Question 1:

$y = e^x + 1 \qquad : \quad y'' - y' = 0$

Answer

$y = e^x + 1$

Differentiating both sides of this equation with respect to *x*, we get:

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x + 1\right)$$

$$\Rightarrow y' = e^x \qquad\qquad \ldots(1)$$

Now, differentiating equation (1) with respect to *x*, we get:

$$\frac{d}{dx}(y') = \frac{d}{dx}\left(e^x\right)$$

$$\Rightarrow y'' = e^x$$

Substituting the values of y' and y'' in the given differential equation, we get the L.H.S.

as:

$$y'' - y' = e^x - e^x = 0 = \text{R.H.S.}$$

Thus, the given function is the solution of the corresponding differential equation.

Question 2:

$y = x^2 + 2x + C \qquad : \quad y' - 2x - 2 = 0$

Answer

$y = x^2 + 2x + C$

Differentiating both sides of this equation with respect to *x*, we get:

$$y' = \frac{d}{dx}\left(x^2 + 2x + C\right)$$

$$\Rightarrow y' = 2x + 2$$

Substituting the value of y' in the given differential equation, we get:

L.H.S. $= y' - 2x - 2 = 2x + 2 - 2x - 2 = 0$ = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 3:

$$y = \cos x + C \qquad : \quad y' + \sin x = 0$$

Answer

$$y = \cos x + C$$

Differentiating both sides of this equation with respect to *x*, we get:

$$y' = \frac{d}{dx}(\cos x + C)$$
$$\Rightarrow y' = -\sin x$$

Substituting the value of y' in the given differential equation, we get:

L.H.S. $= y' + \sin x = -\sin x + \sin x = 0$ = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 4:

$$y = \sqrt{1 + x^2} \quad : \quad y' = \frac{xy}{1 + x^2}$$

Answer

$$y = \sqrt{1 + x^2}$$

Differentiating both sides of the equation with respect to *x*, we get:

$$y' = \frac{d}{dx}\left(\sqrt{1+x^2}\right)$$

$$y' = \frac{1}{2\sqrt{1+x^2}} \cdot \frac{d}{dx}(1+x^2)$$

$$y' = \frac{2x}{2\sqrt{1+x^2}}$$

$$y' = \frac{x}{\sqrt{1+x^2}}$$

$$\Rightarrow y' = \frac{x}{1+x^2} \times \sqrt{1+x^2}$$

$$\Rightarrow y' = \frac{x}{1+x^2} \cdot y$$

$$\Rightarrow y' = \frac{xy}{1+x^2}$$

$\therefore$ L.H.S. = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 5:

$$y = Ax \qquad : \quad xy' = y\,(x \neq 0)$$

Answer

$$y = Ax$$

Differentiating both sides with respect to *x*, we get:

$$y' = \frac{d}{dx}(Ax)$$

$$\Rightarrow y' = A$$

Substituting the value of y' in the given differential equation, we get:

$$\text{L.H.S.} = xy' = x \cdot A = Ax = y = \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 6:

$$y = x\sin x \qquad : \quad xy' = y + x\sqrt{x^2 - y^2}\ (x \neq 0 \text{ and } x > y \text{ or } x < -y)$$

Answer

$y = x \sin x$

Differentiating both sides of this equation with respect to x, we get:

$$y' = \frac{d}{dx}(x \sin x)$$

$$\Rightarrow y' = \sin x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\sin x)$$

$$\Rightarrow y' = \sin x + x \cos x$$

Substituting the value of y' in the given differential equation, we get:

$$\text{L.H.S.} = xy' = x(\sin x + x \cos x)$$

$$= x \sin x + x^2 \cos x$$

$$= y + x^2 \cdot \sqrt{1 - \sin^2 x}$$

$$= y + x^2 \sqrt{1 - \left(\frac{y}{x}\right)^2}$$

$$= y + x\sqrt{y^2 - x^2}$$

$$= \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 7:

$$xy = \log y + C \qquad : \quad y' = \frac{y^2}{1 - xy} \, (xy \neq 1)$$

Answer

$xy = \log y + C$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y)$$

$$\Rightarrow y \cdot \frac{d}{dx}(x) + x \cdot \frac{dy}{dx} = \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow y + xy' = \frac{1}{y}y'$$

$$\Rightarrow y^2 + xy\,y' = y'$$

$$\Rightarrow (xy-1)y' = -y^2$$

$$\Rightarrow y' = \frac{y^2}{1-xy}$$

$\therefore$ L.H.S. = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 8:

$$y - \cos y = x \qquad\qquad : \quad (y\sin y + \cos y + x)y' = y$$

Answer

$$y - \cos y = x \qquad\qquad\qquad ...(1)$$

Differentiating both sides of the equation with respect to *x*, we get:

$$\frac{dy}{dx} - \frac{d}{dx}(\cos y) = \frac{d}{dx}(x)$$

$$\Rightarrow y' + \sin y \cdot y' = 1$$

$$\Rightarrow y'(1 + \sin y) = 1$$

$$\Rightarrow y' = \frac{1}{1 + \sin y}$$

Substituting the value of y' in equation (1), we get:

$$\text{L.H.S.} = \left(y \sin y + \cos y + x\right) y'$$

$$= \left(y \sin y + \cos y + y - \cos y\right) \times \frac{1}{1 + \sin y}$$

$$= y\left(1 + \sin y\right) \cdot \frac{1}{1 + \sin y}$$

$$= y$$

$$= \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 9:

$$x + y = \tan^{-1} y \qquad : \quad y^2 y' + y^2 + 1 = 0$$

Answer

$$x + y = \tan^{-1} y$$

Differentiating both sides of this equation with respect to *x*, we get:

$$\frac{d}{dx}\left(x + y\right) = \frac{d}{dx}\left(\tan^{-1} y\right)$$

$$\Rightarrow 1 + y' = \left[\frac{1}{1 + y^2}\right] y'$$

$$\Rightarrow y'\left[\frac{1}{1 + y^2} - 1\right] = 1$$

$$\Rightarrow y'\left[\frac{1 - \left(1 + y^2\right)}{1 + y^2}\right] = 1$$

$$\Rightarrow y'\left[\frac{-y^2}{1 + y^2}\right] = 1$$

$$\Rightarrow y' = \frac{-\left(1 + y^2\right)}{y^2}$$

Substituting the value of y' in the given differential equation, we get:

$$\text{L.H.S.} = y^2 y' + y^2 + 1 = y^2 \left[\frac{-\left(1+y^2\right)}{y^2} \right] + y^2 + 1$$

$$= -1 - y^2 + y^2 + 1$$

$$= 0$$

$$= \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 10:

$$y = \sqrt{a^2 - x^2}\, x \in (-a, a) \qquad : \quad x + y\frac{dy}{dx} = 0\,(y \neq 0)$$

Answer

$$y = \sqrt{a^2 - x^2}$$

Differentiating both sides of this equation with respect to *x*, we get:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{a^2 - x^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}} \cdot \frac{d}{dx}\left(a^2 - x^2\right)$$

$$= \frac{1}{2\sqrt{a^2 - x^2}}(-2x)$$

$$= \frac{-x}{\sqrt{a^2 - x^2}}$$

Substituting the value of $\dfrac{dy}{dx}$ in the given differential equation, we get:

$$\text{L.H.S.} = x + y\frac{dy}{dx} = x + \sqrt{a^2 - x^2} \times \frac{-x}{\sqrt{a^2 - x^2}}$$

$$= x - x$$

$$= 0$$

$$= \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 11:

The numbers of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4

Answer

We know that the number of constants in the general solution of a differential equation of order n is equal to its order.

Therefore, the number of constants in the general equation of fourth order differential equation is four.

Hence, the correct answer is D.

Question 12:

The numbers of arbitrary constants in the particular solution of a differential equation of third order are:

(A) 3 (B) 2 (C) 1 (D) 0

Answer

In a particular solution of a differential equation, there are no arbitrary constants.

Hence, the correct answer is D.