Ex - 8.1

Q1. In ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine : (i) sin A, cos A (ii) sin C, cos C.

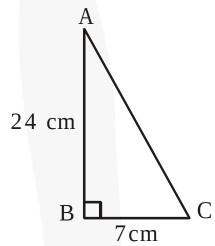
Sol. By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2 = (24)^2 + (7)^2 = 625$

$\Rightarrow$ $AC = \sqrt{625} = 25$ cm.

(i) $\sin A = \frac{BC}{AC}$ $\left\{\text{i.e., } \frac{\text{side opposite to angle A}}{\text{Hyp.}}\right\}$

$= \frac{7}{25}$ ($\because$ BC = 7 cm and AC = 25 cm)



$\cos A = \frac{AB}{AC}$ $\left\{\text{i.e., } \frac{\text{side adjacent to angle A}}{\text{Hyp.}}\right\}$

$= \frac{24}{25}$ ($\because$ AB = 24 cm and AC = 25 cm)

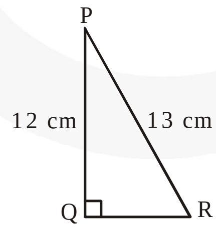
(ii) $\sin C = \frac{AB}{AC}$ $\left\{\text{i.e., } \frac{\text{side opposite to angle C}}{\text{Hyp.}}\right\}$

$= \frac{24}{25}$

$\cos C = \frac{BC}{AC}$ $\left\{\text{i.e., } \frac{\text{side adjacent to angle C}}{\text{Hyp.}}\right\}$

$= \frac{7}{25}$

Q2. In fig, find tan P – cot R.



Sol. In figure, by the Pythagoras Theorem,

$QR^2 = PR^2 - PQ^2 = (13)^2 - (12)^2 = 25$

$\Rightarrow$ $QR = \sqrt{25} = 5$ cm

In ΔPQR right angled at Q, QR = 5 cm is side opposite to the angle P and PQ = 12 cm is side adjacent to the angle P.

Therefore, $\tan P = \frac{QR}{PQ} = \frac{5}{12}$.

Now, QR = 5 cm is side adjacent to the angle R and PQ = 12 cm is side opposite to the angle R.

Therefore, $\cot R = \frac{QR}{PQ} = \frac{5}{12}$

Hence, $\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

Q3. If $\sin A = \frac{3}{4}$, calculate cos A and tan A.

Sol. In figure,

$\sin A = \frac{3}{4}$

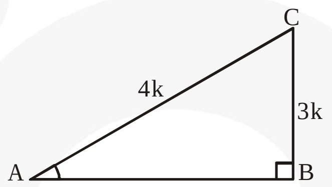
$\Rightarrow \frac{BC}{AC} = \frac{3}{4}$

$\Rightarrow BC = 3k$

and $AC = 4k$

where k is the constant of proportionality.

By Pythagoras Theorem,



$AB^2 = AC^2 - BC^2 = (4k)^2 - (3k)^2 = 7k^2$

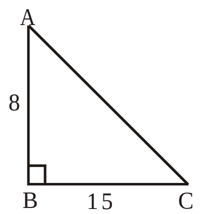
$\Rightarrow AB = \sqrt{7}\,k$

So, $\cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

and $\tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$

Q4. Given 15 cot A = 8, find sin A and sec A.

Sol. $\cot A = \frac{8}{15}$



$\Rightarrow \frac{AB}{BC} = \frac{8}{15}$

$\Rightarrow AB = 8k$

and $BC = 15\,k$

Now, $AC = \sqrt{(8\,k)^2 + (15\,k)^2} = 17\,k$

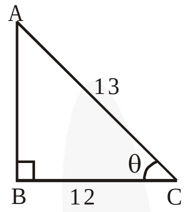
$\sin A = \frac{BC}{AC} = \frac{15\,k}{17\,k} = \frac{15}{17}$, $\quad \sec A = \frac{AC}{AB} = \frac{17\,k}{8\,k} = \frac{17}{8}$

Q5. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Sol. $\sec\theta = \frac{13}{12}$

$\Rightarrow \frac{AC}{BC} = \frac{13}{12}$

By Pythagoras Theorem,



$AC^2 = AB^2 + BC^2$

$(13k)^2 = AB^2 + (12k)^2$

$AB^2 = 169k^2 - 144\ k^2$

$AB = \sqrt{25k^2} = 5\ k$

$\sin\theta = \frac{AB}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{BC}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$\tan\theta = \frac{AB}{BC} = \frac{5k}{12k} = \frac{5}{12}$

$\cot\theta = \frac{BC}{AB} = \frac{12k}{5k} = \frac{12}{5}$

$\operatorname{cosec}\theta = \frac{AC}{AB} = \frac{13k}{5k} = \frac{13}{5}$

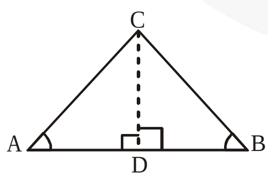
Q6. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Sol. In figure $\angle A$ and $\angle B$ are acute angles of ΔABC.

Draw $CD \perp AB$.

We are given that $\cos A = \cos B$

$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$



$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}\ \left(\text{Each} = \frac{CD}{CD}\right)$

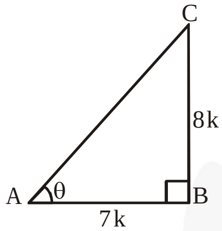
$\Rightarrow \Delta ADC \sim \Delta BDC$ (SSS similarity criterion) $\Rightarrow \angle A = \angle B$

($\because$ all the corresponding angles of two similar triangles are equal)

Q7. If $\cot\theta = \frac{7}{8}$, evaluate :

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

Sol. In figure,



$\cot\theta = \frac{7}{8}$

$\Rightarrow \frac{AB}{BC} = \frac{7}{8}$

$\Rightarrow AB = 7\,k$

and $BC = 8\,k$

Now, $AC^2 = AB^2 + BC^2 = (7k)^2 + (8k)^2$

$= 113\,k^2$

$\Rightarrow AC = \sqrt{113}\,k$

Then $\sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}\,k} = \frac{8}{\sqrt{113}}$

and $\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}\,k} = \frac{7}{\sqrt{113}}$.

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)}$

$\frac{(\sqrt{113}+8)(\sqrt{113}-8)}{(\sqrt{113}+7)(\sqrt{113}-7)} = \frac{(\sqrt{113})^2-(8)^2}{(\sqrt{113})^2-(7)^2}$

$\{\because (a+b)(a-b) = a^2 - b^2\}$

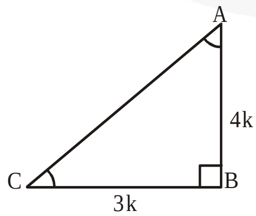
$= \frac{113-64}{113-49} = \frac{49}{64}$

(ii) $\cot\theta = \frac{7}{8}$ $\Rightarrow \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

Q8. If $3\cot A = 4$, check whether

$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Sol. In figure,



$3\cot A = 4$

$\Rightarrow \cot A = \frac{4}{3}$

$\Rightarrow \frac{AB}{BC} = \frac{4}{3}$

$\Rightarrow AB = 4k$ and $BC = 3k$

Now, $AC = \sqrt{(4k)^2+(3k)^2} = 5k$

Then $\sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$,

$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

and $\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

$$\text{LHS} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

$$\text{RHS} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Therefore, LHS = RHS,

i.e., $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

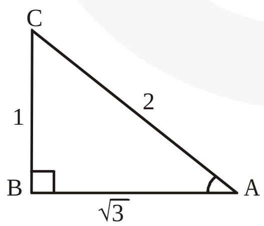
$\left(\because \text{Each side} = \frac{7}{25}\right)$

Q9. In triangle ABC right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of :

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$.

Sol. $\tan A = \frac{1}{\sqrt{3}}$



$\frac{BC}{BA} = \frac{1}{\sqrt{3}}$

$BC = k$ and $BA = \sqrt{3}k$

$AC^2 = BC^2 + BA^2$

$= k^2 + (\sqrt{3}k)^2 = k^2 + 3k^2 = 4k^2$

$AC = \sqrt{4k^2} = 2k$

(i) sin A.cos C + cos A sin C

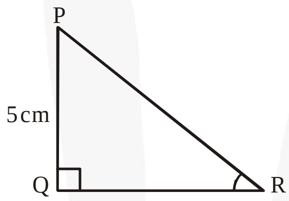
$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$$

(ii) cos A . cos C – sin A . sin C

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Q10. In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Sol. In figure,



PQ = 5 cm

PR + QR = 25 cm

i.e., PR = 25 cm – QR

Now, $PR^2 = PQ^2 + QR^2$

$\Rightarrow (25 - QR)^2 = (5)^2 + QR^2$

$\Rightarrow 625 - 50 \times QR + QR^2 = 25 + QR^2$

$\Rightarrow 50 \times QR = 600 \Rightarrow QR = 12$ cm

and PR = 25 cm – 12cm = 13 cm

We find $\sin P = \frac{QR}{PR} = \frac{12}{13}$, $\cos P = \frac{PQ}{PR} = \frac{5}{13}$

and $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

Q11. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = $\frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) sin θ = $\frac{4}{3}$ for some angle θ.

Sol. (i) False.

We know that 60° = $\sqrt{3} > 1$.

(ii) True.

We know that value of sec A is always ≥ 1.

(iii) False.

Because cos A is abbreviation used for cosine A.

(iv) False, because cot A is not the product of cot and A.

(v) False, because value of sin cannot be more than 1.