Ex - 10.2

Q1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is -

(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm

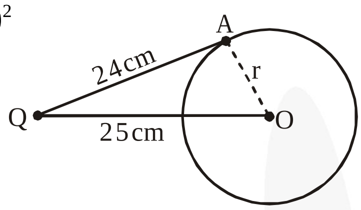
Sol. From figure,

$r^2 = (25)^2 - (24)^2$

$= 625 - 576$

$= 49$

$\Rightarrow r = 7$ cm



Hence, the correct option is (A)

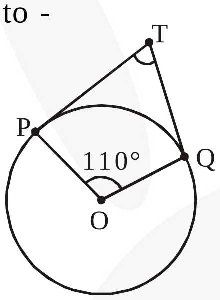
Q2. In fig., if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to -

(A) 60°

(B) 70°

(C) 80°

(D) 90°



Sol. TQ and TP are tangents to a circle with centre O and $\angle POQ = 110°$

$\therefore$ OP $\perp$ PT and OQ $\perp$ QT

$\Rightarrow \angle OPT = 90°$ and $\angle OQT = 90°$

Now, in the quadrilateral TPOQ, we get

$\therefore$ PTQ + 90° + 110° + 90° = 360°

[Angle sum property of a quadrilateral]

$\Rightarrow \angle PTQ + 290° = 360°$

$\Rightarrow \angle PTQ = 360° - 290° = 70°$

Hence, the correct option is (B)

Q3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to

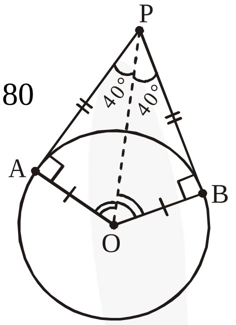
(A) 50° (B) 60° (C) 70° (D) 80°

Sol. In figure,

ΔOAP ≅ ΔOBP (SSS congruence)

⇒ ∠POA = ∠POB

$= \frac{1}{2}$ ∠AOB ...(1)



Also ∠AOB + ∠APB = 180

⇒ ∠AOB + 80° = 180°

⇒ ∠AOB = 100° ...(2)

Then from (1) and (2)

∠POA = $\frac{1}{2}$ ×. 100 = 50°

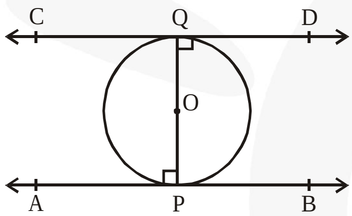
Hence, the correction option is (A)

Q4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Sol. In the figure, PQ is diameter of the given circle and O is its centre.

Let tangents AB and CD be drawn at the end points of the diameter PQ.

Since, the tangents at a point to a circle is perpendicular to the radius through the point.



∴ PQ ⊥ AB

⇒ APQ = 90° and PQ ⊥ CD

⇒ ∠PQD = 90°

⇒ ∠APQ = ∠PQD

But they form a pair of alternate angles.

∴ AB || CD.

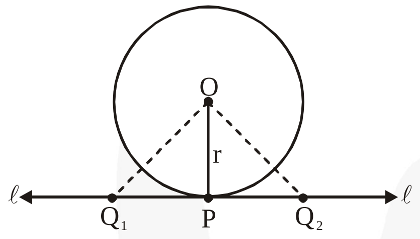
Hence, the two tangents are parallel.

Q5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Sol. In figure, line ℓ is tangent to the circle at P. O is the centre of the circle.

OP = radius of the circle.

If we have some points Q_1, Q_2, etc. on ℓ, then we find that OP is the shortest distance from O in comparison to the distances OQ_1, OQ_2, etc. Therefore, $OP \perp \ell$. Hence, the perpendicular OP drawn to the tangent line at P passes through the centre O of the circle.



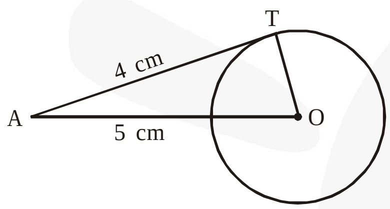
Q6. The length of a tangent from a point A at a distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Sol. The tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OTA = 90°$

Now, in the right ΔOTA, we have :

$OA^2 = OT^2 + AT^2$ [Pythagoras theorem]



$\Rightarrow \quad 5^2 = OT^2 + 4^2$

$\Rightarrow \quad OT^2 = 5^2 - 4^2$

$\Rightarrow \quad OT^2 = (5 - 4)(5 + 4)$

$\Rightarrow \quad OT^2 = 1 \times 9 = 9 = 3^2$

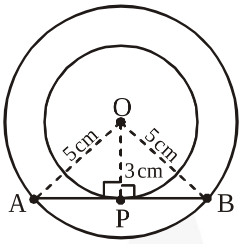
$\Rightarrow \quad OT = 3$

Thus, the radius of the circle is 3 cm.

Q7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Sol. In fig. the two concentric circles have their centre at O. The radius of the larger circle is 5 cm and that of the smaller circle is 3 cm.

AB is a chord of the larger circle and it touches the smaller circle at P.



Join OA, OB and OP.

Now, OA = OB = 5 cm,

OP = 3 cm

and OP ⊥ AB,

i.e., ∠OPA = ∠OPB = 90°

$\Rightarrow$ ΔOAP ≅ ΔOBP (RHS congruence)

$\Rightarrow$ $AP = BP = \frac{1}{2}AB$ or $AB = 2\,AP$

By Pythagoras theorem,

$OA^2 = AP^2 + OP^2$

$\Rightarrow$ $(5)^2 = AP^2 + (3)^2$

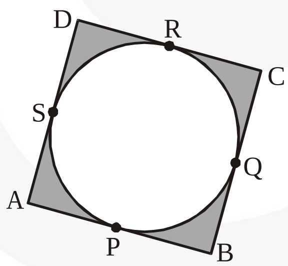
$\Rightarrow$ $AP^2 = 25 - 9 = 16$

$\Rightarrow$ AP = 4 cm

$\Rightarrow$ AB = 2 × 4 cm = 8 cm

Q8. A quadrilateral ABCD is drawn to circumscribe a circle (see fig.).

Prove that AB + CD = AD + BC.



Sol. In fig., we observe that

AP = AS ...(1)

{∵ AP and AS are tangents to the circle drawn from the point A}

Similarly, BP = BQ ...(2)

CR = CQ ...(3)

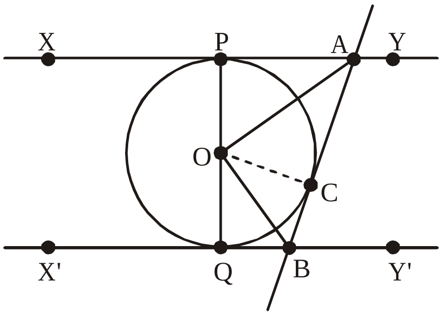
DR = DS ...(4)

Adding (1), (2), (3), (4), we have

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

Q9. In fig., XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that ∠AOB = 90°



Sol. In fig., Join OC and we have Δs AOP and AOC for which

AP = AC (Both tangents from A)

OP = OC (Each = radius)

OA = OA (Common side)

⇒ ΔAOP ≅ ΔAOC (SSS congruence)

⇒ ∠PAO = ∠CAO

⇒ ∠PAC = 2∠OAC ...(1)

Similarly, ∠QBC = 2∠OBC ...(2)

Adding (1) and (2),

∠PAC + ∠QBC = 2 {∠OAC + ∠OBC}

⇒ 180° = 2 {∠OAC + ∠OBC}

(∵ in quadrilateral PABQ, ∠P = ∠Q = 90°}

⇒ $\angle OAC + \angle OBC = \frac{1}{2} \times 180° = 90°$ ...(3)

Now, in ΔAOB we have

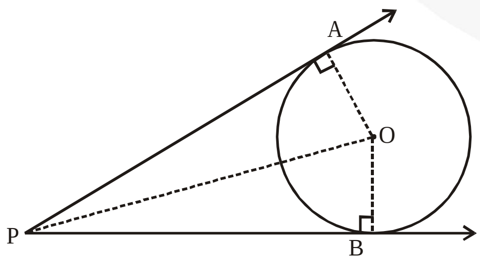
∠AOB + ∠OAC + ∠OBC = 180°

⇒ ∠AOB + 90° = 180° (By (3))

⇒ ∠AOB = 90°

Q10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Sol. Let PA and PB be two tangents drawn from an external point P to a circle with centre O.



Now, in right ΔOAP and right ΔOBP, we have

PA = PB [Tangents to circle from an external point]

OA = OB [Radii of the same circle]

OP = OP [Common]

$\Delta OAP \cong \Delta OBP$ [By SSS congruency]

$\therefore \angle OPA = \angle OPB$ [By C.P.C.T.]

and $\angle AOP = \angle BOP$

$\Rightarrow \angle APB = 2\angle OPA$ and $\angle AOB = 2\angle AOP$

But $\angle AOP = 90° - \angle OPA$

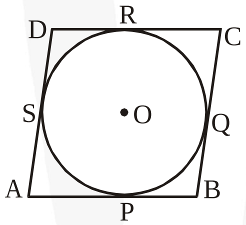
$\Rightarrow 2\angle AOP = 180° - 2\angle OPA$

$\Rightarrow \angle AOB = 180° - \angle APB$

$\Rightarrow \angle AOB + \angle APB = 180°$ (Proved)

Q11. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol. Let ABCD be a parallelogram such that its sides touch a circle with centre O.



AP = AS [Tangents from an external point are equal]

BP = BQ

CR = CQ

DR = DS

Adding these equations

AP + BP + CR + DR = AS + DS + BQ + CQ

AB + CD = AD + BC

2AB = 2BC

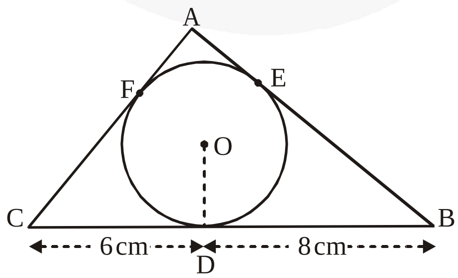
AB = BC

$\Rightarrow$ AB = BC = CD = DA

$\Rightarrow$ ABCD is a rhombus.

Hence proved

Q12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see fig.). Find the sides AB and AC.



Sol. In fig. BD = 8 cm and DC = 6 cm

Then we have BE = 8 cm ($\because$ BE = BD)

and CF = 6 cm ($\because$ CF = CD)

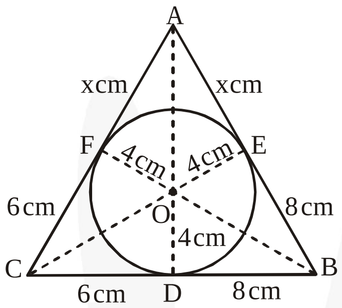
Suppose AE = AF = x cm

In ΔABC, a = BC = 6 cm + 8 cm = 14 cm

b = CA = (x + 6) cm, c = AB = (x + 8) cm

$$s = \frac{a+b+c}{2} = \frac{14+(x+6)+(x+8)}{2} \text{ cm}$$

$$= \frac{2x+28}{2} \text{ cm} = (x + 14) \text{ cm}$$



Area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{(x+14) \times x \times 8 \times 6}$

$= \sqrt{48x \times (x+14)} \text{ cm}^2$ ...(1)

Also, area of ΔABC = area of ΔOBC + area of ΔOCA + area of ΔOAB

$= \frac{1}{2} \times 4 \times a + \frac{1}{2} \times 4 \times b + \frac{1}{2} \times 4 \times c$

$= 2(a + b + c) = 2 \times 2s = 4s$

$= 4(x + 14) \text{ cm}^2$ ...(2)

From (1) and (2), $\sqrt{48x \times (x+14)} = 4 \times (x + 14)$

$\Rightarrow 48x \times (x + 14) = 16 \times (x + 14)^2$

$\Rightarrow 3x = x + 14 \quad \Rightarrow x = 7$ cm

Then AB = c = (x + 8) cm = (7 + 8) cm = 15 cm

and AC = b = (x + 6) cm = (7 + 6) cm = 13 cm