Rightarrow 5ae^{3x} = 2y + y'$

$$\Rightarrow ae^{3x} = \frac{2y + y'}{5}$$

Now, multiplying equation (1) with equation (3) and subtracting equation (2) from it, we get:

$(3ae^{3x} + 3be^{-2x}) - (3ae^{3x} - 2be^{-2x}) = 3y - y'$

$\Rightarrow 5be^{-2x} = 3y - y'$

$$\Rightarrow be^{-2x} = \frac{3y - y'}{5}$$

Substituting the values of ae^{3x} and be^{-2x} in equation (3), we get:

$$y'' = 9 \cdot \frac{(2y + y')}{5} + 4\frac{(3y - y')}{5}$$
$$\Rightarrow y'' = \frac{18y + 9y'}{5} + \frac{12y - 4y'}{5}$$
$$\Rightarrow y'' = \frac{30y + 5y'}{5}$$
$$\Rightarrow y'' = 6y + y'$$
$$\Rightarrow y'' - y' - 6y = 0$$

This is the required differential equation of the given curve.

Question 4:

$$y = e^{2x}(a + bx)$$

Answer

$$y = e^{2x}(a + bx) \qquad ...(1)$$

Differentiating both sides with respect to *x*, we get:

$$y' = 2e^{2x}(a + bx) + e^{2x} \cdot b$$
$$\Rightarrow y' = e^{2x}(2a + 2bx + b) \qquad ...(2)$$

Multiplying equation (1) with equation (2) and then subtracting it from equation (2), we get:

$$y' - 2y = e^{2x}(2a + 2bx + b) - e^{2x}(2a + 2bx)$$
$$\Rightarrow y' - 2 = be^{2x} \qquad ...(3)$$

Differentiating both sides with respect to *x*, we get:

$$y''k - 2y' = 2be^{2x} \qquad ...(4)$$

Dividing equation (4) by equation (3), we get:

$$\frac{y'' - 2y'}{y' - 2y} = 2$$
$$\Rightarrow y'' - 2y' = 2y' - 4y$$
$$\Rightarrow y'' - 4y' + 4y = 0$$

This is the required differential equation of the given curve.

Question 5:

$y = e^x(a\cos x + b\sin x)$

Answer

$$y = e^x(a\cos x + b\sin x) \quad ...(1)$$

Differentiating both sides with respect to *x*, we get:

$$y' = e^x(a\cos x + b\sin x) + e^x(-a\sin x + b\cos x)$$
$$\Rightarrow y' = e^x\left[(a+b)\cos x - (a-b)\sin x\right] \quad ...(2)$$

Again, differentiating with respect to *x*, we get:

$$y'' = e^x\left[(a+b)\cos x - (a-b)\sin x\right] + e^x\left[-(a+b)\sin x - (a-b)\cos x\right]$$
$$y'' = e^x\left[2b\cos x - 2a\sin x\right]$$
$$y'' = 2e^x(b\cos x - a\sin x)$$
$$\Rightarrow \frac{y''}{2} = e^x(b\cos x - a\sin x) \quad ...(3)$$

Adding equations (1) and (3), we get:

$$y + \frac{y''}{2} = e^x\left[(a+b)\cos x - (a-b)\sin x\right]$$
$$\Rightarrow y + \frac{y''}{2} = y'$$
$$\Rightarrow 2y + y'' = 2y'$$
$$\Rightarrow y'' - 2y' + 2y = 0$$

This is the required differential equation of the given curve.

Question 6:

Form the differential equation of the family of circles touching the *y*-axis at the origin.

Answer

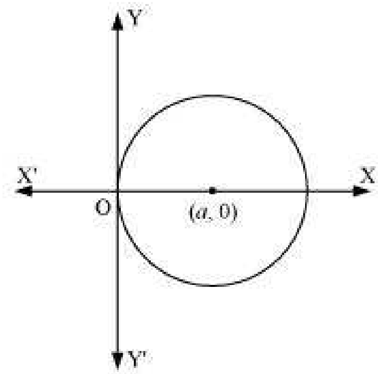
The centre of the circle touching the *y*-axis at origin lies on the *x*-axis.

Let (*a*, 0) be the centre of the circle.

Since it touches the *y*-axis at origin, its radius is *a*.

Now, the equation of the circle with centre (*a*, 0) and radius (*a)* is

$$(x-a)^2 + y^2 = a^2.$$
$$\Rightarrow x^2 + y^2 = 2ax \quad ...(1)$$



Differentiating equation (1) with respect to *x*, we get:

$2x + 2yy' = 2a$

$\Rightarrow x + yy' = a$

Now, on substituting the value of *a* in equation (1), we get:

$x^2 + y^2 = 2(x + yy')x$

$\Rightarrow x^2 + y^2 = 2x^2 + 2xyy'$

$\Rightarrow 2xyy' + x^2 = y^2$

This is the required differential equation.

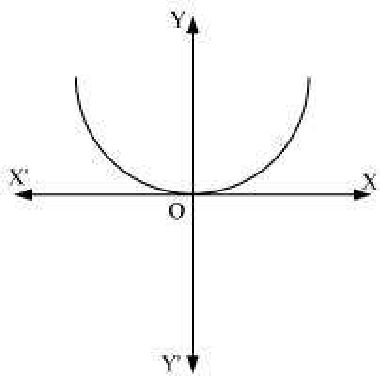
Question 7:

Form the differential equation of the family of parabolas having vertex at origin and axis along positive *y*-axis.

Answer

The equation of the parabola having the vertex at origin and the axis along the positive *y*-axis is:

$x^2 = 4ay \qquad ...(1)$



Differentiating equation (1) with respect to *x*, we get:

$2x = 4ay'$ ...(2)

Dividing equation (2) by equation (1), we get:

$$\frac{2x}{x^2} = \frac{4ay'}{4ay}$$

$$\Rightarrow \frac{2}{x} = \frac{y'}{y}$$

$$\Rightarrow xy' = 2y$$

$$\Rightarrow xy' - 2y = 0$$

This is the required differential equation.

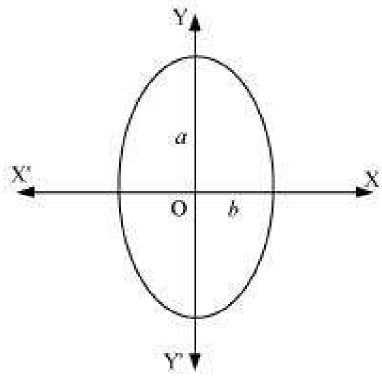
Question 8:

Form the differential equation of the family of ellipses having foci on *y*-axis and centre at origin.

Answer

The equation of the family of ellipses having foci on the *y*-axis and the centre at origin is as follows:

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ . ..(1)



Differentiating equation (1) with respect to *x*, we get:

$$\frac{2x}{b^2}+\frac{2yy'}{b^2}=0$$

$$\Rightarrow \frac{x}{b^2}+\frac{yy'}{a^2}=0 \qquad ...(2)$$

Again, differentiating with respect to *x*, we get:

$$\frac{1}{b^2}+\frac{y'.y'+y.y''}{a^2}=0$$

$$\Rightarrow \frac{1}{b^2}+\frac{1}{a^2}\left(y'^2+yy''\right)=0$$

$$\Rightarrow \frac{1}{b^2}=-\frac{1}{a^2}\left(y'^2+yy''\right)$$

Substituting this value in equation (2), we get:

$$x\left[-\frac{1}{a^2}\left(\left(y'\right)^2+yy''\right)\right]+\frac{yy'}{a^2}=0$$

$$\Rightarrow -x\left(y'\right)^2-xyy''+yy'=0$$

$$\Rightarrow xyy''+x\left(y'\right)^2-yy'=0$$

This is the required differential equation.

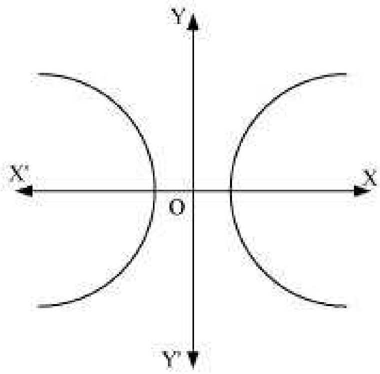
Question 9:

Form the differential equation of the family of hyperbolas having foci on *x*-axis and centre at origin.

Answer

The equation of the family of hyperbolas with the centre at origin and foci along the *x*-axis is:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad ...(1)$$



Differentiating both sides of equation (1) with respect to *x*, we get:

$$\frac{2x}{a^2}-\frac{2yy'}{b^2}=0$$

$$\Rightarrow \frac{x}{a^2}-\frac{yy'}{b^2}=0 \qquad ...(2)$$

Again, differentiating both sides with respect to *x*, we get:

$$\frac{1}{a^2}-\frac{y'\cdot y'+yy''}{b^2}=0$$

$$\Rightarrow \frac{1}{a^2}=\frac{1}{b^2}\left((y')^2+yy''\right)$$

Substituting the value of $\frac{1}{a^2}$ in equation (2), we get:

$$\frac{x}{b^2}\left((y')^2+yy''\right)-\frac{yy'}{b^2}=0$$

$$\Rightarrow x(y')^2+xyy''-yy'=0$$

$$\Rightarrow xyy''+x(y')^2-yy'=0$$

This is the required differential equation.

Question 10:

Form the differential equation of the family of circles having centre on *y*-axis and radius 3 units.

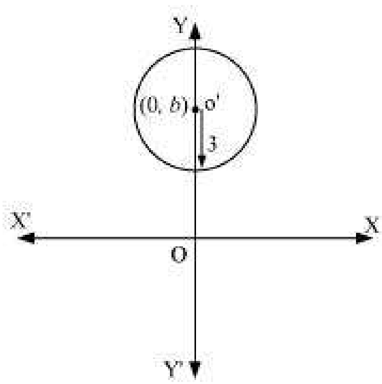
Answer

Let the centre of the circle on *y*-axis be (0, *b*).

The differential equation of the family of circles with centre at (0, *b*) and radius 3 is as follows:

$$x^2+(y-b)^2=3^2$$

$$\Rightarrow x^2+(y-b)^2=9 \qquad ...(1)$$



Differentiating equation (1) with respect to *x*, we get:

$$2x+2(y-b)\cdot y'=0$$

$$\Rightarrow (y-b)\cdot y'=-x$$

$$\Rightarrow y-b=\frac{-x}{y'}$$

Substituting the value of (*y* – *b*) in equation (1), we get:

$$x^2 + \left(\frac{-x}{y'}\right)^2 = 9$$

$$\Rightarrow x^2\left[1 + \frac{1}{(y')^2}\right] = 9$$

$$\Rightarrow x^2\left((y')^2 + 1\right) = 9(y')^2$$

$$\Rightarrow \left(x^2 - 9\right)(y')^2 + x^2 = 0$$

This is the required differential equation.

Question 11:

Which of the following differential equations has $y = c_1e^x + c_2e^{-x}$ as the general solution?

A. $\frac{d^2y}{dx^2} + y = 0$

B. $\frac{d^2y}{dx^2} - y = 0$

C. $\frac{d^2y}{dx^2} + 1 = 0$

D. $\frac{d^2y}{dx^2} - 1 = 0$

Answer

The given equation is:

$$y = c_1e^x + c_2e^{-x} \qquad ...(1)$$

Differentiating with respect to *x*, we get:

$$\frac{dy}{dx} = c_1e^x - c_2e^{-x}$$

Again, differentiating with respect to *x*, we get:

$$\frac{d^2y}{dx^2} = c_1e^x + c_2e^{-x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = y$$

$$\Rightarrow \frac{d^2y}{dx^2} - y = 0$$

This is the required differential equation of the given equation of curve.

Hence, the correct answer is B.

Question 12:

Which of the following differential equation has $y = x$ as one of its particular solution?

A. $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$

B. $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$

C. $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$

D. $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$

Answer

The given equation of curve is $y = x$.

Differentiating with respect to x, we get:

$$\frac{dy}{dx} = 1 \qquad ...(1)$$

Again, differentiating with respect to x, we get:

$$\frac{d^2y}{dx^2} = 0 \qquad ...(2)$$

Now, on substituting the values of y, $\frac{d^2y}{dx^2}$, and $\frac{dy}{dx}$ from equation (1) and (2) in each of the given alternatives, we find that only the differential equation given in alternative **C** is correct.

$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0 - x^2 \cdot 1 + x \cdot x$$
$$= -x^2 + x^2$$
$$= 0$$

Hence, the correct answer is C.