Ex - 7.3

Q1. Find the area of the triangle whose vertices are :

(i) (2,3), (–1, 0), (2, –4) (ii) (– 5, – 1), (3,–5), (5,2)

Sol. (i) Let the vertices of the triangles be A(2, 3), B (–1, 0) and C(2, –4)

Here $x_1 = 2$, $y_1 = 3$,

$x_2 = -1$, $y_2 = 0$

$x_3 = 2$, $y_3 = -4$

$\because$ Area of a Δ

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\therefore$ Area of a Δ

$= \frac{1}{2}[2\{0-(-4)\} + (-1)\{-4-(3)\} + 2\{3-0\}]$

$= \frac{1}{2}[2(0+4) + (-1)(-4-3) + 2(3)]$

$= \frac{1}{2}[8+7+6] = \frac{1}{2}[21] = \frac{21}{2}$ sq.units

(ii) A(– 5, – 1), B (3, –5), C (5, 2) are the vertices of the given triangle.

$x_1 = -5$, $x_2 = 3$, $x_3 = 5$; $y_1 = -1$, $y_2 = -5$, $y_3 = 2$.

Area of the ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-5 \times (-5-2) + 3 \times (2+1) + 5 \times (-1+5)]$

$= \frac{1}{2}[35 + 9 + 20] = \frac{1}{2}[64] = 32$ sq. units

Q2. In each of the following find the value of 'k', for which the points are collinear.

(i) (7, – 2), (5, 1), (3, k) (ii) (8,1), (k – 4), (2,–5).

Sol. The given three points will be collinear if the Δ formed by them has equal to zero area.

(i) Let A(7, –2), B(5, 1) and C(3, k) be the vertices of a triangle.

$\therefore$ The given points will be collinear, if

ar (ΔABC) = 0

or $7(1-k) + 5(k+2) + 3(-2-1) = 0$

$\Rightarrow 7 - 7k + 5k + 10 + (-6) - 3 = 0$

$\Rightarrow 17 - 9 + 5k - 7k = 0$

$\Rightarrow 8 - 2k = 0 \Rightarrow 2k = 8$

$\Rightarrow k = \frac{8}{2} = 4$

The required value of k = 4.

(ii) A(8, 1), B(k, –4), C(2, –5) are the given points.

$x_1 = 8, x_2 = k, x_3 = 2$

$y_1 = 1, y_2 = -4, y_3 = -5$

the condition for the three points to be collinear is

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$8 \times (-4 + 5) + k \times (-5 - 1) + 2 \times (1 + 4) = 0$

i.e. $8 - 6k + 10 = 0$, i.e., $6k = 18$, i.e., $k = 3$

Q3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, – 1), (2, 1) and (0, 3). Find the ratio of this area of the area of the given triangle.

Sol. Let the vertices of the triangle be A(0, –1), B(2, 1) and C(0, 3).

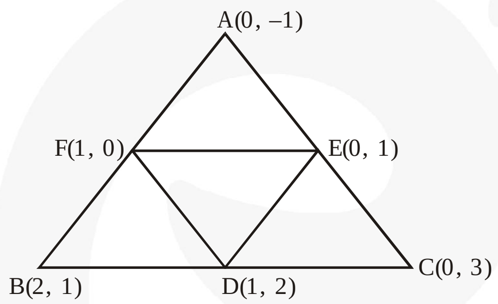
Let D, E and F be the mid-points of the sides BC, CA and AB respectively. Then :

Coordinates of D are

$\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ i.e., $\left(\frac{2}{2}, \frac{4}{2}\right)$ or (1, 2)

Coordinates of E are $\left(\frac{0+0}{2}, \frac{3+(-1)}{2}\right)$ i.e., (0, 1)

Coordinates of F are $\left(\frac{2+0}{2}, \frac{1+(-1)}{2}\right)$ i.e., (1, 0)



Now, ar(ΔABC)

$= \frac{1}{2}[0(1-3) + 2\{3-(-1)\} + 0(-1-1)]$

$= \frac{1}{2}[0(-2) + 8 + 0(-2)]$

$= \frac{1}{2}[0 + 8 + 0] = \frac{1}{2} \times 8 = 4$ sq. units

Now, ar (ΔDEF) $= \frac{1}{2}[1(1-0) + 0(0-2) + 1(2-1)]$

$= \frac{1}{2}[1(1) + 0 + 1(1)]$

$= \frac{1}{2}[1 + 0 + 1] = \frac{1}{2} \times 2 = 1$ sq. unit

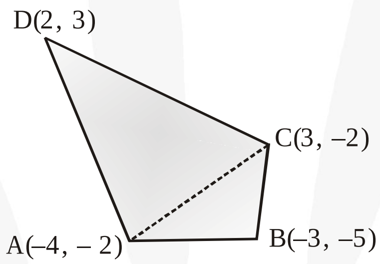
$\therefore \quad \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$

$\therefore \quad ar(\Delta DEF) : ar(\Delta ABC) = 1 : 4.$

Q4. Find the area of the quadrilateral whose vertices taken in order are (–4, –2), (–3, –5), (3, –2) and (2, 3).

Sol. Join A and C. The given points are

A(–4, –2), B(–3, –5), C(3, –2) and D(2, 3)



Area of ΔABC

$= \frac{1}{2}[(-4)(-5 + 2) - 3(-2 + 2) + 3(-2 + 5)]$

$= \frac{1}{2}[12 + 0 + 9] = \frac{21}{2} = 10.5$ sq. units

Area of ΔACD

$= \frac{1}{2}[(-4)(-2 - 3) + 3(3 + 2) + 2(-2 + 2)]$

$= \frac{1}{2}[20 + 15| = \frac{35}{2} = 17.5$ sq. units.

Area of quadrilateral ABCD

= ar. (ΔABC) + ar. (ΔACD)

= (10.5 + 17.5) sq. units = 28 sq. units

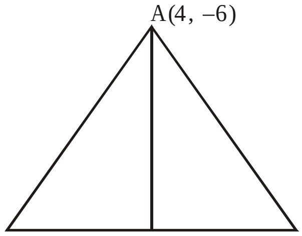
Q5. A median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A(4, – 6), B(3,–2) and C(5,2)

Sol. Here, the vertices of the triangles are A(4, –6), B(3, –2) and C(5, 2).
Let D be the midpoint of BC.

$\therefore$ The coordinates of the mid point D are

$\left\{\frac{3+5}{2}, \frac{-2+2}{2}\right\}$ or (4, 0).

Since, AD divides the triangle ABC into two parts i.e., ΔABD and ΔACD,



Now, ar(ΔABD)

$= \frac{1}{2}[4\{(-2) - 0\} + 3(0 + 6) + 4(-6 + 2)]$

$= \frac{1}{2}[(-8) + 18 + (-16)] = \frac{1}{2}(-6) = -3$ sq. units

= 3 sq. units(numerically) ....(1)

ar(ΔADC) $= \frac{1}{2}[4(0 - 2) + 4(2 + 6) + 5(-6 - 0)]$

$= \frac{1}{2}[-8 + 32 - 30] = \frac{1}{2}[-6] = -3$ sq.units

= 3 sq.units (numerically) ....(2)

From (1) and (2)

ar(ΔABD) = ar(ΔADC)

i.e., A median divides the triangle into two triangles of equal areas.