Ex - 8.3

Q1. Evaluate :

(i) $\frac{\sin 18°}{\cos 72°}$

(ii) $\frac{\tan 26°}{\cot 64°}$

(iii) cos 48° – sin 42°

(iv) cosec 31° – sec 59°

Sol. (i) $\frac{\sin 18°}{\cos 72°} = \frac{\sin 18°}{\cos(90° - 18°)} = \frac{\sin 18°}{\sin 18°} = 1$.

{$\because$ cos (90° – θ) = sin θ}

(ii) $\frac{\tan 26°}{\cot 64°} = \frac{\tan(90° - 64°)}{\cot 64°} = \frac{\cot 64°}{\cot 64°} = 1$

(iii) cos 48° – sin 42° = cos (90° – 42°) – sin 42°

= sin 42° – sin 42° = 0

(iv) cosec 31° – sec 59°

= cosec (90° – 59°) – sec 59°

= sec 59° – sec 59° = 0

Q2. Show that

(i) tan 48° tan 23° tan 42° tan 67° = 1

(ii) cos 38° cos 52° – sin 38° sin 52° = 0.

Sol. (i) LHS = tan 48° tan 23° tan 42° tan 67°

= tan 48° × tan 23° × tan (90° – 48°) × tan (90° – 23°)

= tan 48° × tan 23° × cot 48° × cot 23°

= tan 48° × tan 23° × $\frac{1}{\tan 48°} \times \frac{1}{\tan 23°} = 1$

$\therefore$ LHS = RHS.

(ii) LHS = cos 38° cos52° – sin38° sin52°

= cos(90° – 52°) cos 52° – sin (90° – 52°) sin52°

= sin52° cos52° – cos52° sin52° = 0

Q3. If tan 2 A = cot (A – 18°), where 2 A is an acute angle, find the value of A.

Sol. tan2A = cot(A – 18°)

$\Rightarrow$ cot (90° – 2A) = cot (A – 18°)

$\Rightarrow$ 90° – 2A = A – 18°

$\Rightarrow$ 108° = 3A

A = 36°

Q4. If tan A = cot B, prove that A + B = 90°.

Sol. tan A = cot B

tanA = tan (90°–B)

$\therefore$ A=90°–B

A + B = 90°

Q5. If sec 4 A = cosec (A – 20°), where 4 A is an acute angle, find the value of A.

Sol. sec 4 A = cosec (A – 20°)

$\Rightarrow$ cosec (90° – 4A) = cosec (A – 20°)

{$\because$ cosec (90° – θ) = sec θ}

$\Rightarrow$ 90° – 4A = A – 20°

$\Rightarrow$ 5A = 110° $\Rightarrow$ A = 22°

Q6. If A, B and C are interior angles of a triangle ABC, then show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$.

Sol. A + B + C = 180°

$\Rightarrow$ B + C = 180° – A

$$\Rightarrow \frac{B+C}{2} = \frac{180°-A}{2} \Rightarrow \frac{B+C}{2} = \left(90° - \frac{A}{2}\right)$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right) = \cos\frac{A}{2}$$

{$\because$ sin (90° – θ) = cos θ}

Q7. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Sol. sin 67° + cos 75°

= sin(90° – 23°) + cos (90° – 15°)

= cos 23° + sin 15°