Ex - 4.4

Q1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them :

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Sol. (i) $2x^2 - 3x + 5 = 0$

$a = 2, b = -3, c = 5$

Discriminant $D = b^2 - 4ac = 9 - 4 \times 2 \times 5$

$= 9 - 40 = -31$

$\Rightarrow D < 0$

Hence, no real root.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

$a = 3, b = -4\sqrt{3}, c = 4$

Discriminant $D = b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$

$\Rightarrow D = 0$

$\Rightarrow$ Two roots are equal.

The roots are

$$= \frac{-b \pm \sqrt{D}}{2a} = \frac{4\sqrt{3} \pm 0}{2 \times 3} = \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$$

Hence, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) $2x^2 - 6x + 3 = 0$

$a = 2, b = -6, c = 3$

Discriminant $D = b^2 - 4ac = (-6)^2 - 4(2)(3)$

$= 36 - 24 = 12$

As $D > 0$,

Therefore, roots are distinct and real.

The roots are

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the roots are $\frac{3 + \sqrt{3}}{2}$ or $\frac{3 - \sqrt{3}}{2}$.

Q2. Find the values of k for each of the following quadratic equations, so that they have two real equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

Sol. (i) $2x^2 + kx + 3 = 0$

$a = 2, b = k, c = 3$

$D = b^2 - 4ac = k^2 - 4 \times 2 \times 3 = k^2 - 24$

Two roots will be equal

if $D = 0$, i.e., if $k^2 - 24 = 0$

i.e., if $k^2 - 24$, i.e., if $k = \pm \sqrt{24}$

i.e., if $k = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

$a = k, b = -2k, c = 6$

Discriminant $D = b^2 - 4ac = (-2k)^2 - 4(k)(6)$

$= 4k^2 - 24k$

Two roots will be equal

if $D = 0$,

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

Either $4k = 0$ or $k - 6 = 0$

$k = 0$ or $k = 6$

However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.

Hence $k = 6$.

Q3. Is it possible to design a rectangular mango grove whose length is twice its breadth,

and area is 800 m^2? If so, find its length and breadth.

Sol. Let x be the breadth and 2x be the length of the rectangle.

$x \times 2x = 800$

$\Rightarrow 2x^2 = 800$

$\Rightarrow x^2 = 400 = (20)^2$

$\Rightarrow x = 20$

Hence, the rectangle is possible and it has breadth = 20 m and length = 40 m.

Q4. Is the following situation possible? If so, determine their present ages.
The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in year was 48.

Sol. Let the age of one friend be x years.
Age of the other friend will be (20 – x) years.
4 years ago, age of 1st friend = (x – 4) years
And, age of 2nd friend = (20 – x – 4)
= (16 – x) years
Given that,
$(x - 4)(16 - x) = 48$
$16x - 64 - x^2 + 4x = 48$
$-x^2 + 20x - 112 = 0$
$x^2 - 20x + 112 = 0$
a = 1, b = – 20, c = 112
Discriminant $D = b^2 - 4ac = (-20)^2 - 4(1)(112)$
$= 400 - 448 = -48$
As $b^2 - 4ac < 0$,
Therefore, no real root is possible for this equation and hence, this situation is not possible.

Q5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

Sol. Perimeter of the rectangular park = 80 m
$\Rightarrow$ Length + Breath of the park = $\frac{80}{2}$ m = 40 m.
Let the breadth be x metres, then length
= (40 – x) m.
Here, x < 40.
$x \times (40 - x) = 400$ [Each = area of the park]
i.e., $-x^2 + 40x - 400 = 0$
i.e., $x^2 - 40x + 400 = 0$
i.e., $(x - 20)^2 = 0$
$\Rightarrow$ x = 20
Thus, we have length = breadth = 20 m
Therefore, the park is a square having 20 m side.