Ex - 6.4

Q1. Let $\Delta ABC \sim \Delta DEF$ and their areas be 64 cm^2 and 121 cm^2 respectively. If EF = 15.4 cm, find BC.

Sol. $\Delta ABC \sim \Delta DEF$ (Given)

$\Rightarrow \dfrac{ar(ABC)}{ar(DEF)} = \dfrac{BC^2}{EF^2}$ (By theorem 6.7)

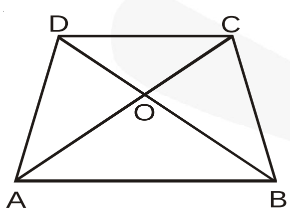
$\Rightarrow \dfrac{64}{121} = \dfrac{BC^2}{EF^2}$ $\Rightarrow \left\{\dfrac{BC}{EF}\right\}^2 = \left\{\dfrac{8}{11}\right\}^2$

$\Rightarrow \dfrac{BC}{EF} = \dfrac{8}{11}$ $\Rightarrow BC = \dfrac{8}{11} \times EF$

$\Rightarrow BC = \dfrac{8}{11} \times 15.4 \text{ cm} = 11.2 \text{ cm}$

Q2. Diagonals of trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

Sol.



In ΔAOB and ΔCOD,

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\angle OBA = \angle ODC$ (Alternate interior angles)

$\therefore$ By AA, similarity

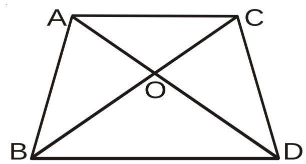
$\Delta AOB \sim \Delta COD$

So, $\dfrac{ar.\Delta AOB}{ar.\Delta COD} = \left(\dfrac{AB}{CD}\right)^2$

$= \left(\dfrac{2}{1}\right)^2$ $\{\because AB = 2CD\}$

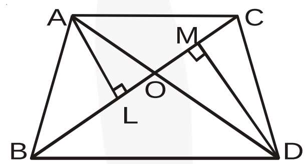
$= 4 : 1$

Q3. In figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$.



Sol. Draw AL ⊥ BC and DM ⊥ BC (see figure)

$\Delta OLA \sim \Delta OMD$ (AA similarity criterion)



$\Rightarrow \frac{AL}{DM} = \frac{AO}{DO}$ ...(1)

Now, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}\times(BC)\times(AL)}{\frac{1}{2}\times(BC)\times(DM)}$

$= \frac{AL}{DM} = \frac{AO}{DO}$ (By 1)

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Q4. If the areas of two similar triangles are equal, prove that they are congruent.

Sol. Let $\Delta ABC \sim \Delta PQR$ and

area (ΔABC) = area (ΔPQR) (Given)

i.e., $\frac{area(\Delta ABC)}{area(\Delta PQR)} = 1$

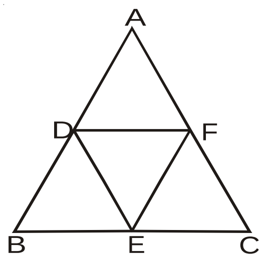
$\Rightarrow \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{PR^2} = 1$

$\Rightarrow$ AB = PQ, BC = QR and CA = PR

$\Rightarrow \Delta ABC \cong \Delta PQR$

Q5. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

Sol.



$DF = \frac{1}{2}BC,\ DE = \frac{1}{2}AC,\ EF = \frac{1}{2}AB$

[By midpoint theorem]

So, $\frac{DF}{BC} = \frac{DE}{AC} = \frac{EF}{AB} = \frac{1}{2}$

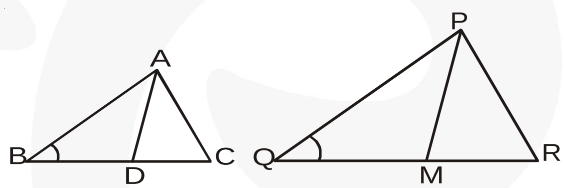
∴ ΔDEF ~ ΔCAB

So, $\frac{ar\,\Delta DEF}{ar\,\Delta ABC} = \left(\frac{DE}{AC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ or 1 : 4

Q6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Sol. In figure, AD is a median of ΔABC and PM is a median of ΔPQR. Here, D is mid-point of BC and M is mid-point of QR.

Now, we have ΔABC ~ ΔPQR.



⇒ ∠B = ∠Q ...(1)

(Corresponding angles are equal)

Also $\frac{AB}{PQ} = \frac{BC}{QR}$

(Ratio of corresponding sides are equal)

⇒ $\frac{AB}{PQ} = \frac{2BD}{2QM}$

(∵ D is mid-point of BC and M is mid-point of QR)

⇒ $\frac{AB}{PQ} = \frac{BD}{QM}$ ...(2)

In ΔABD and ΔPQM

∠ABD = ∠PQM (By 1)

and $\frac{AB}{PQ} = \frac{BD}{QM}$ (By 2)

$\Rightarrow$ $\Delta ABD \sim \Delta PQM$ (SAS similarity)

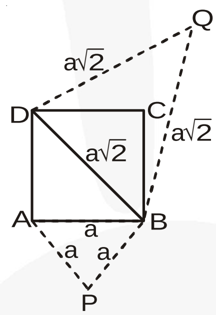
$\Rightarrow$ $\frac{AB}{PQ} = \frac{AD}{PM}$ ...(3)

Now, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$ (By theorem 6.7)

$\Rightarrow$ $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$ $\left(\because \frac{AB}{PQ} = \frac{AD}{PM}\right)$

Q7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Sol. ABCD is a square having sides of length = a.



Then the diagonal BD = $a\sqrt{2}$.

We construct equilateral Δs PAB and QBD

$\Rightarrow$ $\Delta PAB \sim \Delta QBD$ (Equilateral triangles are similar)

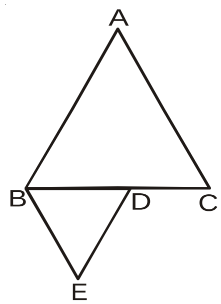
$\Rightarrow$ $\frac{ar(\Delta PAB)}{ar(\Delta QBD)} = \frac{AB^2}{BD^2} = \frac{a^2}{(a\sqrt{2})^2} = \frac{1}{2}$

$\Rightarrow$ ar (ΔPAB) = $\frac{1}{2}$ ar (ΔQBD).

Tick the correct answer and justify

Q8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

(1) 2 : 1 (2) 1 : 2 (3) 4 : 1 (4) 1 : 4

Sol. 

($\because$ BC = 2BD)

Since, both are equilateral triangles.

$\Delta ABC \sim \Delta EBD$

$$\frac{ar\,\Delta ABC}{ar\,\Delta BDE} = \left(\frac{BC}{BD}\right)^2 = \left(\frac{2}{1}\right)^2 = 4 : 1$$

Q9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(1) 2 : 3 (2) 4 : 9 (3) 81 : 16 (4) 16 : 81

Sol. $$\frac{\text{area of } 1^{st}\,\Delta}{\text{area of } 2^{nd}\,\Delta} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$