

Ex - 8.4

Q1. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.

Sol. We have $\operatorname{cosec}^2 A - \cot^2 A = 1$

$$\Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow (\operatorname{cosec} A)^2 = \cot^2 A + 1$$

$$\Rightarrow \left(\frac{1}{\sin A} \right)^2 = \cot^2 A + 1$$

$$\Rightarrow (\sin A)^2 = \frac{1}{\cot^2 A + 1}$$

$$\Rightarrow \sin A = \pm \frac{1}{\sqrt{\cot^2 A + 1}}$$

We reject negative value of $\sin A$ for acute angle

$$\text{A. Therefore, } \sin A = \frac{1}{\sqrt{\cot^2 A + 1}} \quad \tan A = \frac{1}{\cot A}$$

We have $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$= 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Q2. Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

Sol. (i) $\sin A = \sqrt{1 - \cos^2 A}$

$$= \sqrt{1 - \frac{1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\text{(ii) } \cos A = \frac{1}{\sec A}$$

$$\text{(iii) } \tan A = \sqrt{\sec^2 A - 1}$$

$$\text{(iv) } \cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\text{(v) } \operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Q3. Evaluate

- (i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$
 (ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Sol. (i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$$= \frac{\{\sin(90^\circ - 27^\circ)\}^2 + \sin^2 27^\circ}{\cos^2 17^\circ + \{\cos(90^\circ - 17^\circ)\}^2}$$

$$= \frac{\{\cos 27^\circ\}^2 + \sin^2 27^\circ}{\cos^2 17^\circ + \{\sin 17^\circ\}^2}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \sin^2 17^\circ} = \frac{1}{1} = 1$$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$$= \sin(90^\circ - 65^\circ) \cos 65^\circ + \cos(90^\circ - 65^\circ) \sin 65^\circ$$

$$= \cos 65^\circ \cos 65^\circ + \sin 65^\circ \sin 65^\circ$$

$$= \cos^2 65^\circ + \sin^2 65^\circ = 1$$

Q4. Choose the correct option. Justify your choice :

- (i) $9 \sec^2 A - 9 \tan^2 A =$
 (A) 1 (B) 9 (C) 8 (D) 0
- (ii) $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta) =$
 (A) 0 (B) 1 (C) 2 (D) - 1
- (iii) $(\sec A + \tan A) (1 - \sin A) =$
 (A) $\sec A$ (B) $\sin A$ (C) $\operatorname{cosec} A$ (D) $\cos A$
- (iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A} =$
 (A) $\sec^2 A$ (B) - 1 (C) $\cot^2 A$ (D) $\tan^2 A$

Sol. (i) Correct option is (B).

$$9 \sec^2 A - 9 \tan^2 A = 9 (\sec^2 A - \tan^2 A)$$

$$= 9 \times 1 = 9.$$

(ii) Correct option is (C).

$$(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$$

$$= \left\{ 1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \right\} \times \left\{ 1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} \right\}$$

$$\begin{aligned}
 &= \left\{ \frac{\cos\theta + \sin\theta + 1}{\cos\theta} \right\} \times \left\{ \frac{\sin\theta + \cos\theta - 1}{\sin\theta} \right\} \\
 &= \frac{\{(\cos\theta + \sin\theta) + 1\} \times \{(\cos\theta + \sin\theta) - 1\}}{\cos\theta \times \sin\theta} \\
 &= \frac{(\cos\theta + \sin\theta)^2 - (1)^2}{\cos\theta \times \sin\theta} \\
 &\{ \because (a + b)(a - b) = a^2 - b^2 \} \\
 &= \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta \times \sin\theta} \\
 &= \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta\sin\theta} = 2.
 \end{aligned}$$

(iii) Correct option is (D).

$$\begin{aligned}
 &(\sec A + \tan A)(1 - \sin A) \\
 &= \sec A - \tan A + \tan A - \frac{\sin^2 A}{\cos A} \\
 &= \frac{1}{\cos A} - \frac{\sin^2 A}{\cos A} = \frac{1 - \sin^2 A}{\cos A} \\
 &= \frac{\cos^2 A}{\cos A} = \cos A
 \end{aligned}$$

(iv) Correct option is (D).

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \tan^2 A$$

Q5. Prove the following identities, where the angles involved are acute angles for which the following expressions are defined.

(i) $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$.

(ii) $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$.

(iii) $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$.

(iv) $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$.

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$, using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$.

(vii) $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$.

$$\begin{aligned} \text{(viii)} \quad & (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\ & = 7 + \tan^2 A + \cot^2 A. \end{aligned}$$

$$\begin{aligned} \text{(ix)} \quad & (\operatorname{cosec} A - \sin A) (\sec A - \cos A) \\ & = \frac{1}{\tan A + \cot A}. \end{aligned}$$

$$\text{(x)} \quad \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A.$$

Sol. (i) LHS = $(\operatorname{cosec} \theta - \cot \theta)^2$

$$= \left\{ \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right\}^2 = \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2$$

$$= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} = \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}$$

$$= \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta}$$

\therefore LHS = RHS.

(ii) LHS = $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{\cos A(1 + \sin A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{\cos A(1 + \sin A)}$$

$$= \frac{2(1 + \sin A)}{\cos A(1 + \sin A)}$$

$$= 2 \sec A = \text{R.H.S.}$$

(iii) LHS = $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta}$

$$= \frac{\left(\frac{\sin \theta}{\cos \theta} \right)}{\left(1 - \frac{\cos \theta}{\sin \theta} \right)} + \frac{\left(\frac{\cos \theta}{\sin \theta} \right)}{\left(1 - \frac{\sin \theta}{\cos \theta} \right)}$$

$$= \frac{\left(\frac{\sin \theta}{\cos \theta} \right)}{\left(\frac{\sin \theta - \cos \theta}{\sin \theta} \right)} + \frac{\left(\frac{\cos \theta}{\sin \theta} \right)}{\left(\frac{\cos \theta - \sin \theta}{\cos \theta} \right)}$$

$$= \frac{\sin \theta \times \sin \theta}{\cos \theta \times (\sin \theta - \cos \theta)} + \frac{\cos \theta \times \cos \theta}{\sin \theta \times (\cos \theta - \sin \theta)}$$

$$= \frac{\sin^2 \theta}{\cos \theta \times (\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta \times (\sin \theta - \cos \theta)}$$

$$= \frac{\sin \theta \times \sin^2 \theta - \cos \theta \times \cos^2 \theta}{\cos \theta \times \sin \theta \times (\sin \theta - \cos \theta)}$$

$$= \frac{\sin^3 \theta - \cos^3 \theta}{\cos \theta \times \sin \theta \times (\sin \theta - \cos \theta)}$$

$$= \frac{(\sin \theta - \cos \theta) \times (\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\cos \theta \times \sin \theta \times (\sin \theta - \cos \theta)}$$

$$\{ \because a^3 - b^3 = (a - b)(a^2 + b^2 + ab) \}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta}{\cos \theta \times \sin \theta}$$

$$= \frac{1 + \sin \theta \cos \theta}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta} + 1$$

$$= 1 + \left(\frac{1}{\cos \theta} \right) \left(\frac{1}{\sin \theta} \right)$$

$$= 1 + \sec \theta \operatorname{cosec} \theta$$

\therefore LHS = RHS

$$(iv) \text{ L.H.S.} = \frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\cos A + 1}{1}$$

$$\text{R.H.S.} = \frac{\sin^2 A}{1 - \cos A} = \frac{1 - \cos^2 A}{1 - \cos A} = 1 + \cos A$$

\therefore L.H.S. = R.H.S.

$$(v) \text{ LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

(Dividing the numerator and denominator by $\sin A$)

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} = \frac{(\operatorname{cosec} A + \cot A) - 1}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\{1 + \cot A - \operatorname{cosec} A\}}$$

($\because \operatorname{cosec}^2 A = 1 + \cot^2 A$, i.e., $\operatorname{cosec}^2 A - \cot^2 A = 1$)

$$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec} A + \cot A) \times (\operatorname{cosec} A - \cot A)}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$$\{ \because (a + b)(a - b) = a^2 - b^2 \}$$

$$= \frac{(\operatorname{cosec} A + \cot A) \times \{1 - (\operatorname{cosec} A - \cot A)\}}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$$= \frac{(\operatorname{cosec} A + \cot A) \times \{1 + \cot A - \operatorname{cosec} A\}}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$$= \operatorname{cosec} A + \cot A$$

$$= \text{RHS}$$

$$\text{(vi) LHS} = \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$$

$$= \sqrt{\frac{(1 + \sin A)^2}{(1)^2 - (\sin A)^2}} = \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}}$$

$$= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} = \frac{1 + \sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

$$\therefore \text{LHS} = \text{RHS.}$$

$$\text{(vii) L.H.S.} = \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$$

$$= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{\sin \theta (\sin^2 \theta + \cos^2 \theta - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - \sin^2 \theta - \cos^2 \theta)}$$

$$= \frac{\tan \theta (\cos^2 \theta - \sin^2 \theta)}{(\cos^2 \theta - \sin^2 \theta)}$$

$$= \tan \theta = \text{R.H.S.}$$

$$\text{(viii) L.H.S.} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2 + \cos^2 A + \sec^2 A + 2$$

$$= 4 + 1 + 1 + \cot^2 A + 1 + \tan^2 A$$

$$= 7 + \tan^2 A + \cot^2 A = \text{R.H.S.}$$

$$(ix) \text{ LHS} = (\operatorname{cosec} A - \sin A) (\sec A - \cos A)$$

$$= \left(\frac{1}{\sin A} - \sin A \right) \times \left(\frac{1}{\cos A} - \cos A \right)$$

$$= \frac{1 - \sin^2 A}{\sin A} \times \frac{1 - \cos^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A \cos A$$

$$\text{Now, RHS} = \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}} = \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \frac{\sin A \cos A}{1}$$

$$\therefore \text{ LHS} = \text{RHS.}$$

$$(x) \text{ L.H.S.} = \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \tan^2 A = \text{R.H.S.}$$

$$\& \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}} \right)^2$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A = \text{R.H.S.}$$